Linking Leptogenesis and Neutrino Oscillation

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Baryon Number Asymmetry in SM

- Within SM:

  CP violation in quark sector not sufficient to explain the observed matter-antimatter asymmetry of the Universe

- CP phase in CKM matrix: Shaposhnikov, 1986; Farrar, Shaposhnikov, 1993

$$B \simeq \frac{\alpha_w^4 T^3}{s}\delta_{CP} \simeq 10^{-8}\delta_{CP} \qquad \delta_{CP} \simeq \frac{A_{CP}}{T_C^{12}} \simeq 10^{-20}$$

  - effects of CP violation suppressed by small quark mixing

$$A_{CP} = (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \cdot J$$

$$\longrightarrow \quad B \sim 10^{-28}$$

too small to account for the observed $B \sim 10^{-10}$

- various Baryogenesis mechanisms
- neutrino oscillation opens up a new possibility:

Leptogenesis

Fukugita, Yanagida, 1986

Compelling Neutrino Oscillation Evidences

Atmospheric Neutrinos:

SuperKamiokande (up-down asymmetry, L/E, θz dependence of μ-like events)

dominant channel: $\nu_\mu \rightarrow \nu_\tau$

next: K2K, MINOS, CNGS (OPERA)

Solar Neutrinos:

Homestake, Kamiomande, SAGE, GALLEX/GNO, SK, SNO, BOREXINO, KamLAND

dominant channel: $\nu_e \rightarrow \nu_{\mu,\tau}$

next: BOREXINO, KamLAND, ...

LSND:

dominant channel: $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$

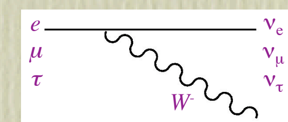
MiniBOONE -- negative result (2007)

Parameters for 3 Light Neutrinos

- three neutrino mixing $$\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j}\nu_{jL} \quad \ell = e,\ \mu,\ \tau$$

- mismatch between weak and mass eigenstates

$$\mathcal{L}_{cc} = (\ \overline{\nu}_1,\quad \overline{\nu}_2,\quad \overline{\nu}_3\ )\gamma^\mu U^\dagger \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} W^+_\mu$$



$$\nu_{1,\,2,\,3} \longrightarrow m_{1,\,2,\,3}$$

- PMNS matrix

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

$$V = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atm}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar}}$$

  - Dirac CP-violating phase: $\delta = [0, 2\pi]$
  - Majorana CP-violating phases: $\alpha_{21},\ \alpha_{31}$

Current Status of Oscillation Parameters

- oscillation probability:

$$P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t \rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

- 3 neutrinos global analysis: Maltoni, Schwetz, Tortola, Valle (updated Sep 2007)

solar+KamLAND+CHOOZ+atmospheric+K2K+Minos

$$\sin^2\theta_{23} = 0.5\ (0.38 - 0.64), \quad \sin^2\theta_{13} = 0\ (< 0.028) \quad \sin^2\theta_{12} = 0.30\ (0.25 - 0.34)$$

$$\Delta m^2_{23} = (2.38^{+0.2}_{-0.16}) \times 10^{-3}\ \mathrm{eV}^2, \quad \Delta m^2_{12} = (8.1 \pm 0.6) \times 10^{-5}\ \mathrm{eV}^2$$

- Tri-bimaximal Neutrino Mixing:

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin\theta_{13,\mathrm{TBM}} = 0.$$

$$\sin^2\theta_{\odot,\mathrm{TBM}} = 1/3 \qquad \tan^2\theta_{\odot,\mathrm{TBM}} \ = \ 1/2$$

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_{\odot,\mathrm{exp}} \ = \ 0.429$$

new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$

Discovery phase into precision phase for some oscillation parameters

Neutrino Mass Spectrum

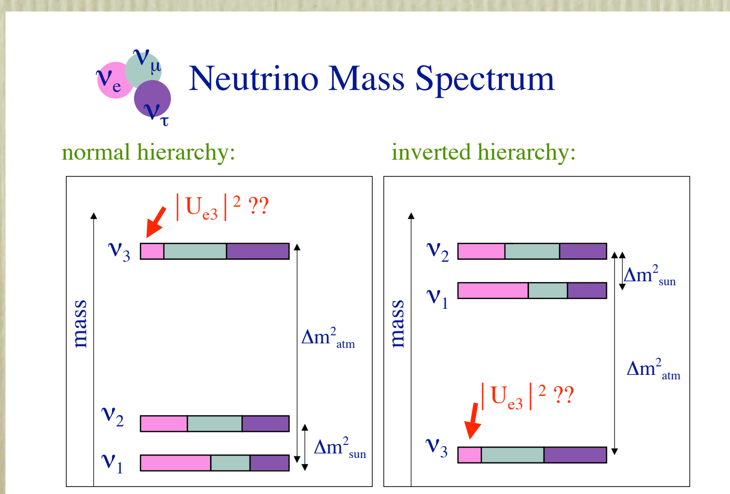
- search for absolute mass scale:
  - end point kinematic of tritium beta decays:

$$\text{Tritium} \rightarrow He^3 + e^- + \bar{\nu}_e$$

Mainz: $m_\nu < 2.2$ eV
KATRIN: increase sensitivity ~ 0.2 eV

  - WMAP + 2dFRGS + Lyα: $\sum(m_{\nu_i}) < (0.7\text{-}1.2)$ eV
  - neutrinoless double beta decay

current bound: $|<m>| < (0.19 - 0.68)$ eV (CUORICINO, Feb 2008)



