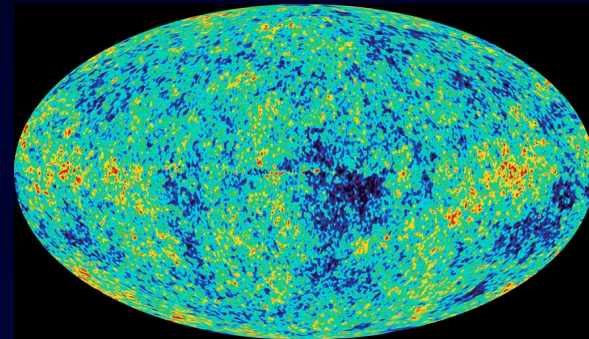
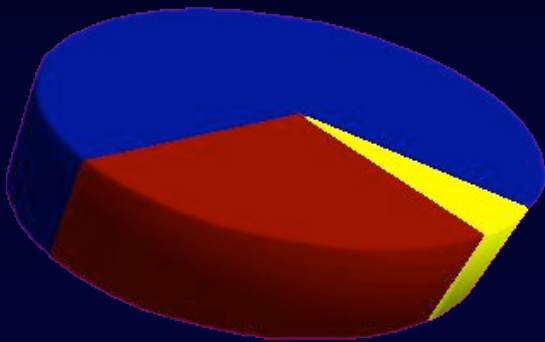


# The Origin of the Universe and the Arrow of Time

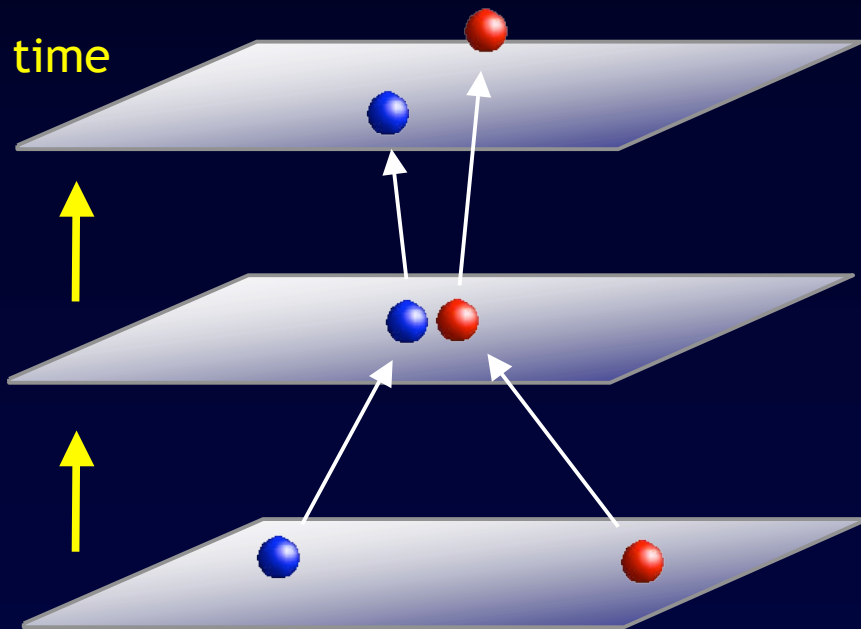
Sean Carroll, Caltech



Why does the history of the universe appear so unnatural?

We need to explain the history of the universe,  
not merely its initial state.

**HISTORY = BOUNDARY CONDITIONS + EVOLUTION LAW**



- not necessarily “initial,” or even “boundary”
- conditions at any one moment specify the entire history

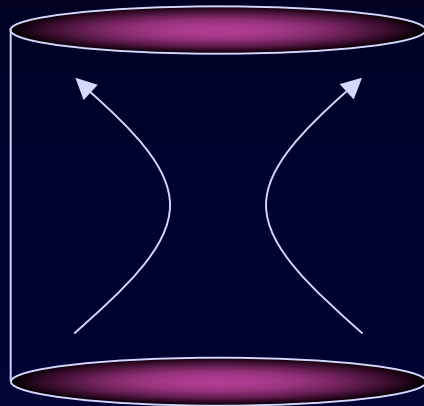
$$\dot{p} = -\partial_q H, \quad \dot{q} = \partial_p H$$

$$\hat{H}|\Psi\rangle = i\partial_t|\Psi\rangle$$

- don't be a temporal chauvinist!

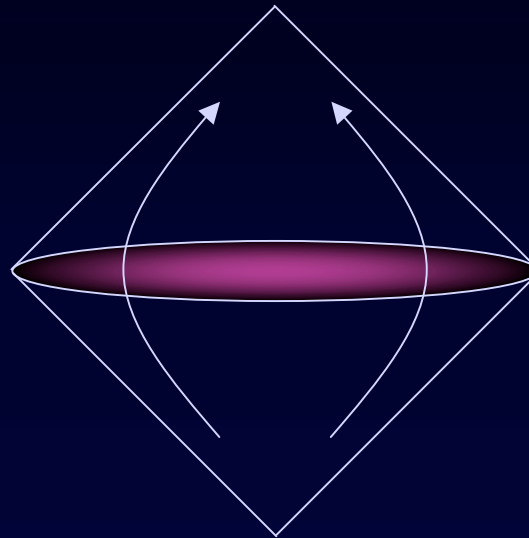
Plausible candidates for “natural” universes: maximally-symmetric vacuum spacetimes.

