Is there eternal inflation in the Cosmic Landscape ?

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hep-th/0611148
ArXiv:0708.4374 [hep-th]
with Qing-Guo Huang, ArXiv:0803.0663 [hep-th]
with Dan Wohns, to appear

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Flux compactification in Type IIB string theory

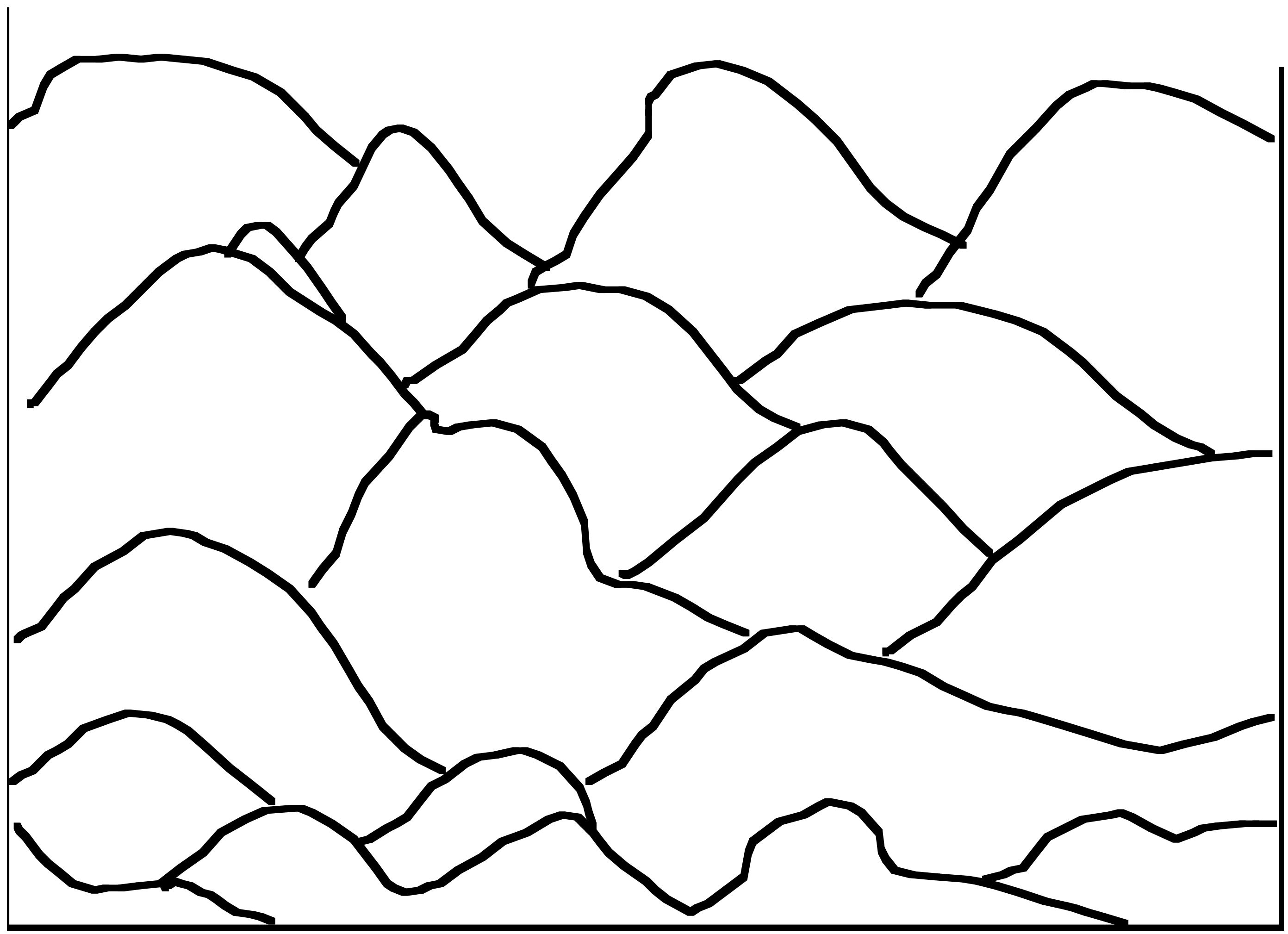
where all moduli of the 6-dim. "Calabi-Yau" manifold are stabilized

- There are many classically meta-stable manifolds/vacua, 10^{500} or more, probably infinite, with positive, zero, as well as negative cosmological constants.
- There are many (tens to hundreds or more) moduli (scalar modes) that are dynamically stabilized.