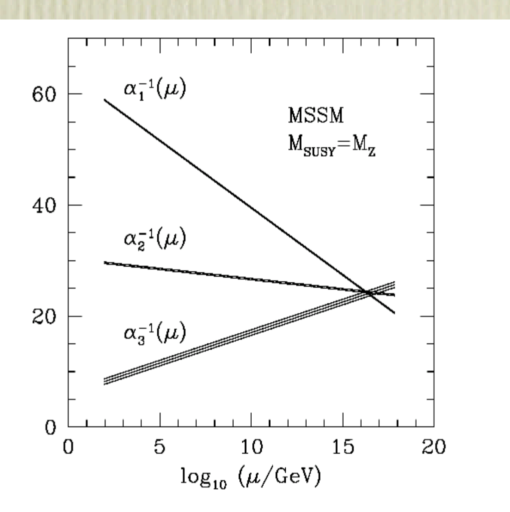
The known unknowns:

- How small is θ_{13}? (ν_e component of ν_3)
- $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$? (ν_3 composition)
- Neutrino mass hierarchy (Δm_{13}^2) ?
- CP violation in neutrino oscillations?

Seesaw Mechanism

- a natural way to generate small neutrino masses
  Minkowski, 1977; Gell-mann, Ramond, Slansky,1981;Yanagida, 1979; Mohapatra, Senjanovic, 1981
- possibility to link origin of BAU to neutrino oscillation through leptogenesis
- Introduce right-handed neutrinos, which are SM gauge singlets
  [predicted in many GUTs, e.g. SO(10)]
- The Lagrangian: $\mathcal{L}_Y = f_{ij}\overline{e}_{R_i}\ell_{L_j}H^\dagger + h_{ij}\overline{\nu}_{R_i}\ell_{L_j}H - \frac{1}{2}(M_R)_{ij}\overline{\nu}^c_{R_i}\nu_{R_j} + h.c.\,.$
- integrating out N_R: effective mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \text{light neutrino mass: } m_\nu \sim \frac{m_D}{M_R} m_D$$



$$m_\nu \sim \sqrt{\Delta m^2_{atm}} \sim 0.05 \text{ eV}, \;\; m_D \sim m_t \sim 172 \text{ GeV}$$

$$\Rightarrow M_R \sim 10^{15}\text{GeV} \quad \sim M_{GUT}$$

seesaw ⇒ Neutrinos are Majorana fermions

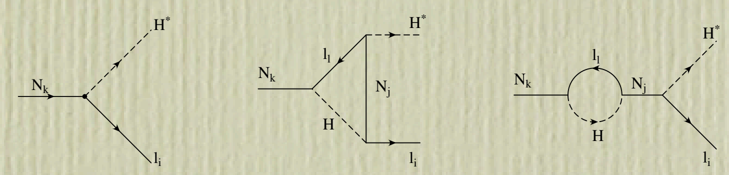
⇒ Lepton Number violation

Leptogenesis

Fukugita, Yanagida, 1986

- implemented in the context of seesaw mechanism
- out-of-equilibrium decays of RH neutrinos produce primordial lepton number asymmetry

Luty, 1992; Covi, Roulet, Vissani, 1996; Flanz et al, 1996; Plumacher, 1997; Pilaftsis, 1997;



$$\epsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}$$

- sphaleron processes: $\Delta L \to \Delta B$

- the asymmetry

Buchmuller, Plumacher, 1998;
Buchmuller, Di Bari, Plumacher, 2004

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} \sim 8.6 \times 10^{-11} \qquad Y_B \simeq 10^{-2} \epsilon \kappa$$

$$\kappa: \text{ efficiency factor} \sim (10^{-1} - 10^{-3})$$

$$\Phi^+ + \ell^- \to N_1, \quad \ell^- + \Phi^+ \to \Phi^- + \ell^+, \text{ etc.}$$

Realizations of Leptogenesis

- Standard Leptogenesis with Type-I seesaw + hierarchycal RH neutrino mass spectrum
  Fukugita, Yanagida, 1986

- Leptogenesis with Type-II seesaw
  Joshipura, Paschos, Rodejohann, 2001; Hambye, Senjanovic, 2004; Antusch, King, 2004, ...

- Resonant leptogenesis: near degenerate RH neutrino mass spectrum
  Pilaftsis, 1997; ...

- soft leptogenesis
  Grossman, Kashti, Nir, Roulet, 2003; D'Ambrosio, Giudice, Raidal, 2003; Boubekeur, 2002; Boubekeur, Hambye, Senjanovic, 2004, ...

$$\epsilon = \left(\frac{4\Gamma_1 B}{\Gamma_1^2 + 4B^2}\right)\frac{\mathrm{Im}(A)}{M_1}\delta_{B-F}$$

A, B: SUSY CP-violating phases
lose connection to neutrino oscillation

- Dirac leptogenesis
  Dick, Lindner, Ratz, Wright, 2000; Murayama, Pierce, 2002; ...