de Sitter



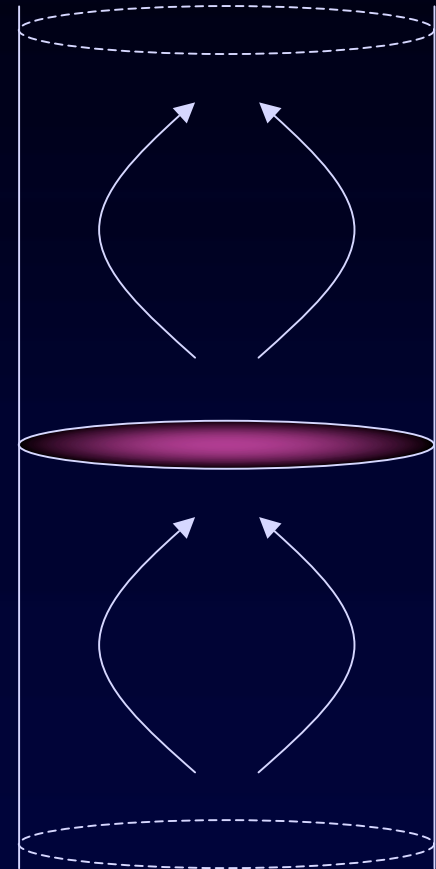
$$\Lambda > 0$$

Minkowski



$$\Lambda = 0$$

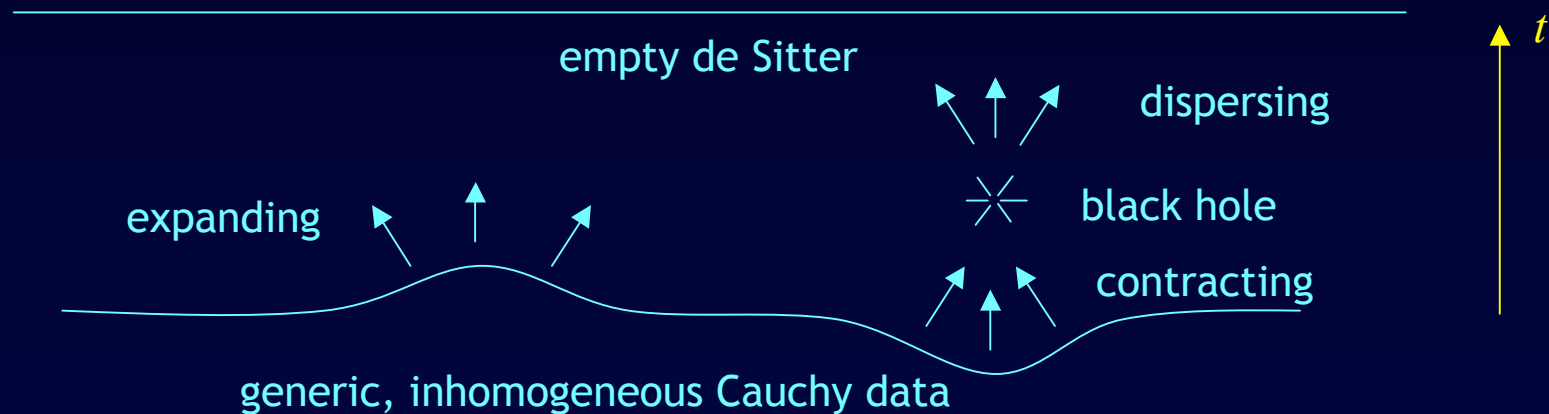
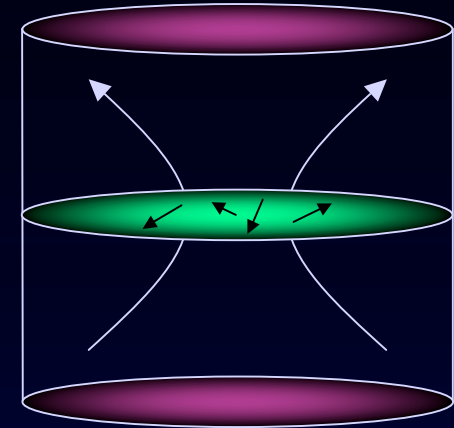
Anti de Sitter



$$\Lambda < 0$$

# de Sitter is an attractive candidate for a natural universe

- stable, or at least metastable;  
small perturbations relax
- other states evolve toward it,  
including the real universe  
(cosmic no-hair theorem)



- high entropy:  $S_{horizon} = A/4G \sim 10^{120}$  for  $\rho_{\Lambda} \sim (10^{-3} \text{ eV})^4$

But we don't live in de Sitter space.

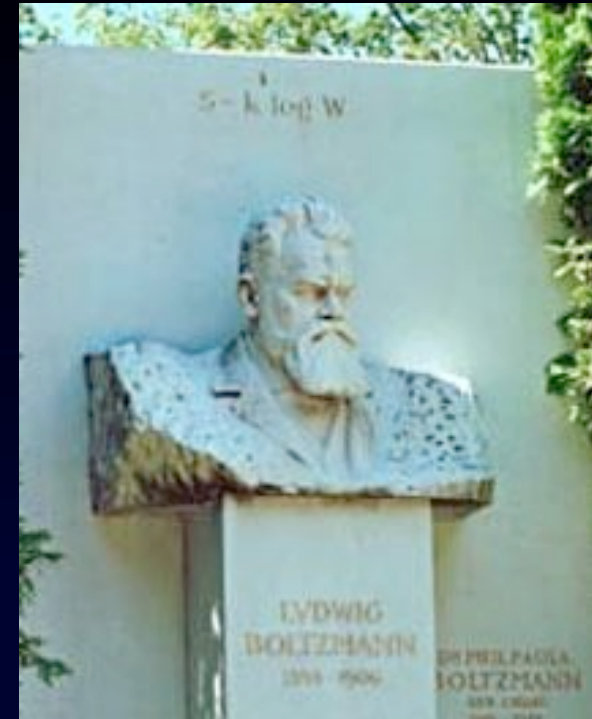
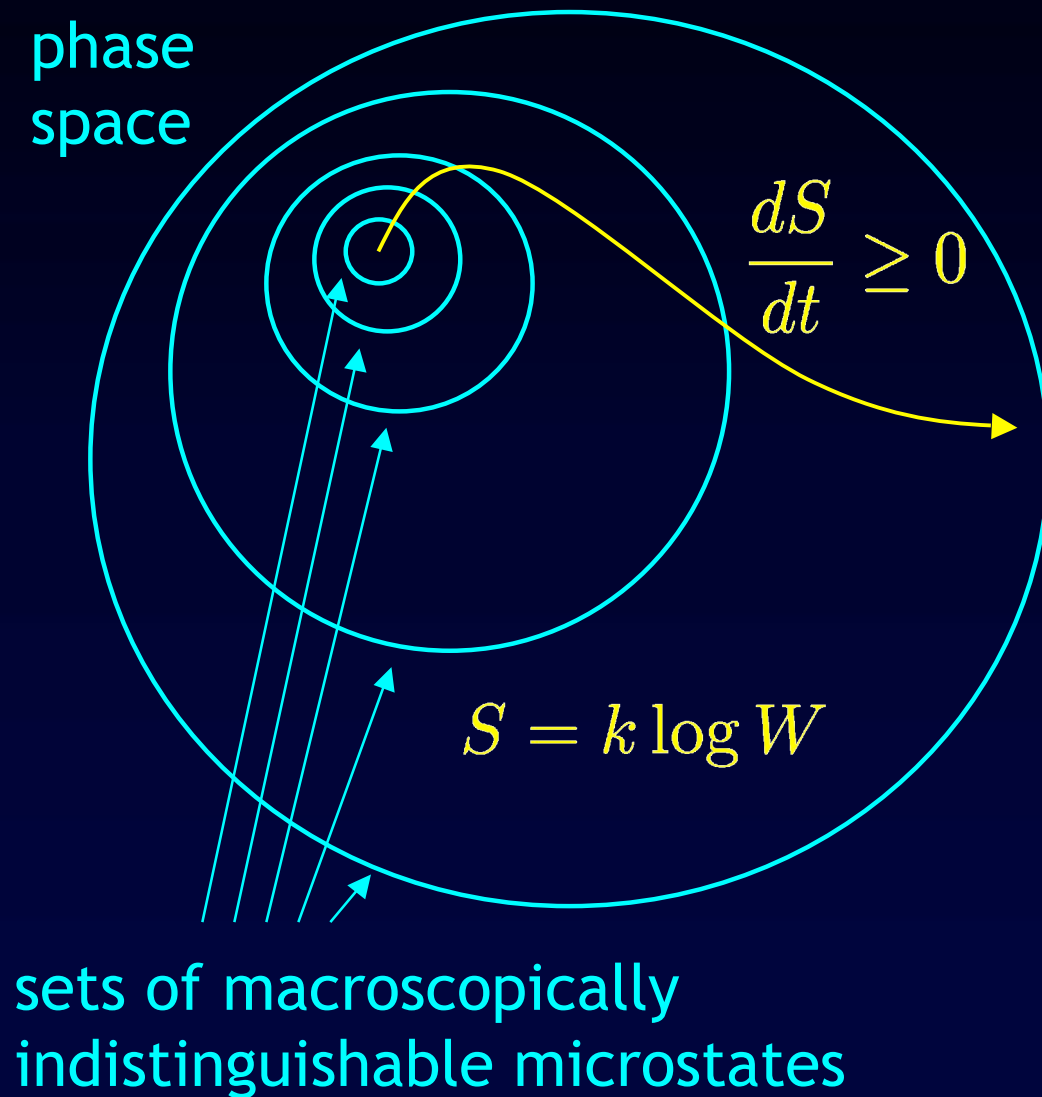
The single most blatant observational fact about our local universe: **entropy is increasing.**

$$\frac{dS}{dt} \geq 0$$



Alan Guth's office

# Entropy measures volumes in phase space.



Boltzmann: entropy increases because there are more high-entropy states than low-entropy ones.

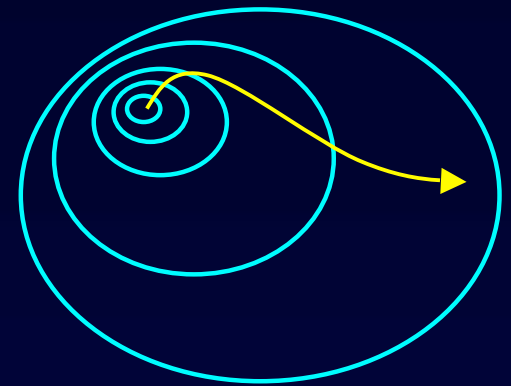
Two ingredients needed to explain why the Second Law of Thermodynamics works in the real world.