KKLT vacua

Giddings, Kachru, Polchinski, Kachru, Kallosh, Linde, Trivedi and many others, 2001....

a cartoon :

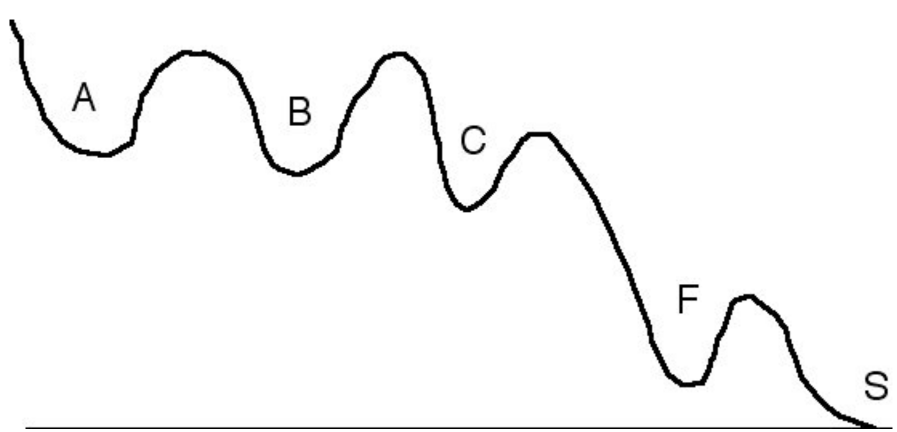


Landscape : looks like
a random potential in multi-dimensions

Eternal Inflation

$$a(t) \simeq e^{Ht} \to V = a(t)^3 \simeq e^{3Ht}$$

Suppose the universe is sitting at a local minimum, with a lifetime larger than the Hubble time 1/H.
Then the number of Hubble patches will increase exponentially. Even after some Hubble patches have decayed, there would be many remaining Hubble patches that continue to inflate.



Eternal inflation implies that somewhere in the universe (outside our horizon), inflation is still happening today.

Pros and cons of eternal inflation

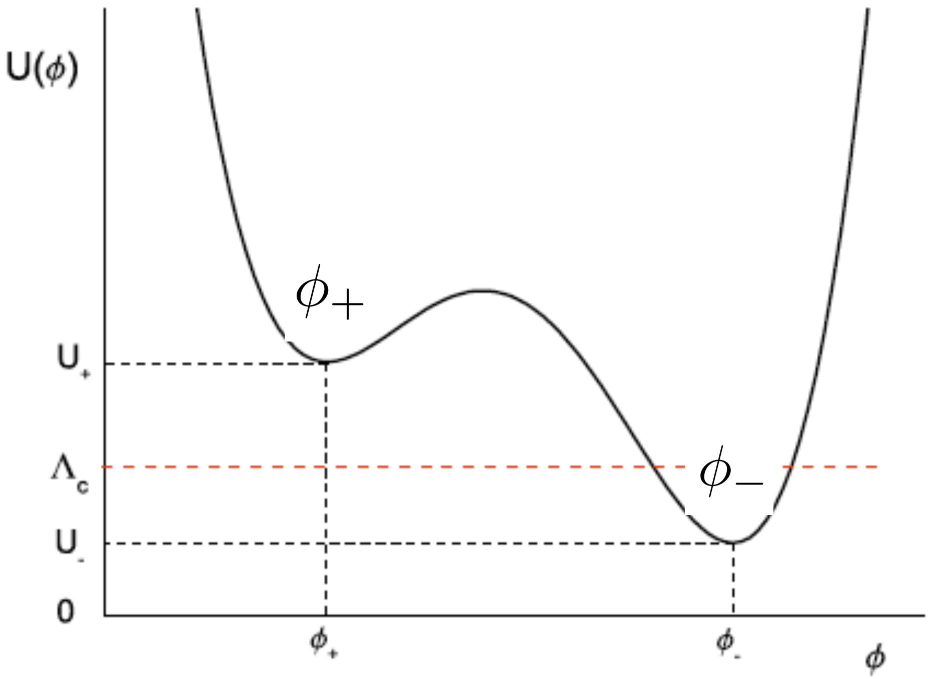
Pros : no matter how small is the tunneling rate from one site to another, it will happen. So the cosmic landscape is populated. In particular, the site we are living in is present somewhere in the universe.

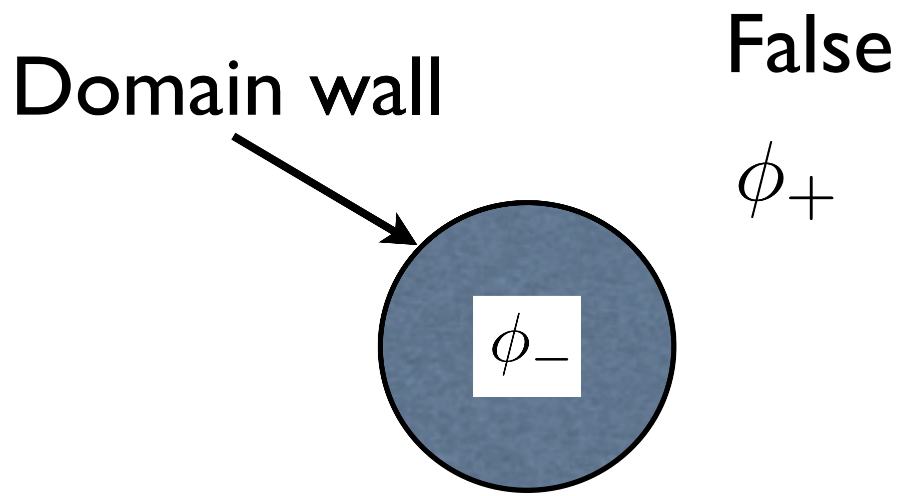
Cons : since there are many (10^{500} or more) vacua, and some parts are still inflating, it is difficult to see why we end up where we are without invoking some strong version of the A principle.
It is very difficult to make any testable prediction.

The scenarios with or without eternal inflation are very different.

