Holographic Systematics of D-brane Inflation

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Based on arXiv:0808.2811 with
D. Baumann (Harvard), A. Dymarsky (Stanford),
I.R. Klebanov (Princeton) and L. McAllister (Cornell)

(and many earlier works...)

It is an idea going back to Guth that to explain the horizon and flatness problems of cosmology, a period of early universe **inflation**

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad a(t) \sim e^{Ht}$$

would be a good thing.

Since we must eventually exit inflation, this expansion should be driven by a dynamical scalar field (not a false vacuum energy which is relaxed by a first order transition).

I. UV sensitivity in inflationary model building

The potentials which support slow-roll inflation are a little bit strange:

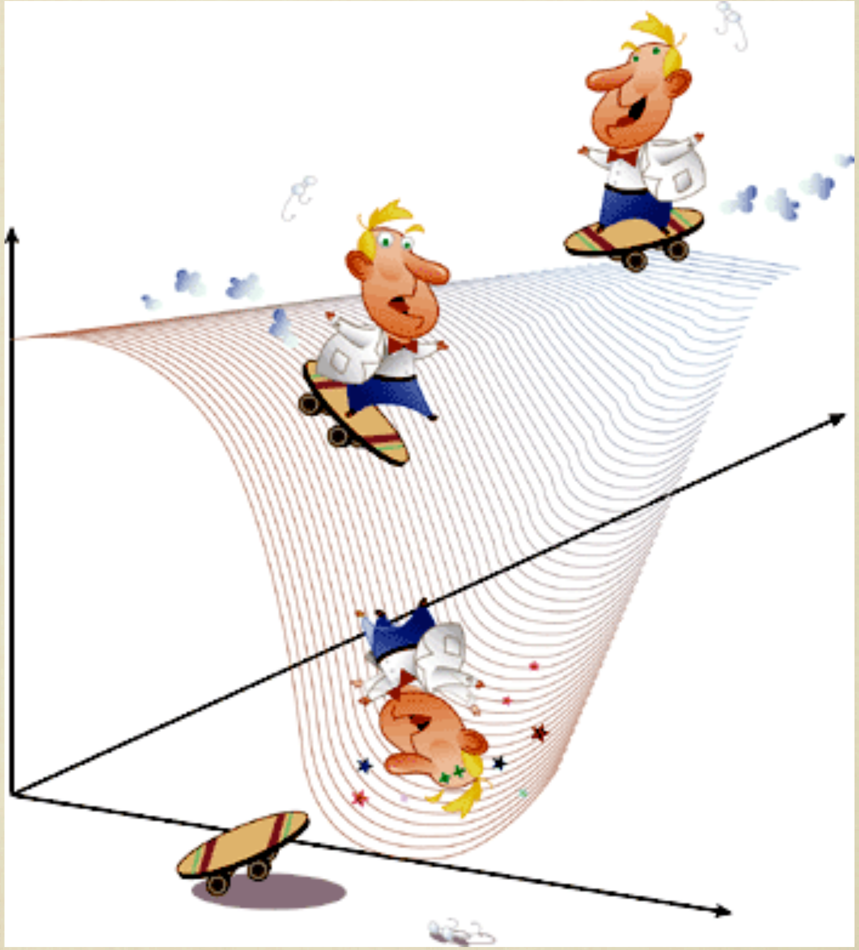
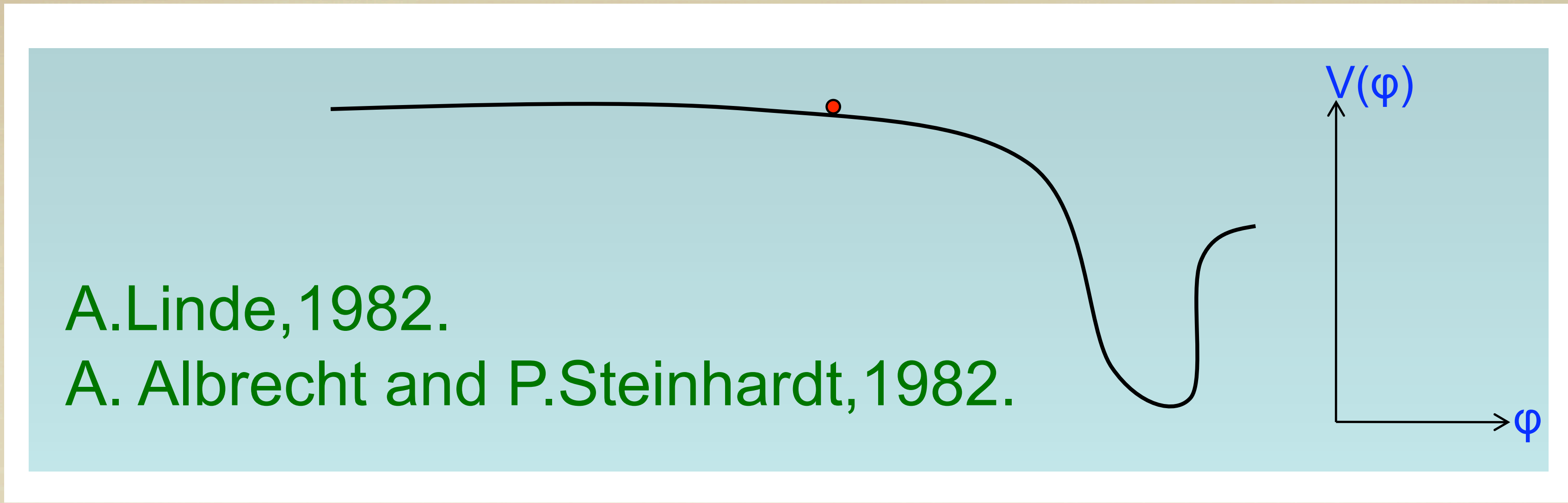