Testing Leptogenesis?

Sakharov conditions:

- out-of-equilibrium (expanding Universe)
- Lepton number violation (neutrinoless double beta decay)
- CP violation

Connection to Low Energy Observables

- Lagrangian at high energy (in the presence of RH neutrinos)

$$\mathcal{L} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + \overline{N}_{R_i} i\gamma^\mu \partial_\mu N_{R_i}$$
$$+ f_{ij} \overline{e}_{R_i} \ell_{L_j} H^\dagger + h_{ij} \overline{N}_{R_i} \ell_{L_j} H - \frac{1}{2} M_{ij} N_{R_i} N_{R_j} + h.c.$$

in f_{ij} and M_{ij} diagonal basis →

h_{ij} general complex matrix: $\left\{ \begin{array}{l} 9\text{-}3 = 6 \text{ mixing angles} \\ 9\text{-}3 = 6 \text{ physical phases} \end{array} \right.$

- Low energy effective Lagrangian (after integrating out RH neutrinos)

$$\mathcal{L}_{eff} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + f_{ii} \overline{e}_{R_i} \ell_{L_i} H^\dagger + \frac{1}{2} \sum_k h^T_{ik} h_{kj} \ell_{L_i} \ell_{L_j} \frac{H^2}{M_k} + h.c.$$

in f_{ij} diagonal basis →

h_{ij} symmetric complex matrix: $\left\{ \begin{array}{l} 6\text{-}3 = 3 \text{ mixing angles} \\ 6\text{-}3 = 3 \text{ physical phases} \end{array} \right.$

- high energy → low energy:
  numbers of mixing angles and CP phases reduced by half

Connection to Low Energy Observables

- diagonal basis for charged lepton and RH neutrino mass matrices
- neutrino Dirac Yukawa interactions: $h = V_R^{\nu\,\dagger} \mathrm{diag}(h_1, h_2, h_3) V_L^\nu$
- CP asymmetry parametrized by (orthogonal parametrization)

(Casas & Ibarra, 2001)

$m = \mathrm{diag}(m_1, m_2, m_3)$ (light neutrino masses)

$M = \mathrm{diag}(M_1, M_2, M_3)$ (RH neutrino masses)

$R = vM^{-1/2} h U m^{-1/2}$ R: phases in RH sector

$$hh^\dagger v^2 = V_R^{\nu\,\dagger} \mathrm{diag}(h_1^2, h_2^2, h_3^2) V_R^\nu v^2 = M^{1/2} R m R^\dagger M^{1/2}$$

$$h = \frac{1}{v} M^{1/2} R m^{1/2} U^\dagger$$

combination relevant for leptogenesis in 1-flavor approximation

R: high energy parameters
U: low energy information

Connection to Low Energy Observables

hierarchical RH neutrinos: $M_1 \ll M_2 \ll M_3$

One Flavor Approximation: T > 10[12] GeV

- individual lepton number asymmetry

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2}, \qquad v = 174 \text{ GeV}$$

where the effective masses $\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau$

- out-of-equilibrium temperature

  Y$_{e,\mu,\tau}$ all small $\quad Y_l \; H^c(x)\overline{l_R}(x)\psi_{lL}$ out of equilibrium at $T \sim M_1 > 10^{12}$ GeV

  ➡ L$_{e,\mu,\tau}$ not distinguishable

- Boltzmann equations for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$
- resulting lepton number asymmetry

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k |R_{1k}|^2} \qquad \widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k |R_{1k}|^2$$

Connection to Low Energy Observables

real R, complex U:
non-vanishing low energy CPV (h)
vanishing leptogenesis

- one-flavor approximation

$\boxed{\text{presence of low energy leptonic CPV (neutrino oscillation, neutrinoless double beta decay)}} \;\cancel{\longleftrightarrow}\; \boxed{\text{leptogenesis} \neq 0}$

- no model independent connection can exist

Flavor matters?

Abada, Davidson, Josse-Michaux, Losada, Riotto, 2006;
Nardi, Nir, Roulet, Racker, 2006

leptogenesis at $T \sim M_1 < 10^{12}$ GeV:

three flavors distinguishable (different $T_{eq} = Y^2 M_{pl}$)

non-universal wash-out factors

Connection to Low Energy Observables

- At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not
- At $M_1 \sim T \sim 10^{9}$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not
- two flavor regime: $M_1 \sim 10^9 - 10^{12}$ GeV