Coleman-de Luccia Tunneling

$$\Gamma(CDL) \sim e^{-B}$$





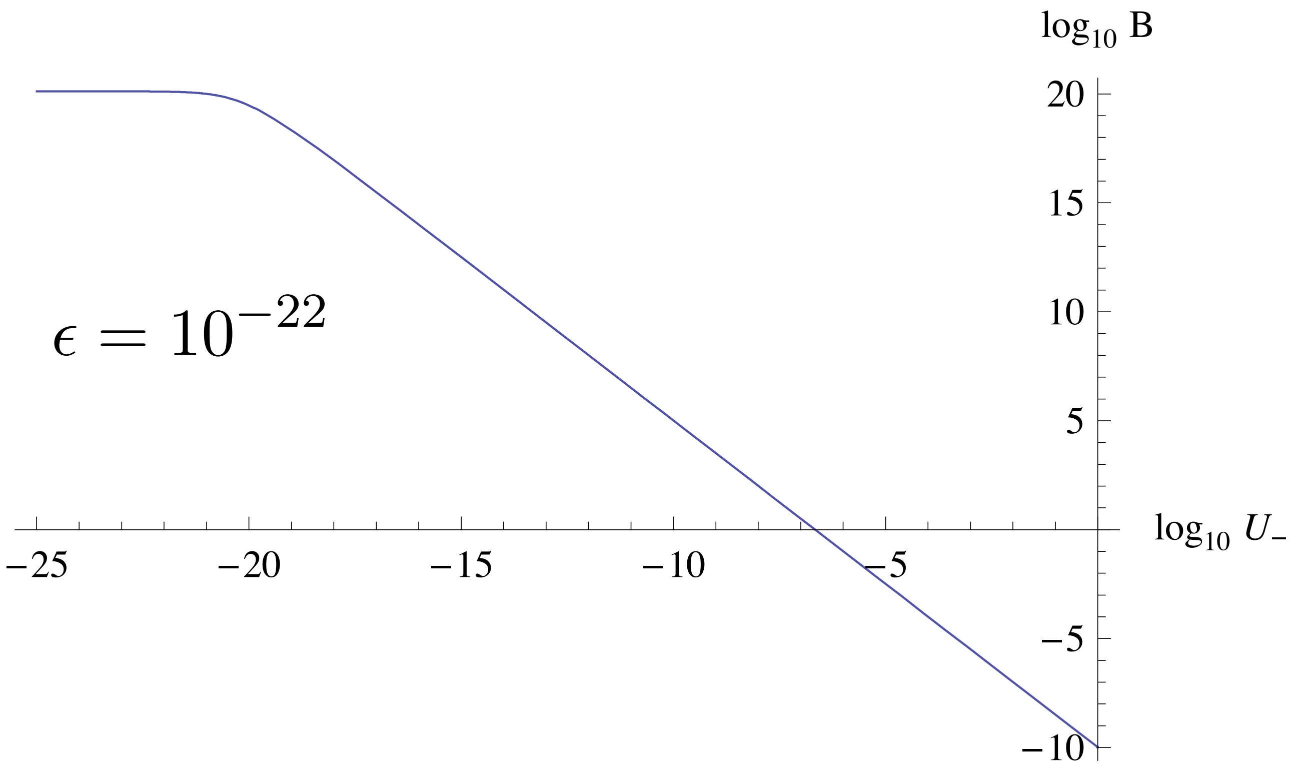
$$\epsilon = U(\phi_+) - U(\phi_-)$$

In Coleman-de Luccia Tunneling

$$\Gamma(CDL) \sim e^{-B}$$

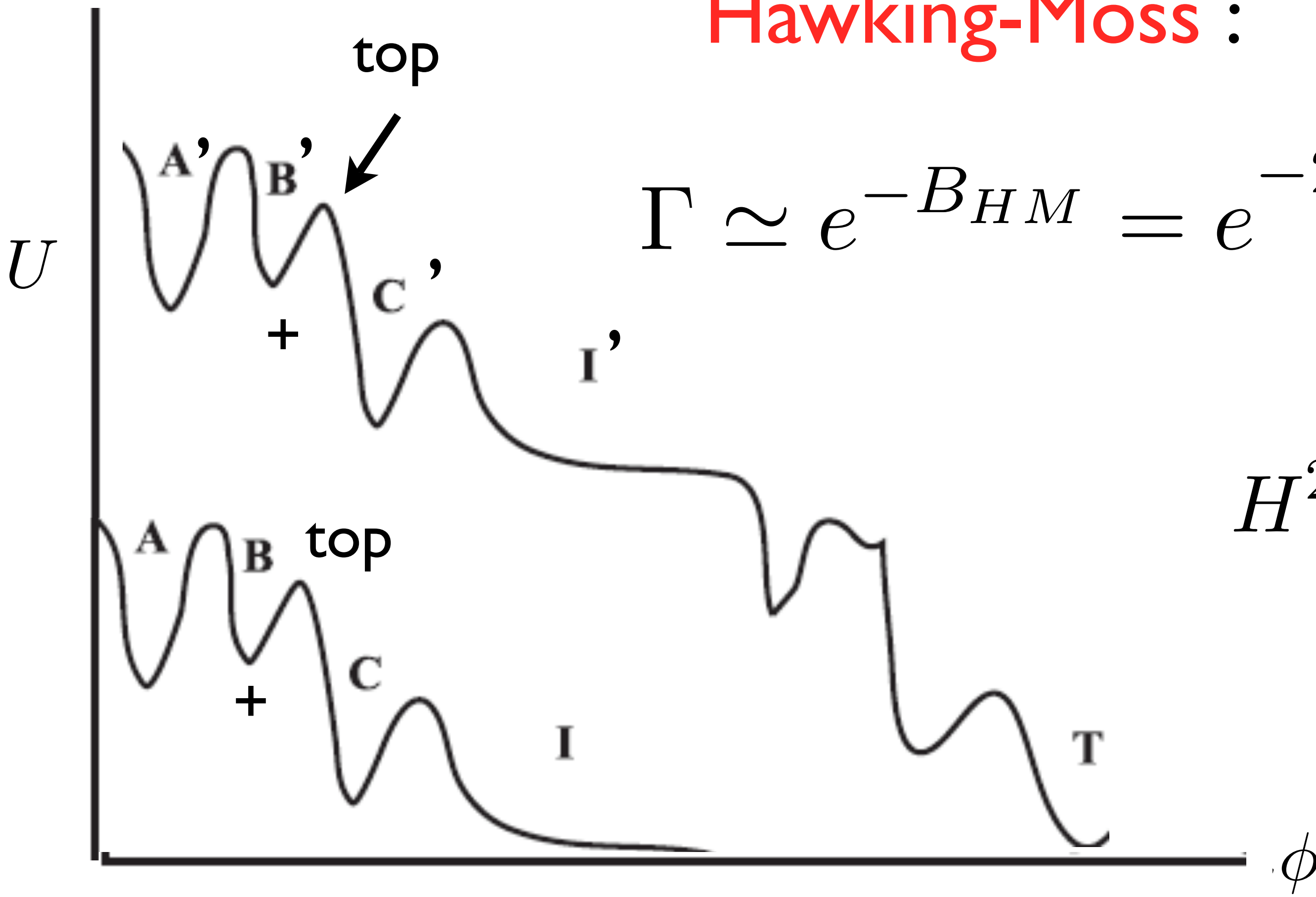
In the thin wall approximation :

$$\tau = 10^{-12}, \qquad \epsilon = 10^{-22}$$



for fixed domain wall tension τ and
(false-true) energy density difference $\epsilon = U(\phi_+) - U(\phi_-)$

Tunneling is much faster at high C.C.



Hawking-Moss :

$$\Gamma \simeq e^{-B_{HM}} = e^{-24\pi^2[\frac{M_P^4}{U(\phi_+)} - \frac{M_P^4}{U(\phi_t)}]}$$

$$H^2 = U/3M_P^2$$

$$B_{HM} = \frac{8\pi^2}{3}\frac{\delta U}{H_+^2 H_t^2}, \qquad \delta U = U(\phi_t) - U(\phi_+)$$

Hawking-Moss

Coleman-de Luccia Tunneling

$$B_{HM} = \frac{8\pi^2}{3}\frac{\delta U}{H_+^2 H_t^2}$$

$$B \simeq \frac{27\pi^2\tau^4}{2\epsilon^3} \rightarrow \frac{2\pi^2\tau}{H^3}$$

Brown and E.Weinberg
arXiv: 0706.1573

Interpret this gravitational effect as a finite (Hawking) temperature effect.

$$\Gamma \sim e^{-E/T_H}$$

$$T_H = H/2\pi$$

$$E/T_H \rightarrow \frac{8\pi^2\tau}{H^3}$$

S^3 with size $2\pi^2$ **vs** $S^2 \times S^1$ with size $4\pi \times 2\pi = 8\pi^2$

$$\sim A_1 e^{-B} + A_T e^{-4B}$$

Finite Hawking temperature effect

$$U(\phi_i) \text{ should be replaced by } U(\phi_i, T)$$

Expanding about the top of a symmetric barrier :

$$U(\phi, T=0) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

$$U(\phi, T_H) = \frac{1}{2}\left(\frac{\lambda T_H^2}{24} - m^2\right)\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots + H^2\phi^2$$

d ~100

No trapping if for any modulus :

$$T_H^2 > \frac{24 m_i^2}{\lambda_i}, \qquad i = 1, 2, \ldots., d$$

A comment :

- Tunneling of a D3-brane can be significantly enhanced by the Dirac-Born-Infeld action.

CDL :

Brown, Sarangi, Shlaer and Weltman

Hawking-Moss :