Figure stolen from A. Linde



The slow-roll conditions which ensure accelerated expansion of sufficient duration:

$$\epsilon = \frac{1}{2} M_P^2 (\frac{V'}{V})^2 << 1$$

$$\eta = M_P^2 \frac{V''}{V} << 1$$

are sensitive to dimension six, Planck suppressed corrections to the potential.

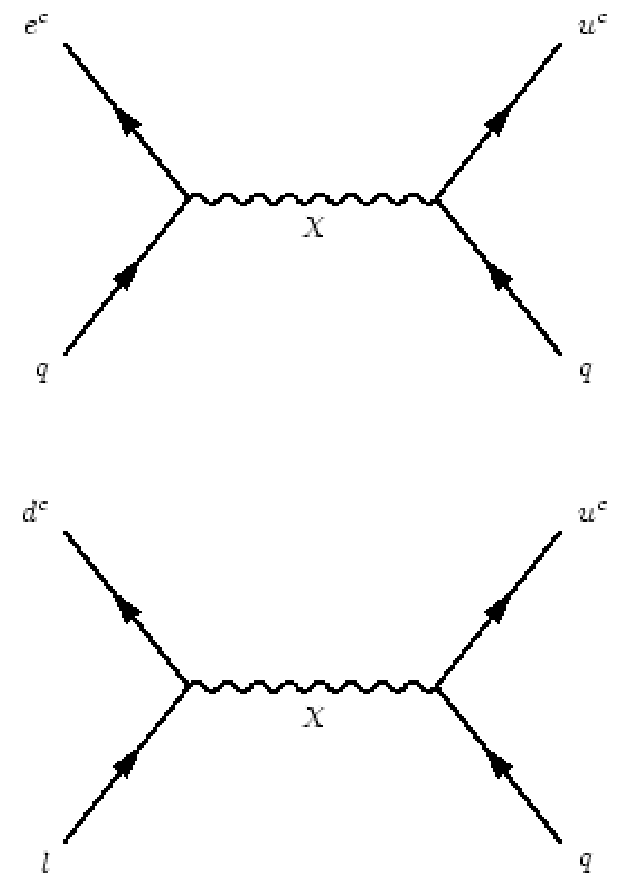
I.e., if a quantum correction induces a small shift to the potential of the form:

$$\Delta V = V_0 \frac{\phi^2}{M_P^2} \rightarrow$$

$$\Delta\eta \sim \mathcal{O}(1)$$

Therefore, one must understand all of the possible contributions to the potential to a very high degree of accuracy, to discuss inflationary model building in a reasonable way.

This degree of sensitivity to high scale physics is rare in model building. Even proton decay in GUTs only depends on dimension six, GUT scale suppressed operators:



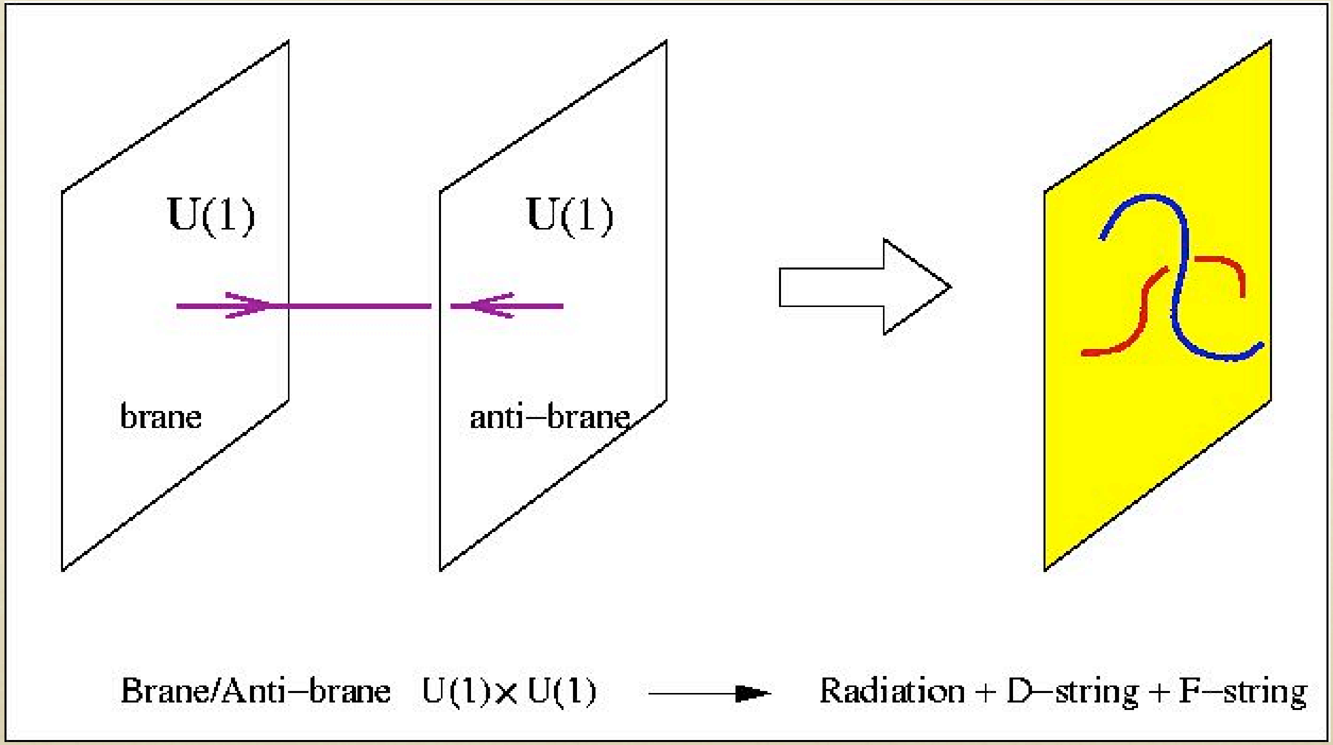
Dimension 6 proton decay mediated by the X boson $(3,2)_{-\frac{5}{6}}$ in $SU(5)$ GUT.