  $\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently

- three flavor regime $M_1 < 10^9$ GeV

  $\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently

  - asymmetry associated with each flavor

Pascoli, Petcov, Riotto, 2006

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}$$

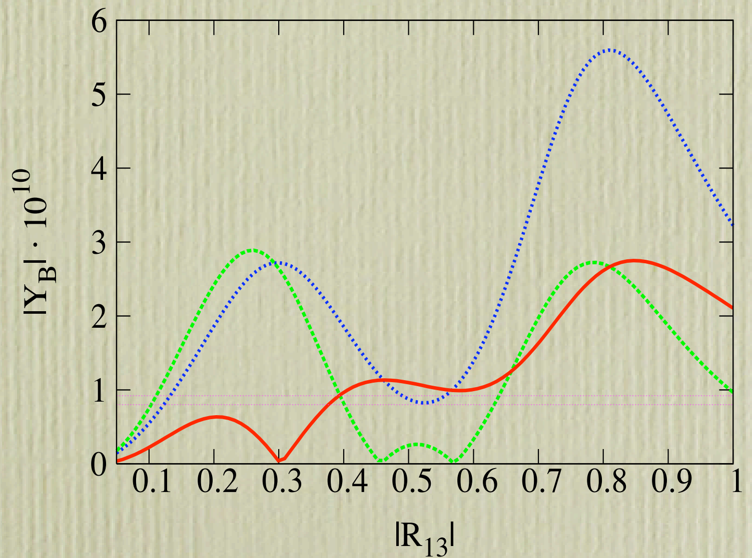
$\boxed{\text{leptogenesis} \neq 0} \Longrightarrow \boxed{\text{low energy CPV} \neq 0}$

Connection to Low Energy CPV

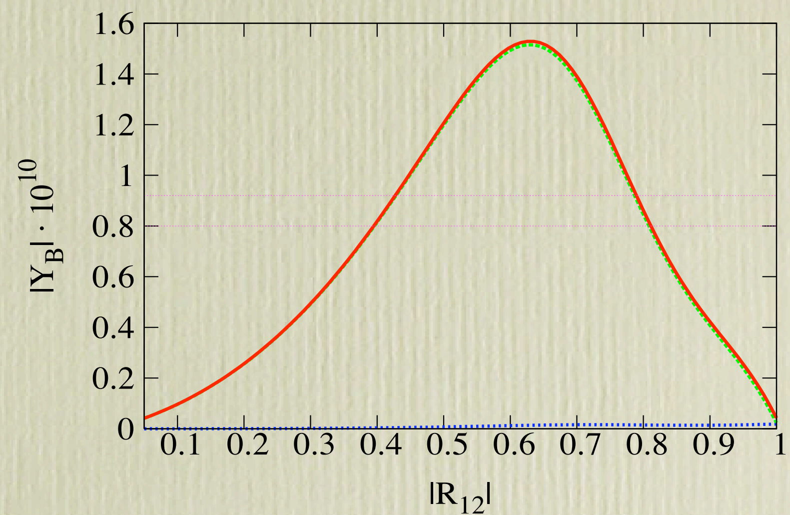
- including flavor effects: for inverted mass spectrum, low energy CP phases can be important under certain conditions

Molinaro, Petcov, 2008

$m_1 < m_2 < m_3$ (NO(NH))



$m_3 \ll m_1 < m_2$ (IH))



$\alpha_{21} = \pi/2$, $|R_{11}| \cong 1.0$

$M_1 = 10^{11}$ GeV $\quad s_{13} = 0.2$ and $\delta = \pi$

R CPV, blue

U CPV, green

total $|Y_B|$ (red line)

CPV: α21 in U and R phases

Connection in Specific Models

- models for neutrino masses:
  additional symmetries or textures
  - ➡ reduce the number of parameters
  - ➡ connection can be established

- texture assumption
  2x3 seesaw model

- all CP violation can come from a single source
  minimal left-right model with spontaneous CP violation

- implications of tri-bimaximal neutrino mixing
  A4 model

Seesaw with 2 RH Neutrinos

- cancellation of Witten anomaly
  - ➡ leptonic SU(2) horizontal symmetry
  - ➡ two RH neutrinos
  - ➡ 2x3 seesaw mechanism

Kuchimanchi & Mohapatra, 2002

- Lagrangian

Frampton, Glashow, Yanagida, 2002

$$\mathcal{L} = \frac{1}{2}(N_1 N_2)\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + (N_1 N_2)\begin{pmatrix} a & a' & 0 \\ 0 & b & b' \end{pmatrix}\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} H + h.c. \,,$$

- effective neutrino mass matrix

$$\begin{pmatrix} \frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\ \frac{aa'}{M_1} & \frac{a'^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\ 0 & \frac{bb'}{M_2} & \frac{b'^2}{M_2} \end{pmatrix} \qquad a, b, b' \text{ are real and } a' = |a'|e^{i\delta}$$

Seesaw with 2 RH Neutrinos

- bi-large mixing angle $a' = \sqrt{2}a \qquad b = b' \qquad a^2/M_1 \ll b^2/M_2$

$$m_{\nu_1} = 0, \quad m_{\nu_2} = \frac{2a^2}{M_1}, \quad m_{\nu_3} = \frac{2b^2}{M_2}$$

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \qquad \theta \simeq m_{\nu_2}/\sqrt{2}m_{\nu_3}$$

- relation between sign of baryonic asymmetry and sign of CP violation in neutrino oscillation

$$\xi_{osc} = -\frac{a^4 b^4}{M_1^3 M_2^3}(2 + Y^2)\xi_B \propto -B$$

$$(B \propto \xi_B = Y^2 a^2 b^2 \sin 2\delta)$$

Sources of CP Violation

- Manifestations of CP violation
  - weak scale CPV (kaon, B-meson, neutrino oscillation, ...)
  - cosmological BAU
  - strong CP problem

  $\Rightarrow$ can they come from a common origin??