**1. A statistical basis for entropy.**

Entropy counts (the logarithm of) the number of microstates that are macroscopically indistinguishable.

**2. A “Past Hypothesis.”**

The observable universe began, not too long ago, in a state of very low entropy.

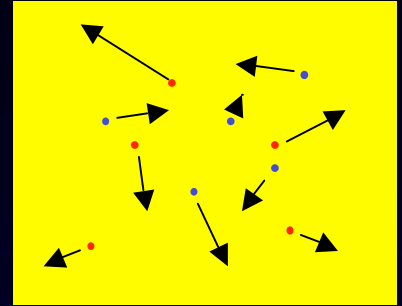


Boltzmann gave us the first; it's up to cosmology to provide the second.

We don't have a general formula for entropy, but we do understand some special cases.

Thermal gas  
(early universe):

$$S_{\text{therm}} = \left( \frac{\rho + p}{T} \right) V \sim N \sim 10^{88}$$



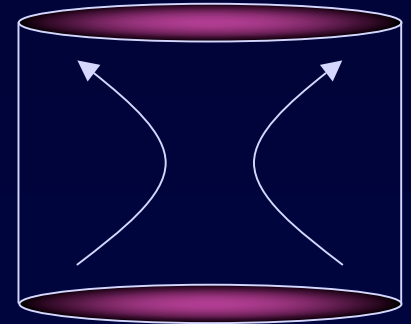
Black holes  
(today):

$$S_{\text{BH}} = \frac{A}{4G} \sim 10^{90} \left( \frac{M_{\text{BH}}}{10^6 M_{\odot}} \right)^2 \sim 10^{100}$$



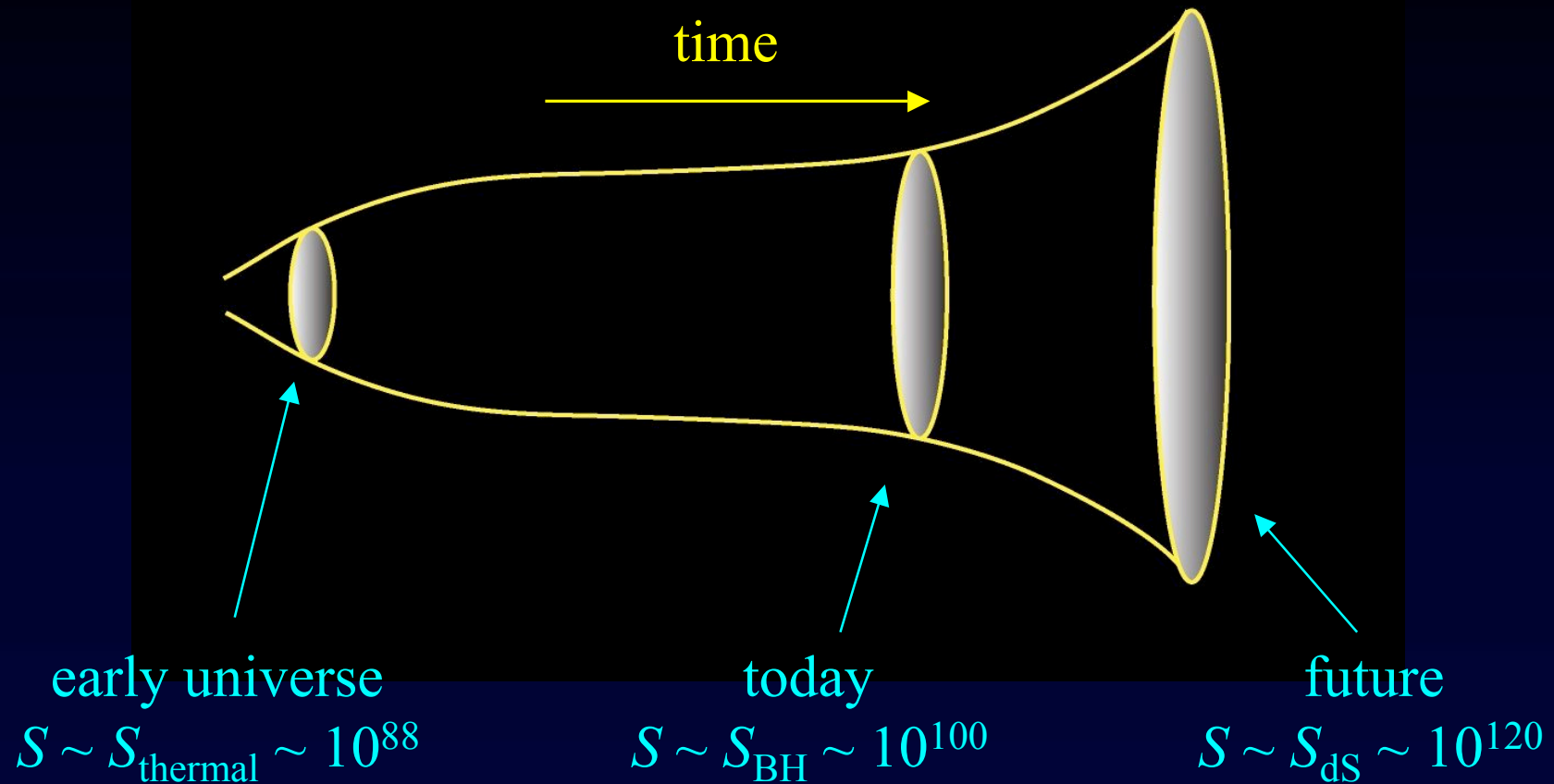
de Sitter space:  
(future universe)

$$S_{\text{dS}} = \frac{A}{4G} \sim \left( \frac{L_{\text{dS}}}{M_{\text{P}}} \right)^2 \sim 10^{120}$$





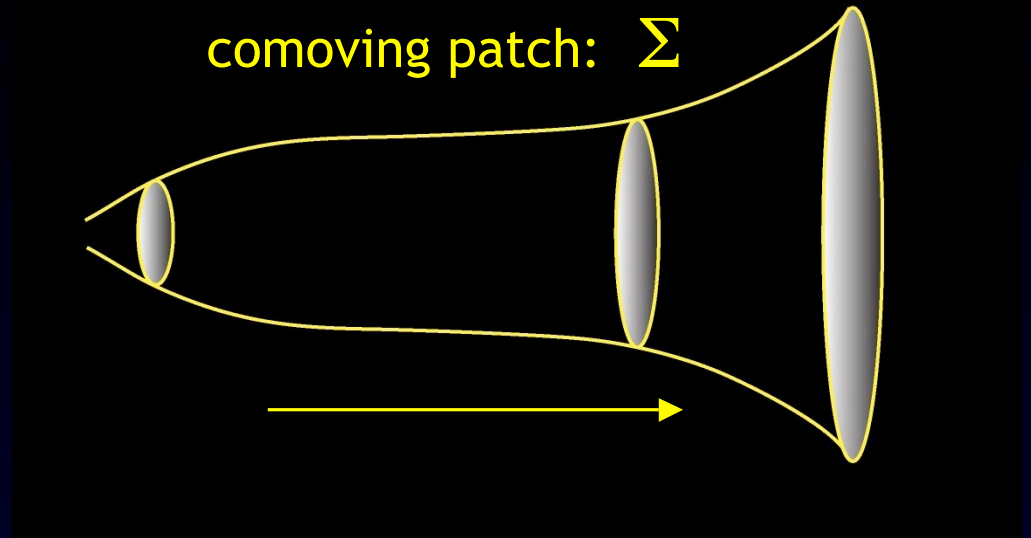
Entropy goes up as the universe expands -- the 2nd law works! Consider our comoving patch.



Our entropy today is much smaller than it could be.  
The reason why is that it was even smaller in the past.  
Why was it ever so small?