In this talk, we discuss in detail one way that people have tried to construct models where the relevant effects can be understood in detail and, potentially, computed in a very wide class of string theory compactifications.

Happily, these models or their close relatives also have potential striking observable signatures (low-tension cosmic string networks; Non-Gaussianity).

II. Building inflation with D-branes

One well studied class of scenarios postulates that the inflaton is the modulus controlling a brane / anti-brane separation. Its potential arises from Coulomb attraction between the oppositely charged branes:



Dvali, Tye

Burgess, Martineau, Quevedo, Rajesh, Zhang

In its earliest form, this idea suffers from the following problem. The Coulomb potential for branes separated by a distance d is

$$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3}{M_{10}^8 d^4}\right)$$

Or in terms of a canonically normalized field:

$$V(\phi) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3^3}{M_{10}^8 \phi^4}\right)$$

Then using the standard definition of the slow-roll parameters, we see that if the radius of the compactification manifold is L (this enters in determining the 4d Planck mass from the 10d one):

$$\eta = \mathcal{O}(1)(L/d)^6$$

So you run out of space in the extra dimensions, before you can separate enough to inflate!

A second problem, which is more serious, is that even if one did find a model where eta has no (very) negative eigenvalue, the Einstein-frame potential energy is basically

$$V \sim \frac{2T_3}{L^{12}}$$

This sources rapid runaway to large L, not slow roll of the brane separation mode. A similar problem typically occurs with other compactification "moduli" (e.g. the dilaton).

So we learn a general lesson:

If one wishes to inflate at some hubble scale H in string theory, it is important to give dangerous moduli a mass which is larger than H.

In particular, high scale inflation (which can generate observable B-modes) requires moduli stabilization at a very high scale.

Various ways to solve the problems of the Brane/Anti-brane inflation model have been discovered.

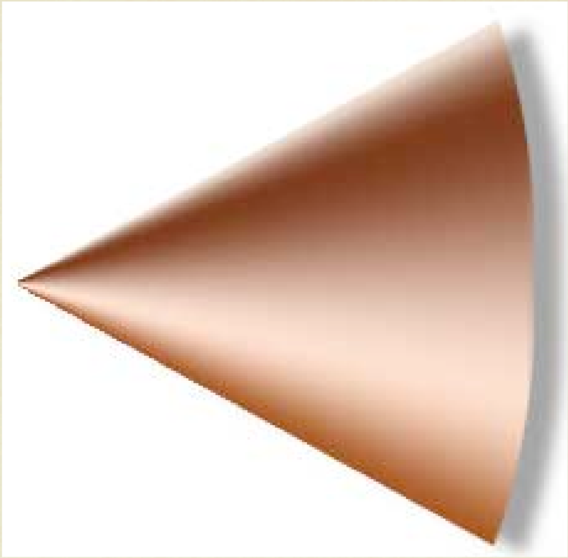
In one variant, one places the branes in a warped compactification geometry with approximate metric:

S.K., Kallosh, Linde, Maldacena, McAllister, Trivedi

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(dr^2 + r^2 ds^2_{T^{1,1}})$$

$$h(r) = \frac{27\pi}{4}\frac{1}{r^4}(\alpha')^2 g_s N$$

The picture you should have in mind for the extra dimensions is a cone with 5d base (compactified at some large value of the radial coordinate):



The tip is actually smoothed out in a way that will not be important for us.

This geometry arises in a canonical example of the AdS/CFT correspondence. It is called the (warped) conifold; the dual field theory is an N=1 supersymmetric conformally invariant gauge theory.