- Explicit CP violation
  - complex Yukawa couplings
- Spontaneous CP violation
  - complex VEV

Models with Spontaneous CP Violation

M-C.C & Mahanthappa, 2005

- minimal left-right model:
- gauge symmetry

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\ \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \ ,$$

- particle content

$$Q_{i,L} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,L} \sim (1/2, 0, 1/3), \qquad Q_{i,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,R} \sim (0, 1/2, 1/3)$$

$$L_{i,L} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,L} \sim (1/2, 0, -1), \qquad L_{i,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,R} \sim (0, 1/2, -1)$$

- minimal higgs sector

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1/2,\ 1/2,\ 0) \qquad \Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} \sim (1,\ 0,\ 2) \qquad \Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} \sim (0,\ 1,\ 2)$$

- in general, 4 complex VEV's

$$\langle \Phi \rangle = \begin{pmatrix} \kappa e^{i\alpha_\kappa} & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \ \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\alpha_R} & 0 \end{pmatrix}$$

Models with Spontaneous CP Violation

- Lagrangian invariant under unitary transformations

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \qquad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}$$

$$\psi_L \to U_L \psi_L, \qquad \psi_R \to U_R \psi_R$$

$$\Phi \to U_R \Phi U_L^\dagger, \qquad \Delta_L \to U_L^* \Delta_L U_L^\dagger, \qquad \Delta_R \to U_R^* \Delta_R U_R^\dagger$$

- VEVs transform accordingly

$$\kappa \to \kappa e^{-i(\gamma_L - \gamma_R)}, \quad \kappa' \to \kappa' e^{i(\gamma_L - \gamma_R)}, \quad v_L \to v_L e^{-2i\gamma_L}, \quad v_R \to v_R e^{-2i\gamma_R}$$

- only two physical phases:

$$< \Phi > = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \qquad < \Delta_L > = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \quad < \Delta_R >= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$\boxed{\alpha_{\kappa'} \Rightarrow \text{ all CPV in quark sector}}$ (contributions to lepton sector negligible for high seesaw scale)

$\boxed{\alpha_L \Rightarrow \text{ all CPV in lepton sector}}$

Models with Spontaneous CP Violation

M-C.C & Mahanthappa, 2005

- all leptonic CP violation from a single phase
  - the three CP-violating phases in MNS matrix are functions of the intrinsic phase α_L
- the phase α_L enters
  - neutrino oscillation

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}) \sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \; +2\sum_{i>j} J^{\mathrm{lep}}_{\mathrm{CP}} \sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right)$$

$$J^{\mathrm{lep}}_{\mathrm{CP}} = -\frac{Im(H_{12}H_{23}H_{31})}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}}, \quad H \equiv (M^{eff}_\nu)(M^{eff}_\nu)^\dagger$$

$$\boxed{J^{lep}_{CP} \sim \sin\alpha_L}$$

  - neutrino-less double beta decay

$$|\langle m_{ee}\rangle|^2 = m_1^2\,|U_{e1}|^4 + m_2^2\,|U_{e2}|^4 + m_3^2\,|U_{e3}|^4 \; +2m_1m_2\,|U_{e1}|^2\,|U_{e2}|^2 \cos\alpha_{21}$$
$$+2m_1m_3\,|U_{e1}|^2\,|U_{e3}|^2 \cos\alpha_{31} +2m_2m_3\,|U_{e2}|^2\,|U_{e3}|^2 \cos(\alpha_{31}-\alpha_{21})$$

  - leptogenesis

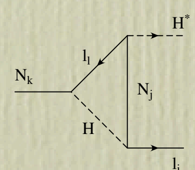
Models with Spontaneous CP Violation

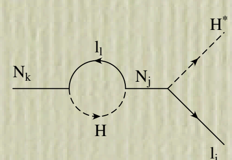
- triplet leptogenesis:
  - $N_1 \to \ell + H^\dagger$ $\qquad \epsilon = \dfrac{\Gamma(N_1 \to \ell + H^\dagger) - \Gamma(N_1 \to \overline{\ell} + H)}{\Gamma(N_1 \to \ell + H^\dagger) + \Gamma(N_1 \to \overline{\ell} + H)}$
  - $\Delta^* \to \ell + \ell$ $\qquad \epsilon = \dfrac{\Gamma(\Delta_L^* \to \ell + \ell) - \Gamma(\Delta_L \to \overline{\ell} + \overline{\ell})}{\Gamma(\Delta_L^* \to \ell + \ell) + \Gamma(\Delta_L \to \overline{\ell} + \overline{\ell})}$
- natural scenario: Δ^* heavier than N_1 ➔ N_1 decay dominant
- two contributions

M-C.C & Mahanthappa, 2005

  - usual diagrams (type I contribution)

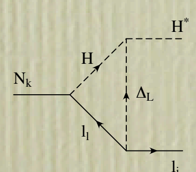
$$\mathcal{M}_D = O_R M_D$$





$$\epsilon^{N_1} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}\left(\mathcal{M}_D \left(M_\nu^I\right)^* \mathcal{M}_D^T\right)_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}} = 0$$