Tolley and Wyman
Wohns

Harder to trap in higher dimensions

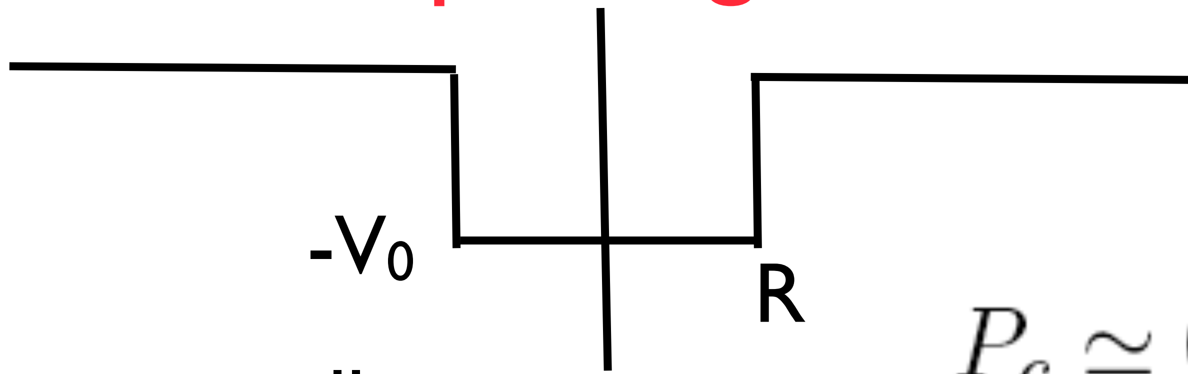
$$\eta(r)'' + \left(2m[E - V(r)] - \frac{(d-1)(d-3)}{4r^2}\right)\eta(r) = 0$$

Repulsive

A 1-dim. attractive delta-function potential always has a bound state but not a 3-dim one.

$$\psi(r) = r^{-(d-1)/2}\eta(r)$$

Harder to trap in higher dimensions

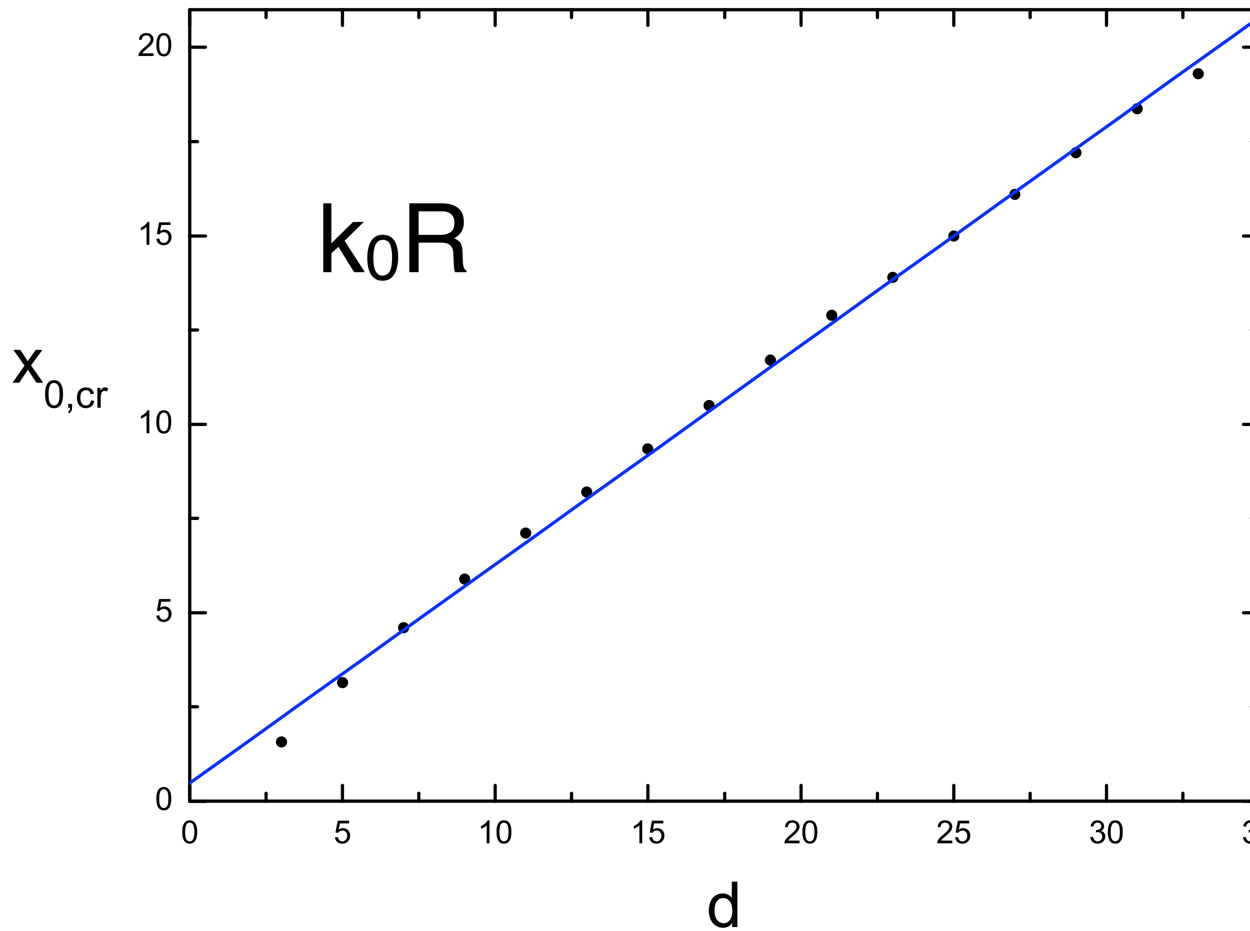


$$P_c \simeq 0.58d + 0.5$$

Spherical square well :