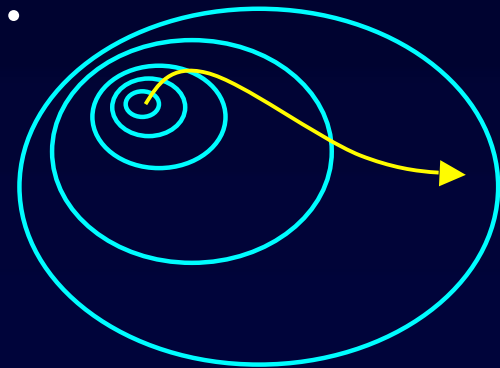
Don't get hung up on:

- “gravitational” vs. “non-gravitational” entropy
- “effective” vs. “true” degrees of freedom



These distinctions don't matter, because the 2nd Law doesn't care what the Hamiltonian is.

$$\partial_t \hat{\rho}_\Sigma = -i \left[ \hat{H}_\Sigma, \hat{\rho}_\Sigma \right]$$



We assume evolution of our patch is **local** and **unitary** in a fixed Hilbert space; degrees of freedom don't change.

# Why did the early universe have low entropy?

1. It just did.

# Why did the early universe have low entropy?

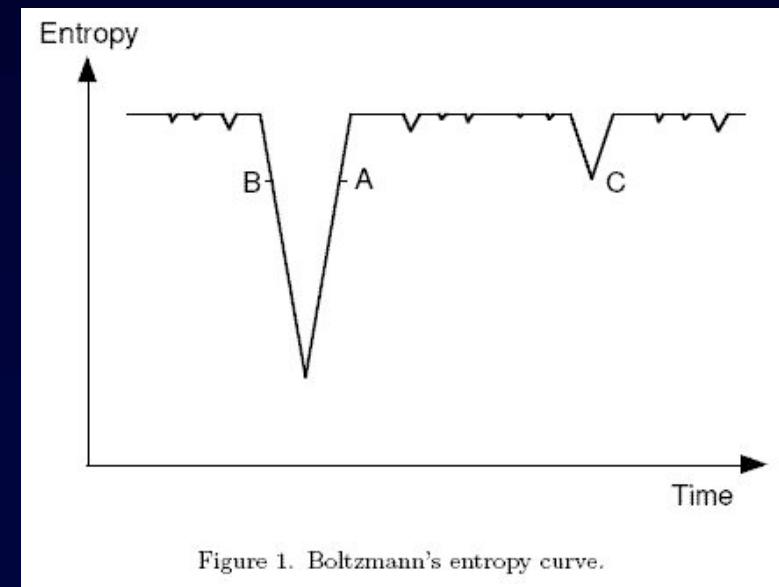
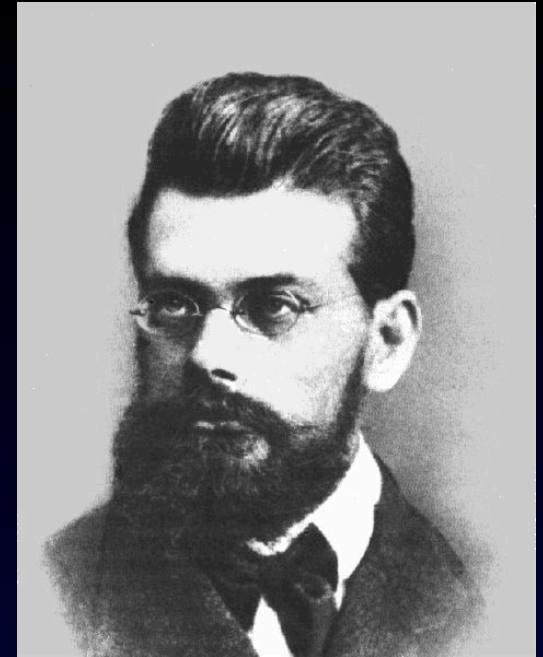
1. It just did.  
→ consistent, but cowardly.
2. Anthropic selection.

Boltzmann: maybe the **multiverse** is in thermal equilibrium, and we can explain our local environment by invoking the **anthropic principle**.

“There must then be in the universe, which is in thermal equilibrium as a whole and therefore dead, here and there relatively small regions of the size of our galaxy (which we call *worlds*), which during the relatively short time of eons deviate significantly from thermal equilibrium.

Among these worlds the state probability increases as often as it decreases.” (1895)

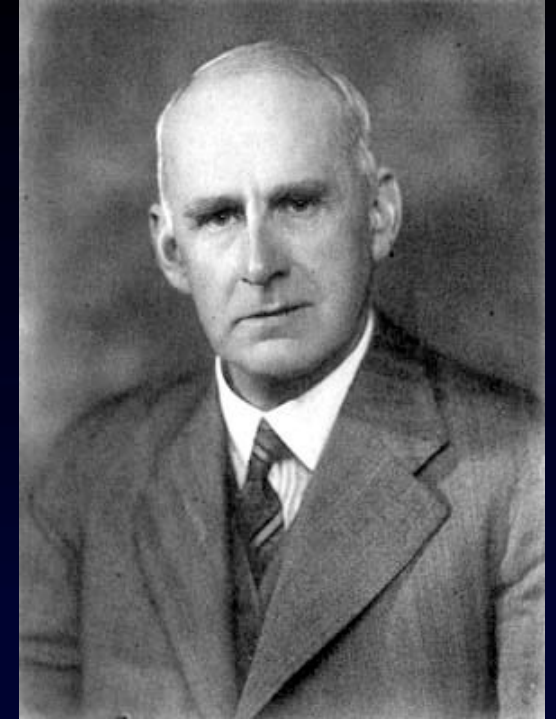
Maybe the observable universe is just a thermal fluctuation.



**Eddington** explained why we cannot be a thermal fluctuation.

“It is practically certain that at any assigned date the universe will be almost in the state of maximum disorganization...

A universe containing mathematical physicists will at any assigned date be in the state of maximum disorganization which is not inconsistent with the existence of such creatures.” (1931)



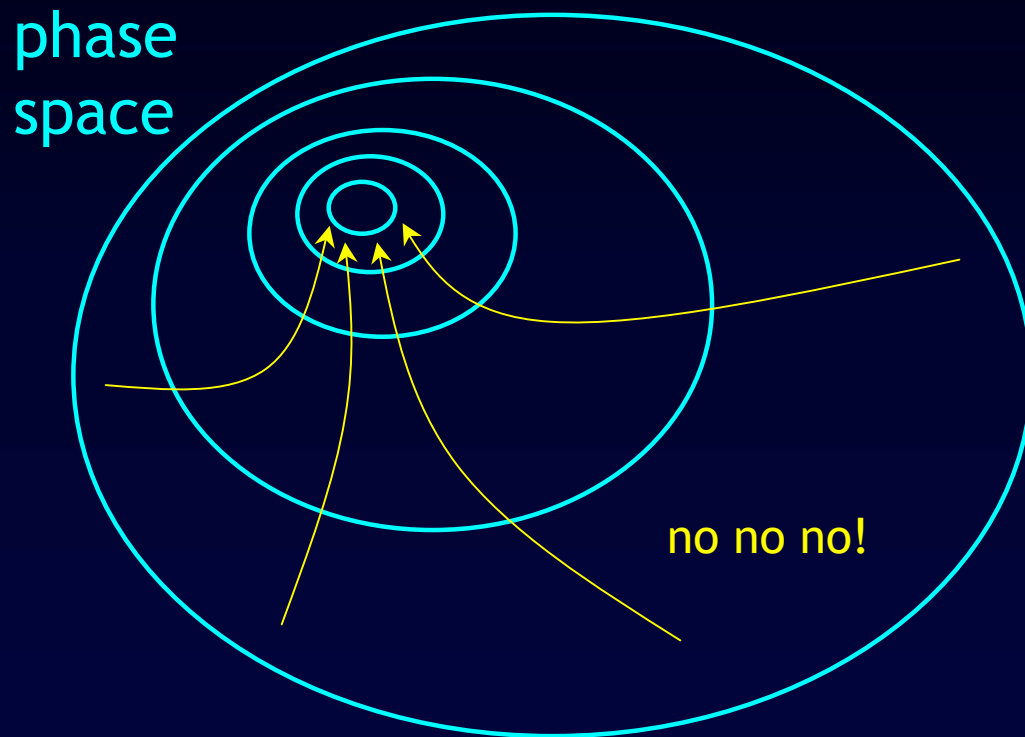
Fluctuations are rare, and large fluctuations are very rare:  $P \sim e^{-\Delta S}$ . This scenario predicts that we should be the minimum possible fluctuations -- “Boltzmann Brains.”