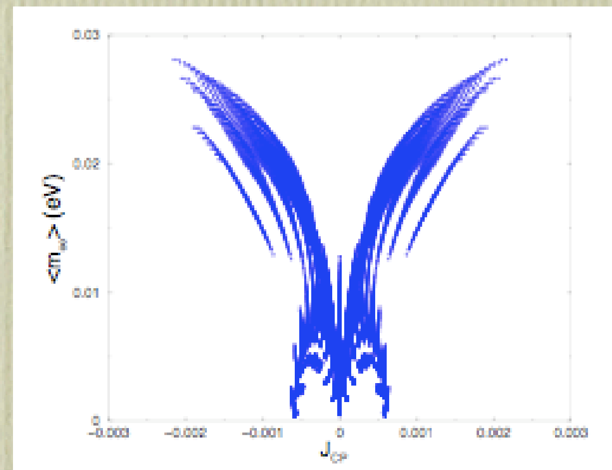
  - new diagram (type II contribution)

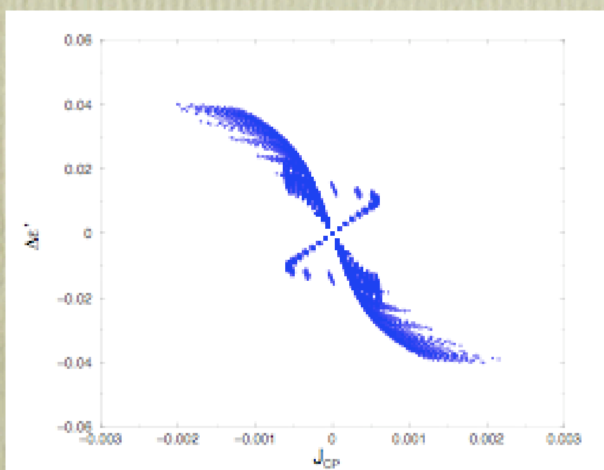


$$\epsilon^{\Delta_L} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}\left(\mathcal{M}_D \left(M_\nu^{II}\right)^* \mathcal{M}_D^T\right)_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}} \sim \sin\alpha_L$$

independent of choice of $U_{L,R}$

Models with Spontaneous CP Violation





M.-C.C & Mahanthappa, 2005

- predict small θ_{13}
- in large J_{cp} regime:
  strong correlation between J_{cp} and $\langle m_{ee} \rangle$
- J_{cp}: $(0 - 10^{-3})$
- $\langle m_{ee} \rangle$: $(10^{-4} \sim 10^{-2})$ eV; current limit ~ 0.1 eV

- symmetry between 2nd & 4th quadrants
- in large J_{cp} regime: strong correlation between J_{cp} and $\Delta\epsilon'$ (even without flavor effects)

- total amount of lepton number asymmetry

$$\epsilon = 10^{-2} \times \Delta\epsilon' < (10^{-5} - 10^{-4})$$

- no wash-out $\quad \dfrac{\Gamma_{N_1}}{H|_{T=M_1}} = \dfrac{1}{0.01\ \mathrm{eV}} \dfrac{(M_D M_D^\dagger)_{11}}{M_1} < 1$

$$\frac{(M_D M_D^\dagger)_{11}}{M_1} \propto \left(\frac{m_c}{m_t}\right)^2 v_L < \mathcal{O}(10^{-7})\ \mathrm{eV}$$

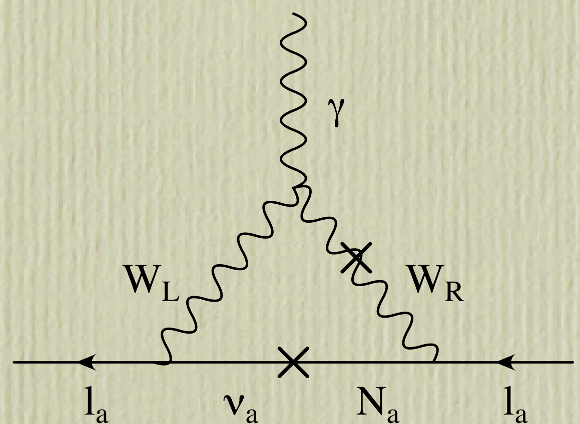
$\boxed{\text{observed BAU} \Rightarrow J_{cp} \sim 10^{-5}}$

Models with Spontaneous CP Violation

M.-C.C & Mahanthappa, 2007

- with an additional U(1) symmetry

  ⇒ can lower seesaw scale to 10^6 GeV (and below)



$$d_e \simeq -\frac{e\alpha}{4\pi M_W^2}\frac{\kappa\kappa'}{v_R^2 - v_L^2}\mathrm{Im}(e^{-i\alpha_{\kappa'}}M_D)_{ee}$$

$$\simeq 10^{-19} \times r\left(\frac{GeV}{v_R}\right)^2\left(\frac{|(M_{\nu_D})_{ee}|}{MeV}\right)\sin(2\alpha_{\kappa'}) \quad \text{e-cm}$$