Klebanov, Witten; Klebanov, Strassler

The warping arises from backreaction of background gauge fluxes threading the extra dimensions. These help to stabilize the problematic moduli fields.

Gukov, Vafa, Witten; Dasgupta, Rajesh, Sethi; Giddings, S.K., Polchinski

$$W = \int_M (F - \tau H) \wedge \Omega)$$

The Coulomb attraction between a brane and an anti-brane in this warped geometry takes the form:

$$V(r_1) = 2T_3 \frac{r_0^4}{R^4} \left(1 - \frac{1}{N} \frac{r_0^4}{r_1^4}\right)$$

where:

r_0 is the anti-D location (end of throat)

r_1 is the D brane location (inflaton)

and the warped geometry naturally allows r_0 to be exponentially small.

This class of models then offers promise of evading the most basic problems outlined earlier:

* The fluxes together with other effects (e.g. non-perturbative dynamics to fix the volume modulus) allow one to fix the fast-rolling moduli.

S.K., Kallosh, Linde, Trivedi

* The warping softens the Coulomb potential enough so that at attainable brane separations, one can achieve slow roll.

Subtleties from inflaton/modulus mixing

Unfortunately, new more refined problems arise as old problems are solved.

Suppose we call the chiral multiplet containing the volume modulus

$$\rho \sim L^4.$$

Dimensional reduction on the compact Calabi-Yau manifold shows that the 4d effective theory has Kahler potential:

$$K(\rho, \bar{\rho}, \phi, \bar{\phi}) = -3 \log \left(\rho + \bar{\rho} - k(\phi, \bar{\phi}) \right)$$

A stabilization mechanism that fixes ρ (or any combination of fields other than the one appearing in the argument of the log) then generically imparts a mass to the D-brane position modes!

You can see this because the resulting supergravity potential has the form:

$$V(r,\phi) = \frac{X(\rho)}{r^\alpha} = \frac{X(\rho)}{(\rho - \phi\bar{\phi}/2)^\alpha}$$

Expanding about some vacuum for ρ

$$V = V_0 \left(1 + \alpha \frac{\phi\bar{\phi}}{2r} + ... \right).$$

This is a Hubble-scale mass for the would-be inflaton. The problem here is very analogous to the SUGRA eta problem.

Copeland, Liddle, Lyth, Stewart, Wands

III. Holographic Systematics

So far, we have written the D3-brane potential in the throat as:

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + H^2\phi^2$$

* The first term includes the brane tensions and the Coulomb attraction to the anti-D3

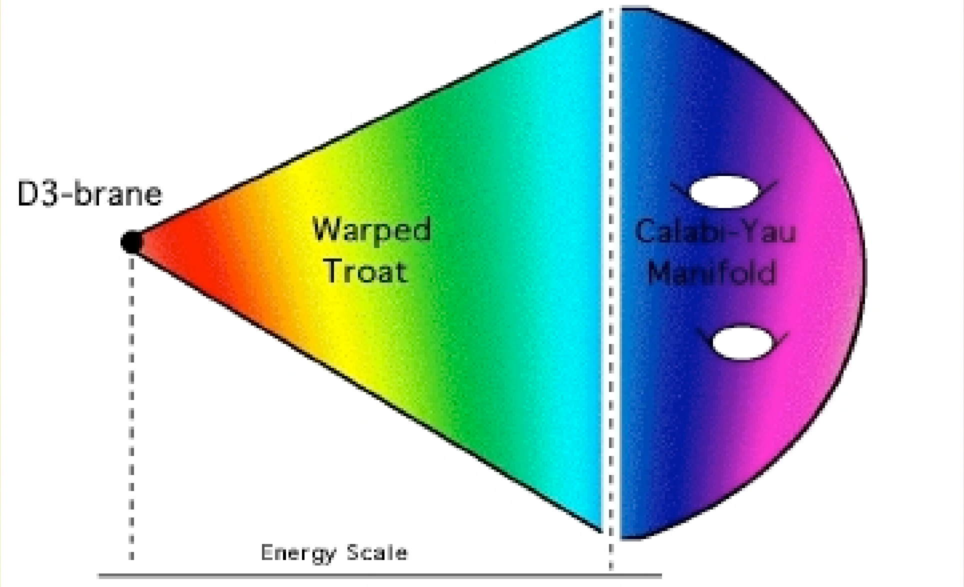
* The second term $H^2\phi^2$ arises from the minimal effect of stabilizing the volume of the extra dimensions, as we just described.