[Albrecht & Sorbo]

# Why did the early universe have low entropy?

1. It just did.  
→ consistent, but cowardly.
2. Anthropic selection.  
→ can't explain how low our entropy is.
3. Local dynamics (including inflation).

Local, unitary dynamics can **never**, in principle, explain why a system was “naturally” in a state of low entropy -- that depends on how state space is coarse-grained, not on the particular choice of Hamiltonian.

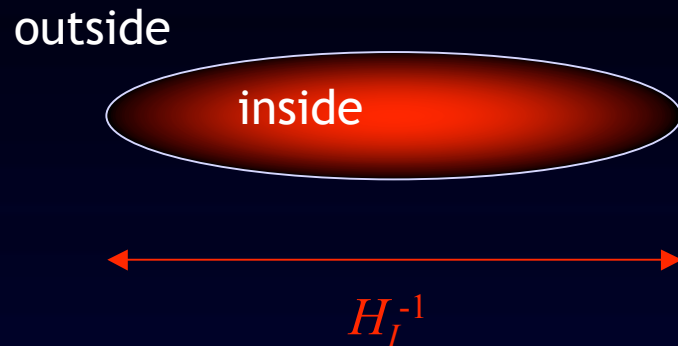


Liouville's theorem:  
volume in phase space  
is conserved under  
Hamiltonian evolution.

There is no clever choice of dynamics which naturally makes the early universe small, dense, and smooth.



Inflation is algorithmically simple: a tiny patch of smooth space, where potential energy dominates gradient energy.



But that's a lower-entropy (more finely-tuned) initial condition than conventional Big-Bang cosmology.

$$\mathcal{H} = \mathcal{H}^{(\text{in})} \otimes \mathcal{H}^{(\text{out})}$$

$$\rho^{(\text{in})} = \text{Tr}_{(\text{out})} \rho$$

$$S^{(\text{in})} = -\text{Tr} \rho^{(\text{in})} \log \rho^{(\text{in})} \sim S_{\text{dS}} \sim (M_P/E_I)^4 \sim 10^{12} \ll 10^{88} !$$

A proto-inflationary patch is less likely to arise “randomly” as a configuration of the universe’s degrees of freedom. Inflation makes the entropy problem significantly worse.

## Degrees of freedom in an expanding universe

Decompose the Hilbert space of fields in our current observable universe into long-wavelength and short-wavelength modes:  $\mathcal{H}^{(\text{in})} = \mathcal{H}^{(l)} \otimes \mathcal{H}^{(s)}$ .

If we choose the cutoff to be  $\lambda_c = 0.1 \text{ Mpc } (E_I/10^{15} \text{ GeV})^{-4}$ ,

then all short modes were trans-Planckian at the onset of inflation. They had to be in their vacuum state at the start, or inflation would not have occurred.

$$\rho^{(\text{in})} = \left( \sum \rho_{\alpha\beta} |\ell_\alpha\rangle \langle \ell_\beta| \right) \otimes \underbrace{|0^{(s)}\rangle \langle 0^{(s)}|}_{\text{unique vacuum state for short-wavelength modes}}$$

unique vacuum  
state for short-  
wavelength modes

## Inflation does not:

- explain homogeneity, isotropy, flatness (low entropy)
- remove sensitive dependence on initial conditions.

## On the other hand, inflation does:

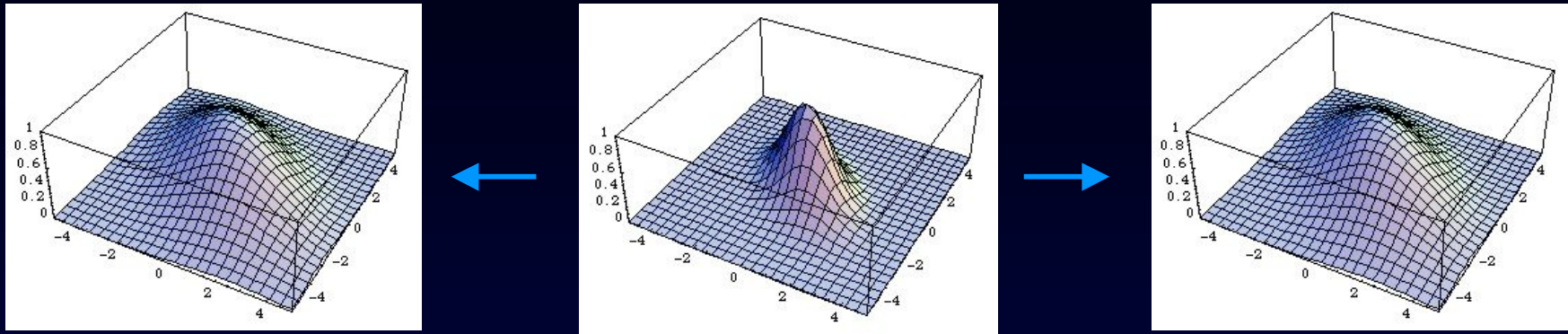
- produce scale-free primordial density perturbations
- create a lot of particles from almost nothing.

So inflation is definitely worth salvaging. But it does not remove the need for a theory of initial conditions: it makes that need more urgent than ever!

# Why did the early universe have low entropy?

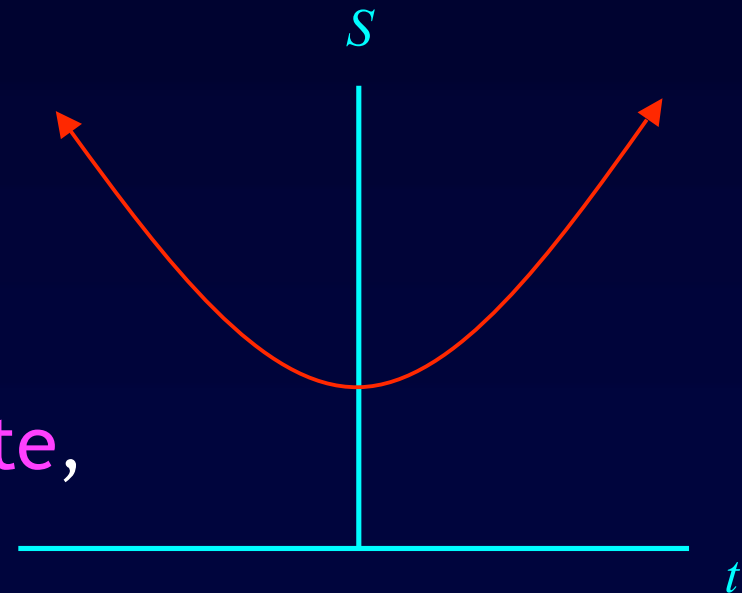
1. It just did.  
→ consistent, but cowardly.
2. Anthropic selection.  
→ can't explain how low our entropy is.
3. Local dynamics (including inflation).  
→ cannot in principle explain low entropy.
4. Global dynamics (of the multiverse).

We want a theory where **entropy increases forever**.  
Not very hard: consider a 2-dim wave packet,  
projected onto one dimension.