- relation between CPV in quark & lepton sectors
- electron EDM ~ 10^{-32} e-cm

Models with Spontaneous CP Violation

- SM + D° (vectorial quark) + S (singlet scalar) Branco, Parada, Rebelo, 2003

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \qquad \langle S \rangle = \frac{V \exp(i\alpha)}{\sqrt{2}}$$

$$(f_q S + f_q{}' S^*)\overline{D^0_L} d^0_R + \tilde{M}\overline{D^0_L} D^0_R$$ →quark CPV

$$\frac{1}{2}\nu_R^{0T} C(f_\nu S + f_\nu{}' S^*)\nu^0_R$$ →leptonic CPV

- SCPV in SO(10) Achiman, 2004, 2008

<126> complex: break (B-L) $\overline{\Delta} =< \overline{\Sigma}(1,1,0) > \; = \frac{\sigma}{\sqrt{2}} e^{i\alpha}$

$$Y_\ell^{ij} \nu_R^i \overline{\Delta} \nu_R^j$$

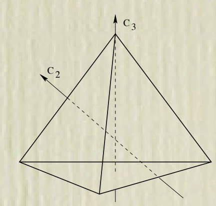
- no symmetry reason why <S> is the only complex VEV

Models with Tri-bimaximal Neutrino Mixing

- global neutrino oscillation data strongly suggests TBM mixing pattern

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \qquad \boxed{\sin\theta_{13,\rm TBM} = 0} \longleftrightarrow \text{Leptogenesis?}$$

- TBM mixing can arise from underlying symmetry
  - S_3 : less constrained Mohapatra, Nasri, Yu, 2006
  - $Z_7 \times Z_3$ Luhn, Nasri, Ramond, 2007
  - A_4: Ma, 2004; Altarelli, Feruglio, 2006
    - tri-bimaxmal mixing results from group theory!
    - no CKM mixing
  - $^{(d)}T$: double covering of A_4 Carr, Frampton, 2007; Feruglio, Hedgedorn, Lin, Merlo, 2007
    - retain predictivity of A_4 in neutrino sector
    - realistic CKM in $SU(5) \times {}^{(d)}T$ M.-C.C & Mahanthappa, 2007



Models with Tri-bimaximal Neutrino Mixing

- TBM neutrino mixing from group theoretical CG coefficients

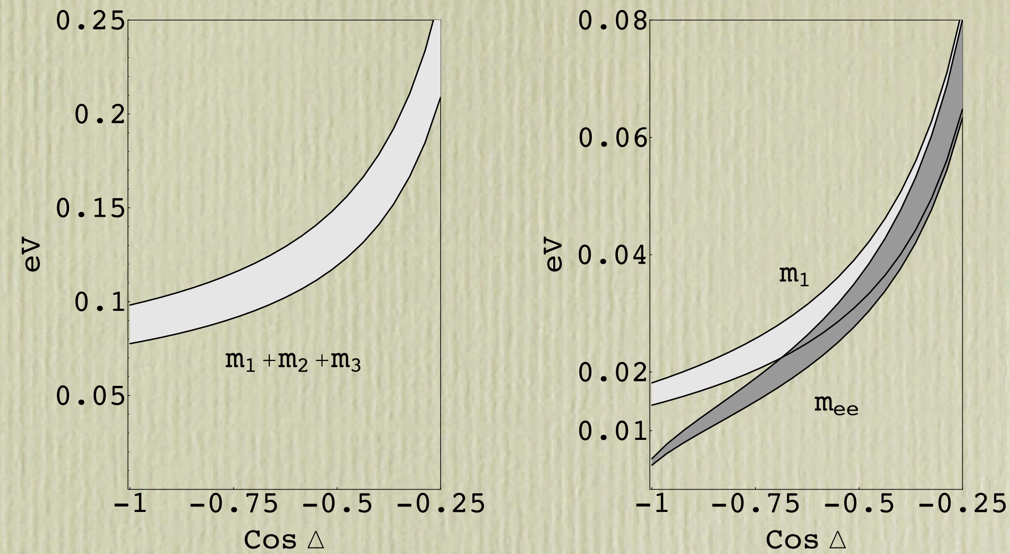
$$M_\nu = \frac{\lambda v^2}{M_x}\begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

Form diagonalizable!

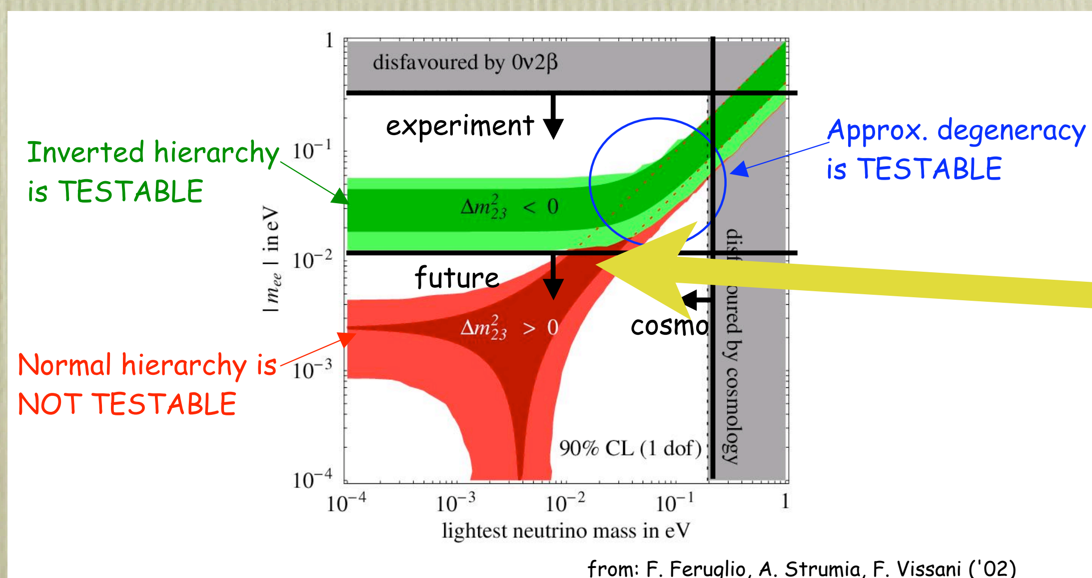
$$V_\nu^{\mathrm{T}} M_\nu V_\nu = \mathrm{diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0)\frac{v_u^2}{M_x}$$