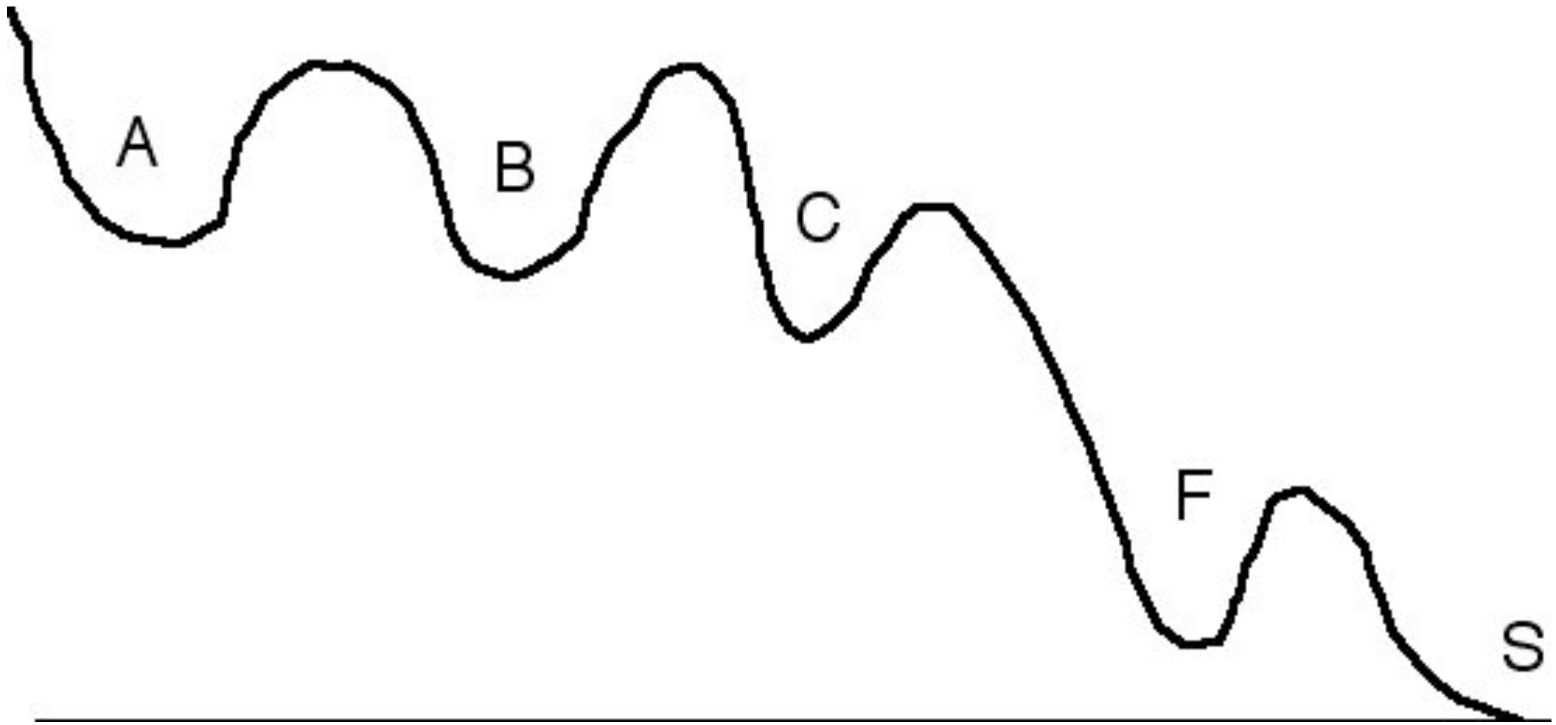
$$V(r) = -V_0 \quad r < R$$
$$= 0 \quad r > R$$

$$k_0^2 = 2mV_0$$



$$\psi(r) \sim e^{-ar}$$

$$a^2 = 2m|E|$$



Resonance Tunneling in QM:

Tunneling probabilities $T_{A\rightarrow B} \sim T_{B\rightarrow C}$ are exponentially small

When the condition is right : $T_{A\rightarrow C} \sim 1$

Resonance tunneling in QFT : Saffin, Padilla and Copeland
Sarangi, Shiu, Shlaer

Tunneling from a typical meta-stable site below the Planck (or string) scale.

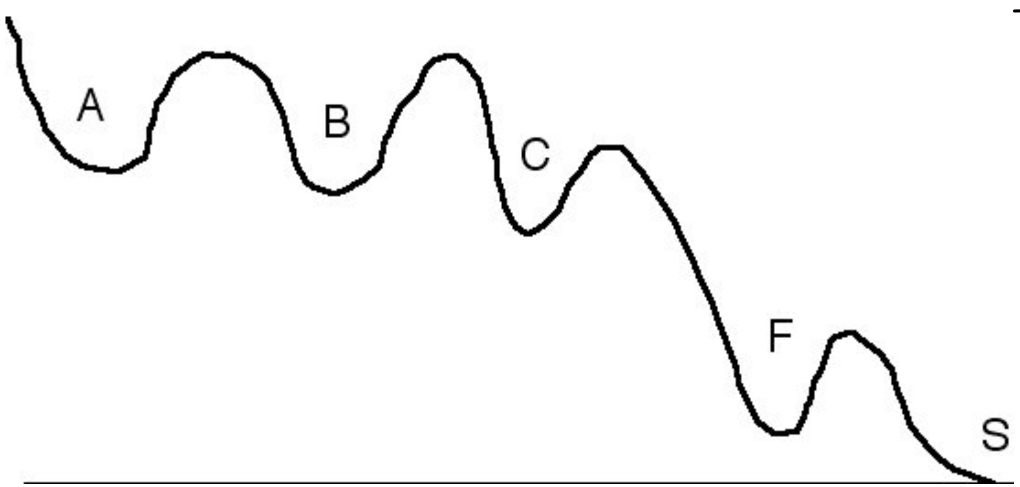
Why fast tunneling is possible ?