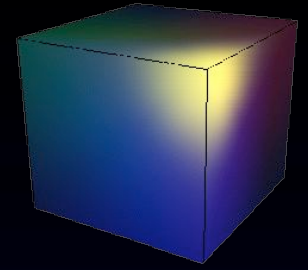


$$\rho(x_1, x_2) = \int dy \Psi(x_1, y) \Psi^*(x_2, y)$$
$$S = - \int dx \rho \log \rho$$

Spectrum of  $H$  must be **non-discrete**,  
or we just get ergodic evolution.



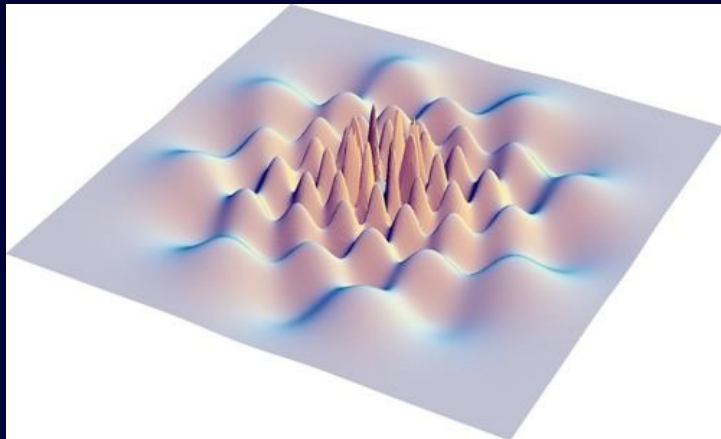
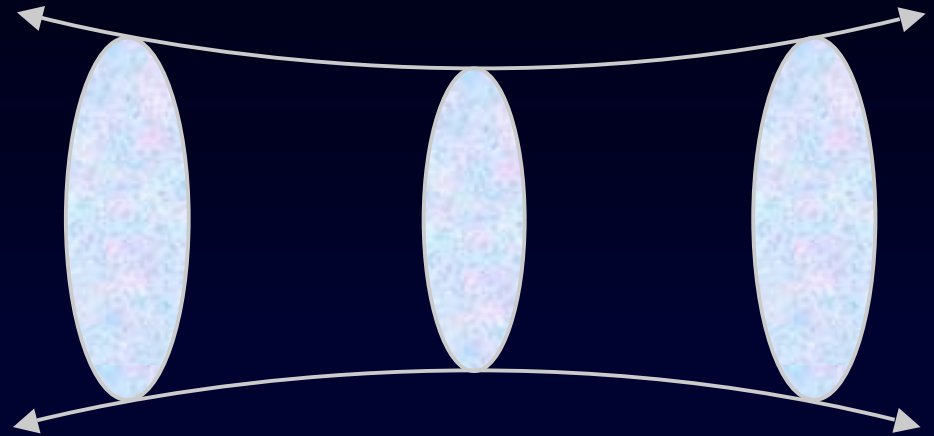
The hard part: matching this story onto a spacetime description.



Start with **de Sitter space**, where our universe is heading.

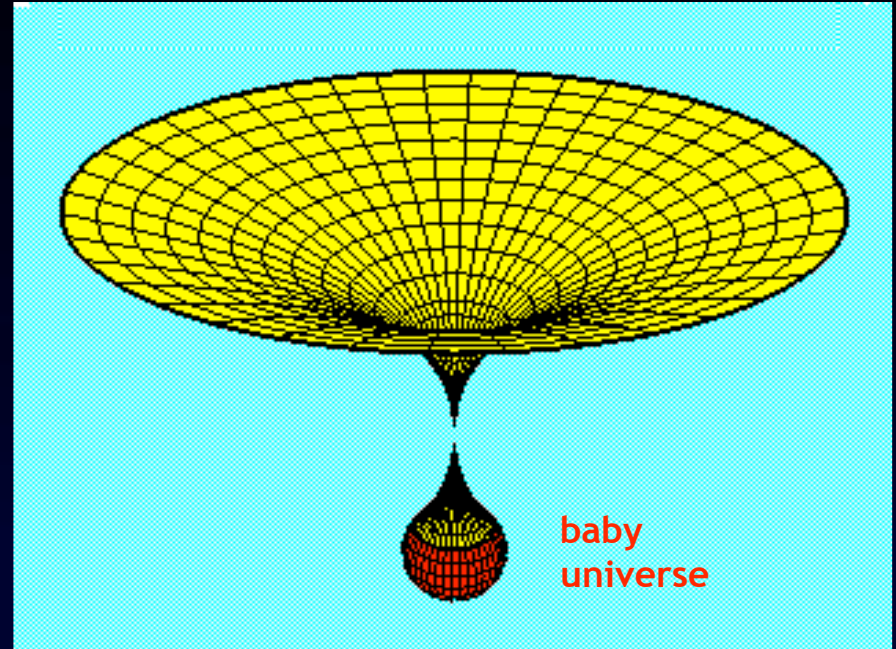
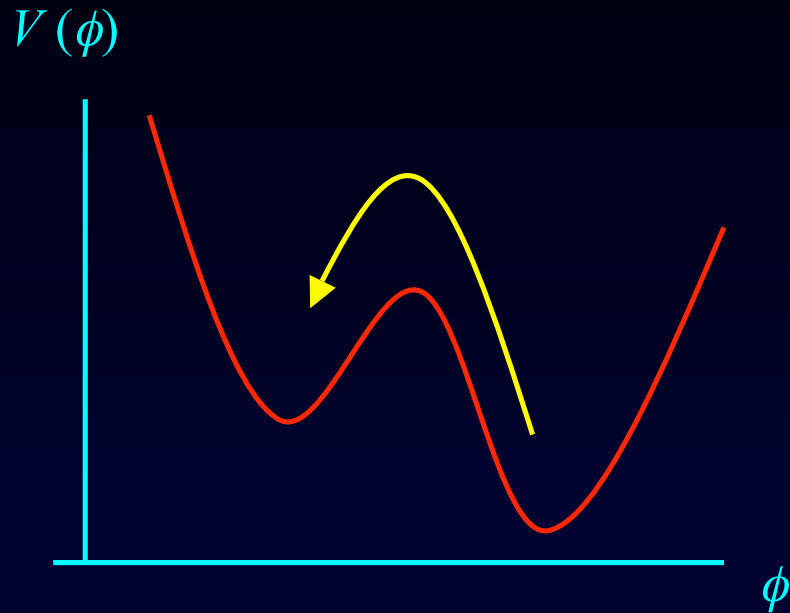
de Sitter has a nonzero **temperature**.

$$T_{\text{dS}} = \frac{H}{2\pi} \sim 10^{-33} \text{ eV}$$



In such a space, the quantum fields of which matter is composed will constantly be fluctuating, even though space is “empty.”

# Quantum fluctuations can produce new universes. (Maybe.)

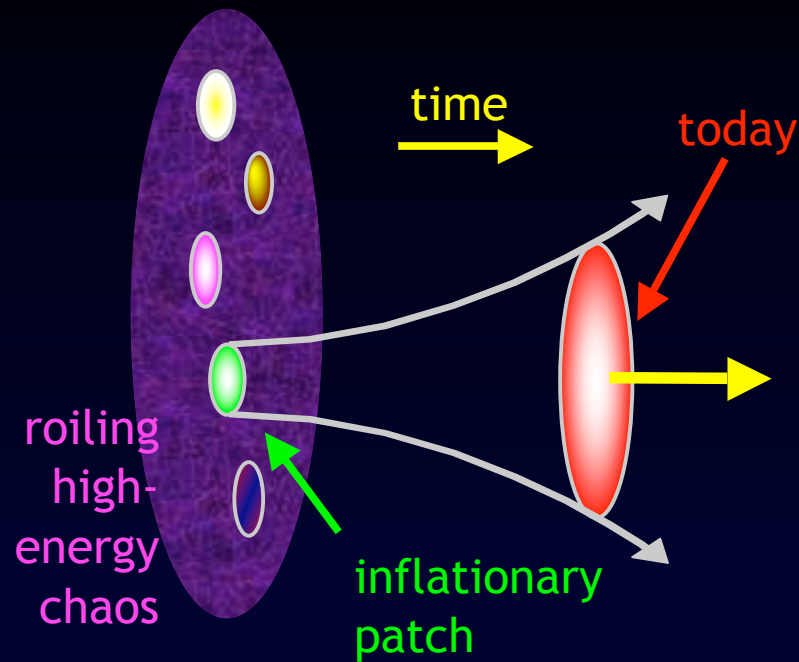


[Farhi, Guth, et al.]