But the true formula for V should look like this:

$$V = V_{D3/\overline{D3}} + H^2\phi^2 + \Delta V$$

where the last term includes all of the corrections to the potential that come from embedding the system into a compact Calabi-Yau space.

How on earth can we estimate the effects that contribute to something so general?



We know the explicit solution for the warped throat region. Such a region could arise (and rather frequently does) as part of many distinct compact Calabi-Yau spaces with different bulk fluxes, brane contents, etc.

There is a completely systematic way to estimate the leading corrections to the throat solution!

We'll first describe this in the throat gravity solution, then use AdS/CFT duality to describe it in a dual gauge theory.

A. The perturbed throat geometry

The Type IIB string theory has a metric and various p-form antisymmetric tensor fields (generalized gauge fields).

However, the Dirac-Born-Infeld action of a D3-brane moving in a IIB background couples to a very specific combination of background fields as a potential. If:

$$ds^2 = e^{2A}(y) g_{\mu\nu} dx^\mu dx^\nu + e^{-2A}(y) \tilde{g}_{mn}(y) dy^m dy^n$$

(where the unwarped compact metric is Calabi-Yau) and if the 5-form field strength

$$F_5 = (1 + \star_{10})[d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3]$$

then the D3-brane "sees" a potential:

$$V = T_3 \left(e^{4A} - \alpha\right)$$

So, we need to understand the leading possible perturbations to the object

$$\Phi_- = e^{4A} - \alpha$$

induced by coupling the throat to a compact Calabi-Yau geometry.

[Note that e.g. the potential attracting the D3 to the anti-D3 can be understood in this language as well; we will not go through that exercise here.]

DeWolfe, S.K., Mulligan

Our leading-order throat solution involves compactification on the Calabi-Yau cone over $T^{1,1}$. Luckily, the KK harmonics of the Φ_- field on this space have been solved for by Ceresole, Dall'Agata, D'Auria and Ferrara.

Their results (after some massaging) yield:

$$\Phi_-(r,\Psi) = \sum_{L,M} \Phi_{LM} \left(\frac{r}{r_{\rm UV}}\right)^{\Delta(L)} Y_{LM}(\Psi) + c.c.\,,$$

where $L = (J_1, J_2, R),\ M = (m_1, m_2)$ label $SU(2) \times SU(2) \times U(1)$ global quantum numbers under the isometries of $T^{1,1}$ (and Ψ represents the 5 angular variables there).

THE DELTA VALUES THAT APPEAR ARE:

$$\Delta \equiv -2 + \sqrt{6\Big[J_1(J_1+1) + J_2(J_2+1) - R^2/8\Big] + 4}\,.$$

AND GROUP-THEORETIC SELECTION RULES RESTRICT WHICH QUANTUM NUMBERS ARE ALLOWED.

THIS ALL LOOKS COMPLICATED BUT THE UPSHOT IS VERY SIMPLE.