Naive

$$T_{A\rightarrow B} = T_{B\rightarrow C} = T_0$$

$$T_{A\rightarrow C} = T_0/2$$

$$T(n) \simeq T_0/n$$



In a d-dimensional hyper-cubic lattice

naive : $$\Gamma_t^{nr} \sim 2d\,\Gamma_0$$

$$\text{time for one e-fold of inflation is Hubble time } 1/H$$

So the lifetime of a typical site is longer than the Hubble scale, and eternal inflation seems unavoidable.

actually : $$\Gamma_t \sim n^d\,\Gamma_0$$

$n \sim 1/Hs$

For large enough *d* (and maybe *n*) the tunneling can be fast.

Is there eternal inflation ?

- Hawking temperature makes it harder to trap the wave function of the universe at a classical local vacuum site in the landscape.
- The presence of many moduli (tens to hundreds) makes it harder to trap the wavefunction.
- Even if trapped, both effects shorten the decay time of any metastable site in the landscape.
- Resonance tunneling effects will further shorten the decay time of any metastable site.
- Treating the cosmic landscape as a random potential, one can borrow (renormalization group) techniques developed in condensed matter physics to argue that there is no eternal inflation in the landscape until one reaches an exponentially small CC site.

The vastness of cosmic landscape

- At a typical meta-stable site, count the number of parameters or the number of light scalar fields. This gives the number of moduli, or directions in the field space.
- This number at any neighborhood in the landscape may be taken as the dimension d of the landscape around that neighborhood.
- The landscape potential is not periodic (though it may be close to periodic along some axion directions). It is very complicated.

The wavefunction of the universe

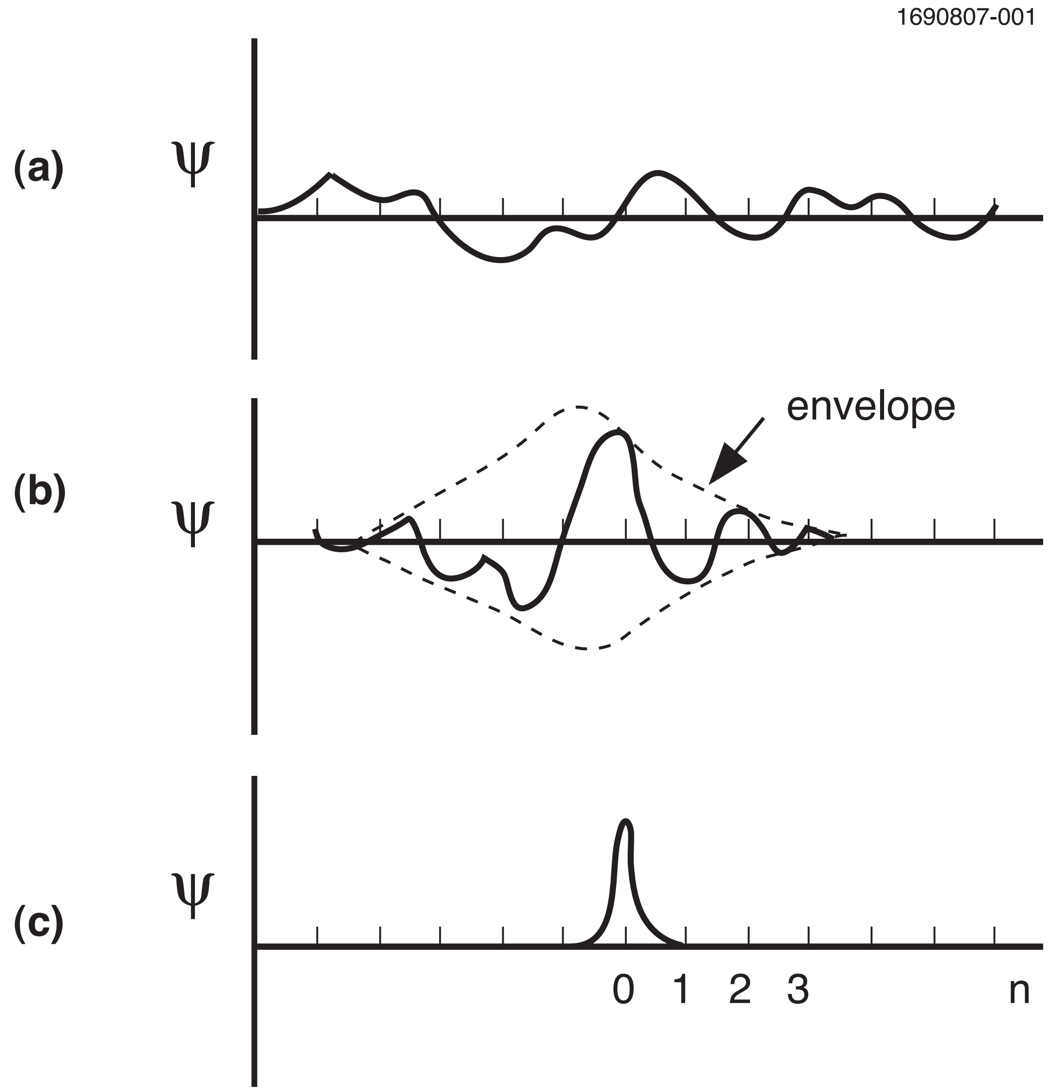
may be crudely approximated by that of a D3-brane