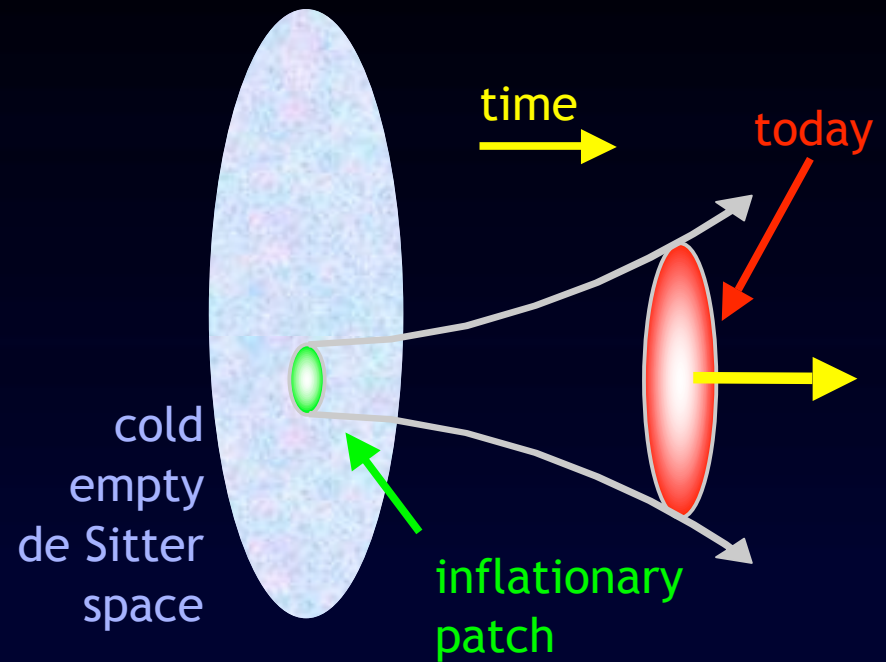
Rarely, but inevitably, fluctuating fields will conspire to create a patch of false vacuum, ready to inflate.

A **baby universe** will pinch off from the background spacetime, expanding and creating more entropy.

We're replacing this:



with this:

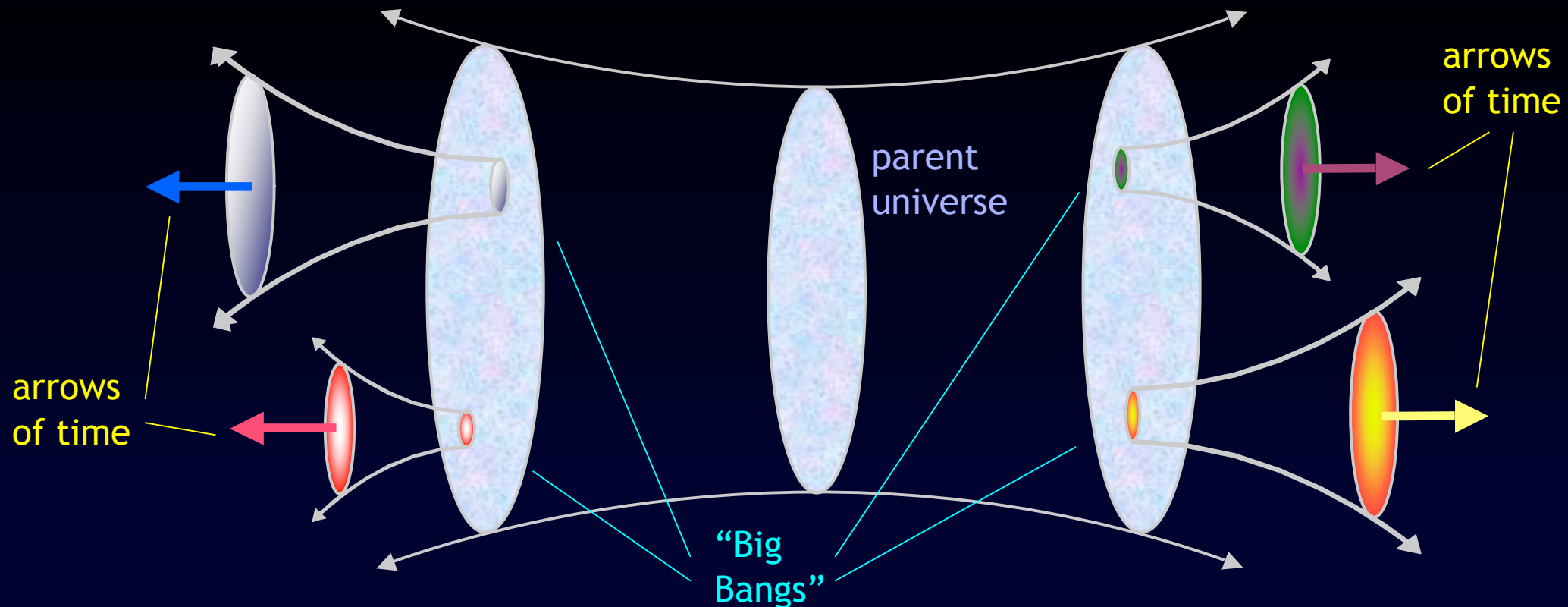


Why bother?

The point is that cold empty space is not a finely-tuned initial condition; it's high-entropy, generic, "natural."



Bonus: This story can be told in both directions in time.



Evolving empty space to the past, we would also see baby-universes created; their arrow of time would be reversed with respect to ours. The multiverse can be perfectly time-symmetric; we just don't see all of it.

[Carroll & Chen, 2004; cf. Aguirre & Gratton;  
Garriga & Vilenkin; Hartle, Hawking, & Hertog]

# Why did the early universe have low entropy?

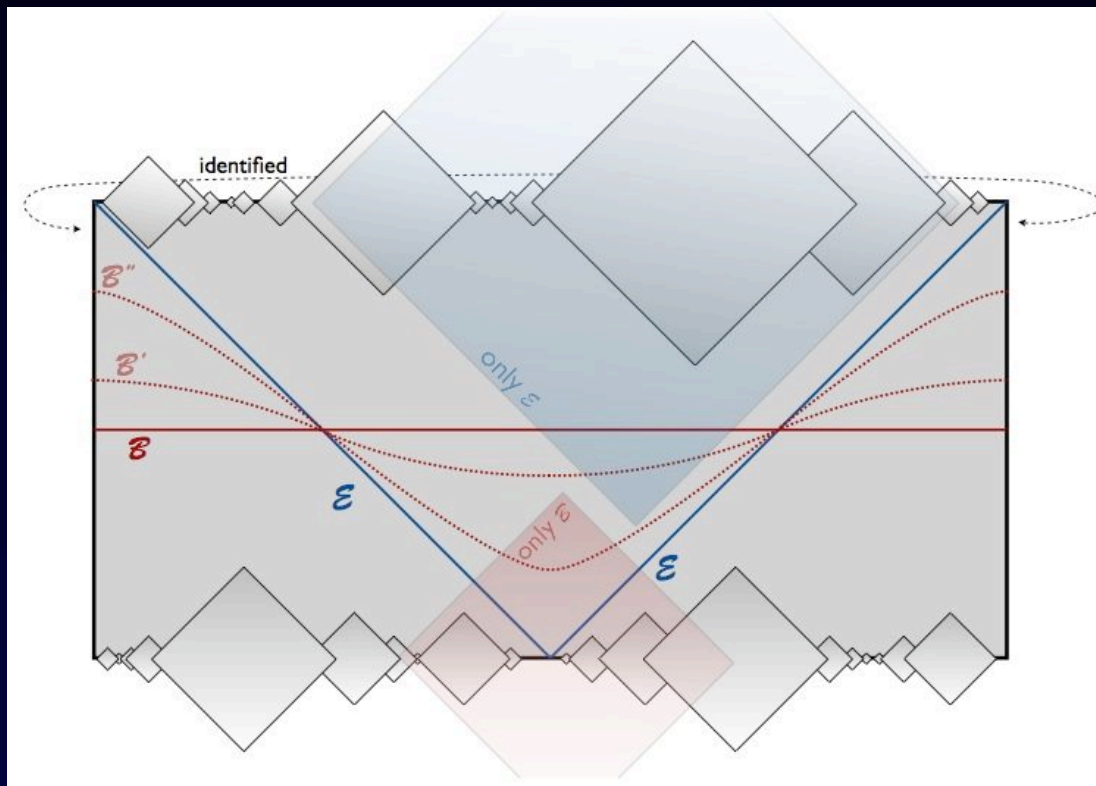
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3. Local dynamics (including inflation).  
→ cannot in principle explain low entropy.
4. Global dynamics (of the multiverse).  
→ maybe.  
state of the art needs a bit of work.