* THE LOWEST EIGENVALUES THAT ARE ALLOWED BY SELECTION RULES HAVE

$$(J_1, J_2, R) = (1/2, 1/2, 1), (1, 0, 0), (0, 1, 0) \rightarrow \Delta = 3/2, 2, 2$$

So, there are two cases to consider:

"Fractional" case:

$$\Delta = 3/2 \text{ perturbation present}$$
$$\rightarrow \Phi_-(r) \sim \left((\frac{r}{r_{UV}})^{3/2} + ... \right)$$

"Quadratic" case:

$$\Delta = 2 \text{ is leading perturbation present}$$
$$\rightarrow \Phi_-(r) \sim \left((\frac{r}{r_{UV}})^{2} + ... \right)$$

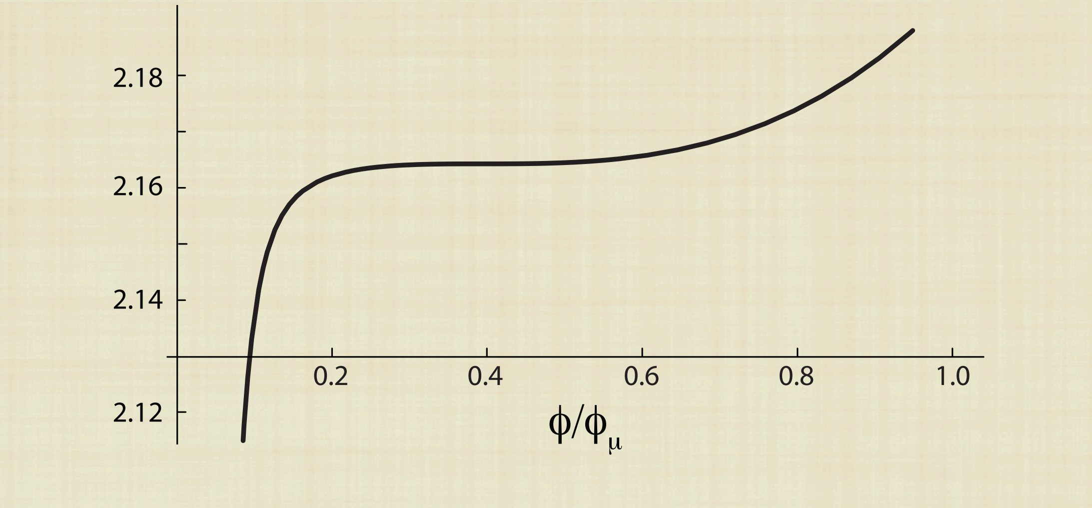
Obviously, the first case is generic. However, the second possibility is also quite realistic, because unbroken global symmetries of the full compact manifold can forbid the fractional case.

The phenomenology in these two cases is very different.

In the "fractional" case, one needs to inflate in a potential of the form:

$$V(\phi) = V_0 \left[1 + \frac{1}{3} \left(\frac{\phi}{M_{\rm pl}} \right)^2 - c_{3/2} \left(\frac{\phi}{M_{\rm UV}} \right)^{3/2} \right]$$

to obtain inflation, one needs to inflate near an inflection point.



This is very similar to models that were studied in the very specific case that D7-branes (generating non-perturbative corrections to the potential) stretched down the throat:

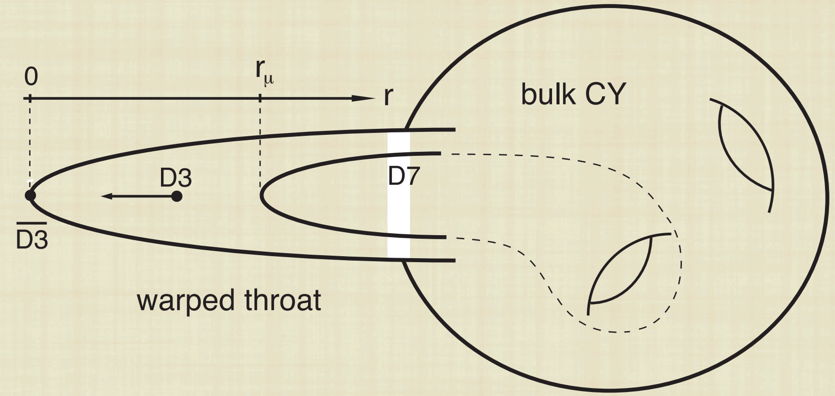


Figure 1: Cartoon of an embedded stack of D7-branes wrapping a four-cycle Σ_4, and a mobile D3-brane, in a warped throat region of a compact Calabi-Yau. The D3-brane feels a force from the D7-branes and from an anti-D3-brane at the tip of the throat.