$$,V_\nu = U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Models with Tri-bimaximal Neutrino Mixing



For A4: Altarelli et al, 2006



prediction in
A_4 and $^{(d)}T$ models

Models with Tri-bimaximal Neutrino Mixing

- TBM mixing arises from underlying broken discrete symmetries (A_4, Z_7 x Z_3, $^{(d)}T$) through type-I seesaw Jenkins, Manohar, 2008
  - ➡ exact TBM mixing

    $\sin\theta_{13} = 0 \;\Rightarrow\; J_{CP}^{lep} \propto \sin\theta_{13} = 0$

    CP violation through Majorana phases: $\alpha_{21},\ \alpha_{31}$
  - ➡ no leptogenesis as $Im(y_D y_D^\dagger) = 0$
  - ➡ true even when flavor effects included
  - corrections to TBM pattern due to high dim operators

    small symmetry breaking parameter $\eta \ll 1$:

    $$\sin\theta_{13} \sim \eta \sim 10^{-2},\ \epsilon \sim 10^{-6} \text{ can be generated}$$
- type-II seesaw contribution in S_3 Mohapatra, Yu, 2006
  - exact TBM limit: $\varepsilon_2^{II} \simeq -\dfrac{3}{8\pi}\dfrac{m_1 M_2 \sin\varphi_1}{v^2\sin^2\beta}$ φ_1 : one of the Majorana phases

Quantum Boltzmann Equations

Buchmuller, Fredenhagen, 2000;
Simone, Riotto 2007;
Lindner, Muller 2007

- Classical vs Quantum Boltzmann equations:
  - collision terms: involving quantum interference
  - time evolution: quantum mechanical treatment

- Classical Boltzmann equations:

  scattering independent from previous one

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1} \rangle \left(n_{N_1} - n_{N_1}^{\rm eq} \right),$$

$$\langle \Gamma_{N_1} \rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\rm eq}}{n_{N_1}^{\rm eq}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1 \to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} (2\pi)\delta\left(\omega_{N_1} - \omega_\ell - \omega_H\right)$$

Quantum Boltzmann Equations

Schwinger, 1961; Mahanthappa, 1962; Bakshi, Mahanthappa, 1963; Keldysh, 1965

- Quantum Boltzmann equations:
  - Closed-Time-Path (CTP) formulation for non-equilibrium QFT
  - involve time integration for scattering terms

➡ "memory effects": time-dependent CP asymmetry

$$
\begin{aligned}
\frac{\partial n_{N_1}}{\partial t} &= -\langle \Gamma_{N_1}(t) \rangle n_{N_1} + \langle \widetilde{\Gamma}_{N_1}(t) \rangle n_{N_1}^{\mathrm{eq}}, \\
\langle \Gamma_{N_1}(t) \rangle &= \int_0^t dt_z \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\mathrm{eq}}}{n_{N_1}^{\mathrm{eq}}} \Gamma_{N_1}(t), \\
\Gamma_{N_1}(t) &= 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\left| \mathcal{M}(N_1 \to \ell H) \right|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} \cos\left[\left(\omega_{N_1} - \omega_\ell - \omega_H \right) (t - t_z) \right]
\end{aligned}
$$

Quantum Boltzmann Equations

- time scale of Kernel << relaxation time scale $\sim 1/\Gamma_{N1}$

  Classical Boltzmann eqs ≈ Quantum Boltzmann equation

- In resonant leptogenesis: $\Delta M = (M_2\text{-}M_1) \sim \Gamma_{N2}$

  Kernel time scale $\sim 1/\Delta M > 1/\Gamma_{N1}$ possible

  ⇒ quantum Boltzmann equations important!!

Conclusions

- Leptogenesis: promising mechanism for BAU
- connection between leptogenesis & low energy CPV processes generally does not exist in a model independent way
  - statement weakened when flavor effects included
- models for neutrino mass: reduced number of parameters, allowing connection
  - 2x3 seesaw
  - models with SCPV: single source for all CPV
  - TBM mixing pattern compatible with leptogenesis, if
    - higher order corrections included; or
    - type-II seesaw
- Quantum Boltzmann equations?