Why should we care?





# Observable remnants of the multiverse: colliding bubbles

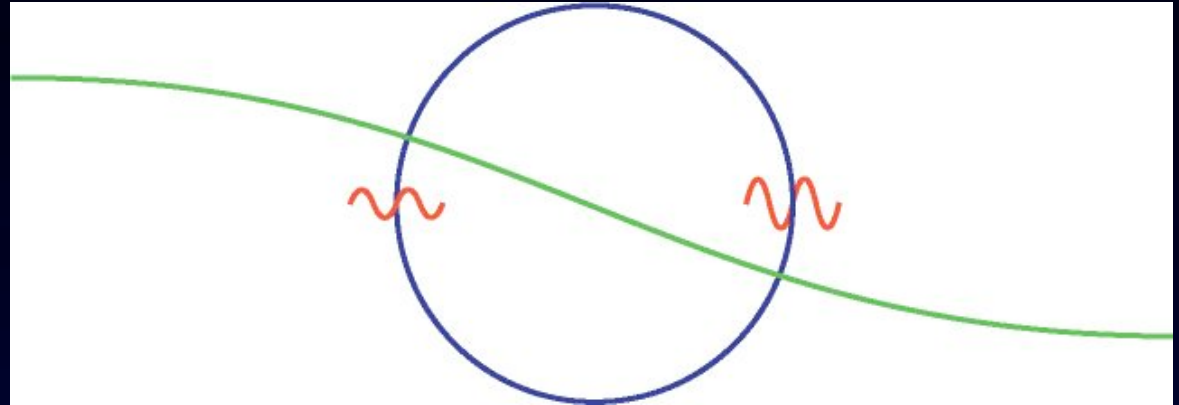


[Aguirre & Johnson; Chang, Kleban & Levi]

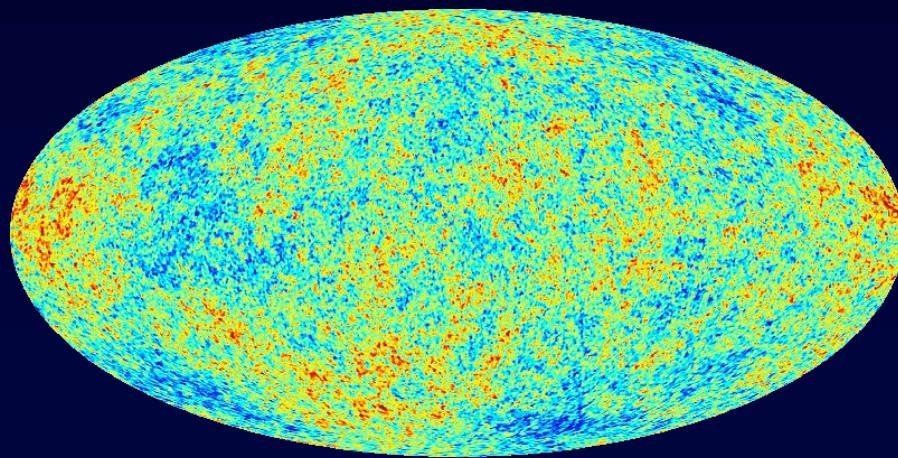
# Observable remnants of the multiverse: tilted CMB

A pre-inflationary  
**supermode** can lead  
to a hemispherical  
power modulation  
in the CMB -- which  
apparently exists!

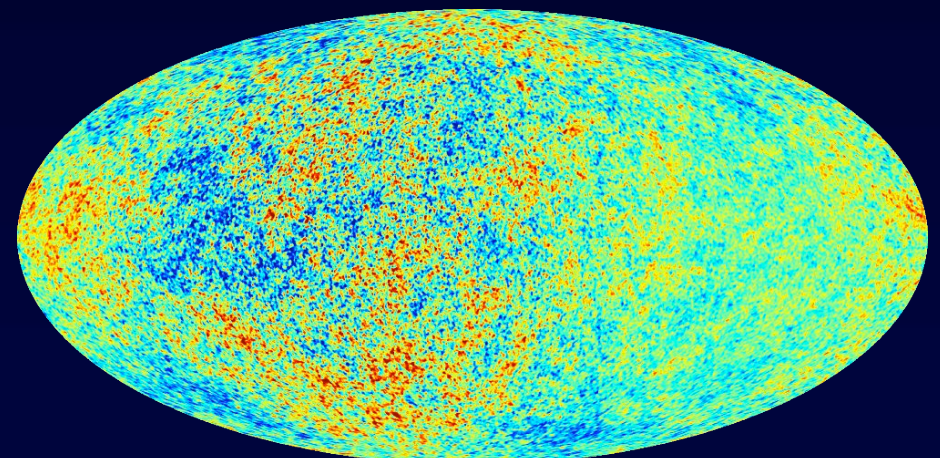
[Ericksen et al; Hansen et al]



[Erickcek, Kamionkowski  
and Carroll]



Unmodulated CMB

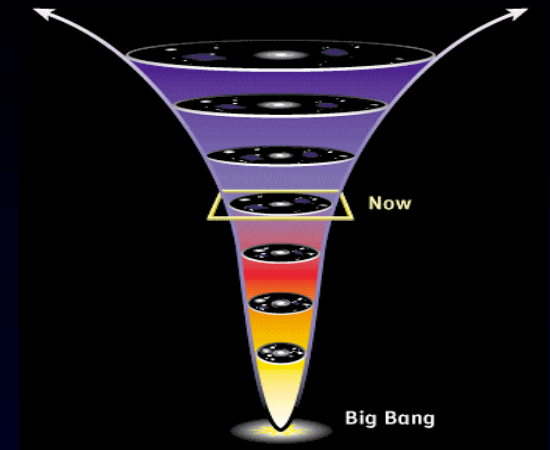
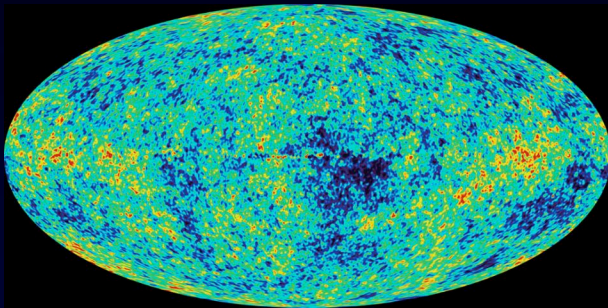


Dipole Power Modulation

[Images courtesy  
H.K. Eriksen]

## Take-home messages

The observable universe features a finely-tuned **boundary condition in the past**, responsible for the arrow of time.



**Inflation is an ingredient, but not the final answer.** Inflation can't succeed without a theory of initial conditions.

**A symmetric universe with unbounded entropy** might be the answer. But we can't really address the problem without understanding quantum gravity. And we can't understand the multiverse without confronting this problem.