Baumann, Dymarsky, Klebanov, McAllister, Steinhardt; Krause, Pajer

However we have seen that the form of the potential is generic (even without D7s in the throat).

The tuning required to get an inflection point in the regime where one trusts the throat geometry looks mild, $\mathcal{O}(\frac{1}{100})$

In the "quadratic" case, things are rather different.

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + H^2 \underbrace{\left[1 - c_2\left(\frac{M_{\rm pl}}{M_{\rm UV}}\right)^2\right]}_{\equiv\, \beta} \phi^2$$

One tries to tune $\beta < 1$ to achieve inflation. This was the original class of warped models; the phenomenology was studed in detail by Firouzjahi and Tye.

In both classes of models, one can obtain a red spectrum with very small tensor modes easily. Cosmic Strings may be present, with

$$10^{-12} < G_N \mu < 10^{-8}$$

Sarangi, Tye;
Copeland, Myers, Polchinski

B. Dual gauge theory perspective

For the experts, I briefly describe why these results are obvious from the perspective of the dual gauge theory.

String theory on the conical geometry we studied, is dual to an N=1 supersymmetric gauge theory with

$$SU(N) \times SU(N)$$

gauge group, and bi-fundamental matter fields

$$A_{1,2}, B_{1,2}$$

governed by a superpotential:

$$W = h\epsilon^{ij}\epsilon^{kl}A_iB_kA_jB_l$$

Klebanov, Witten

The leading perturbations when we couple the throat to a compact Calabi-Yau are going to be operators of the form:

$$\Delta\mathcal{K} = c \int d^4\theta \; M_{\rm UV}^{-\Delta} \; X^\dagger X \, \mathcal{O}_\Delta \qquad \Rightarrow \qquad \Delta V = c \; M_{\rm UV}^{-\Delta} |F_X|^2 \, \mathcal{O}_\Delta \,,$$

A few lines of argument suggests that in KKLT-type models, the scale of bulk F-terms then gives effects of the size we mentioned, for the specific cases:

"Fractional": $\mathcal{O}_{3/2} = \mathrm{Tr}\,(A_i B_j) + c.c.\,,$

"Quadratic": $\mathcal{O}_2 = \quad \mathrm{Tr}\left(A_1 \bar{A}_2\right) \;, \quad \mathrm{Tr}\left(A_2 \bar{A}_1\right) \;, \quad \frac{1}{\sqrt{2}}\mathrm{Tr}\left(A_1\bar{A}_1 - A_2\bar{A}_2\right)$

I would like to close by stressing three important points:

i) The quadratic operators (in this case) are lowest components of symmetry currents of the throat. That is why their dimension is protected and stays at 2 even at strong coupling. Such currents are present in all of the known throats, though there could well be throats without them that we haven't written down yet.

ii) The fractional operator is easily forbidden by a discrete symmetry, e.g.

$$A_i \to -A_i$$

It is easy to find compact Calabi-Yau spaces with a conifold throat that would preserve this symmetry; inflation in these cases would fall into the quadratic case.

iii) Our analysis immediately generalizes to infinite classes of throat geometries, given sufficient information about the KK spectrum. For instance, as we discuss in the paper, we expect the full infinite class of $Y^{p,q}$ cones falls into the quadratic case.

(I say "expect" because this relies on the assumption that no un-protected non-chiral operators have $\Delta < 2$).