moduli open string modes

$$\Psi(a, \ldots, \varphi_j, \phi_i, \ldots)$$

cosmic scale factor

The behavior of the wavefunction in the landscape is a quantum diffusion and percolation problem.



$$|\psi(\mathbf{r})| \sim \exp(-|\mathbf{r} - \mathbf{r}_0|/\xi)$$

, $\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$

Scaling theory :

(A renormalization group approach)

for a random potential (disordered medium)
in d dimensions

Generic decay rate to nearest neighbor in the landscape :

d ~ 100

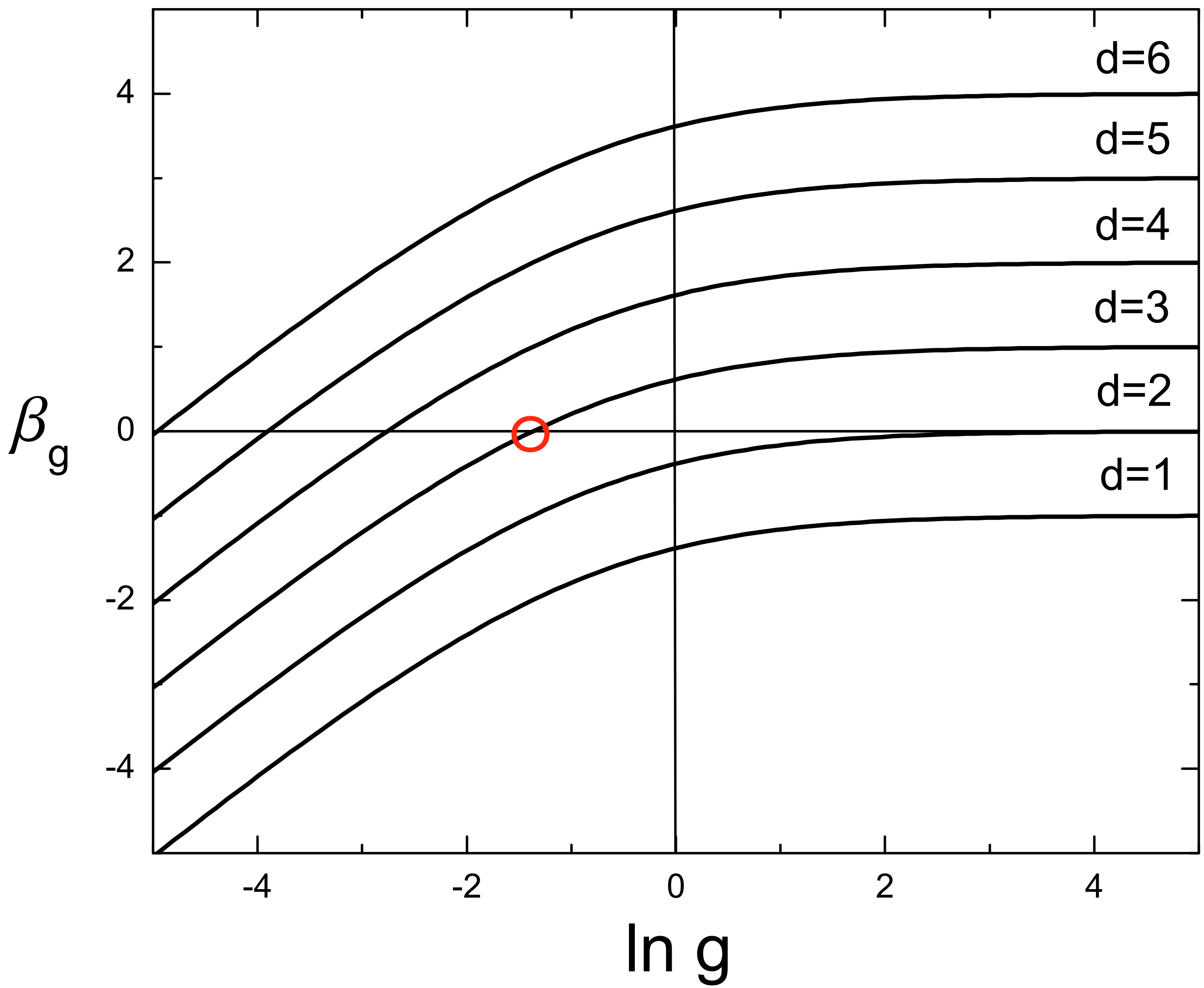
Mobility condition :

$$\Gamma_0 > e^{-2(d-1)}$$

No mobility in d=1,2.

Mobility in the landscape implies no eternal inflation.

$$\beta_d(g_d(L)) = \frac{d \ln g_d(L)}{d \ln L}$$

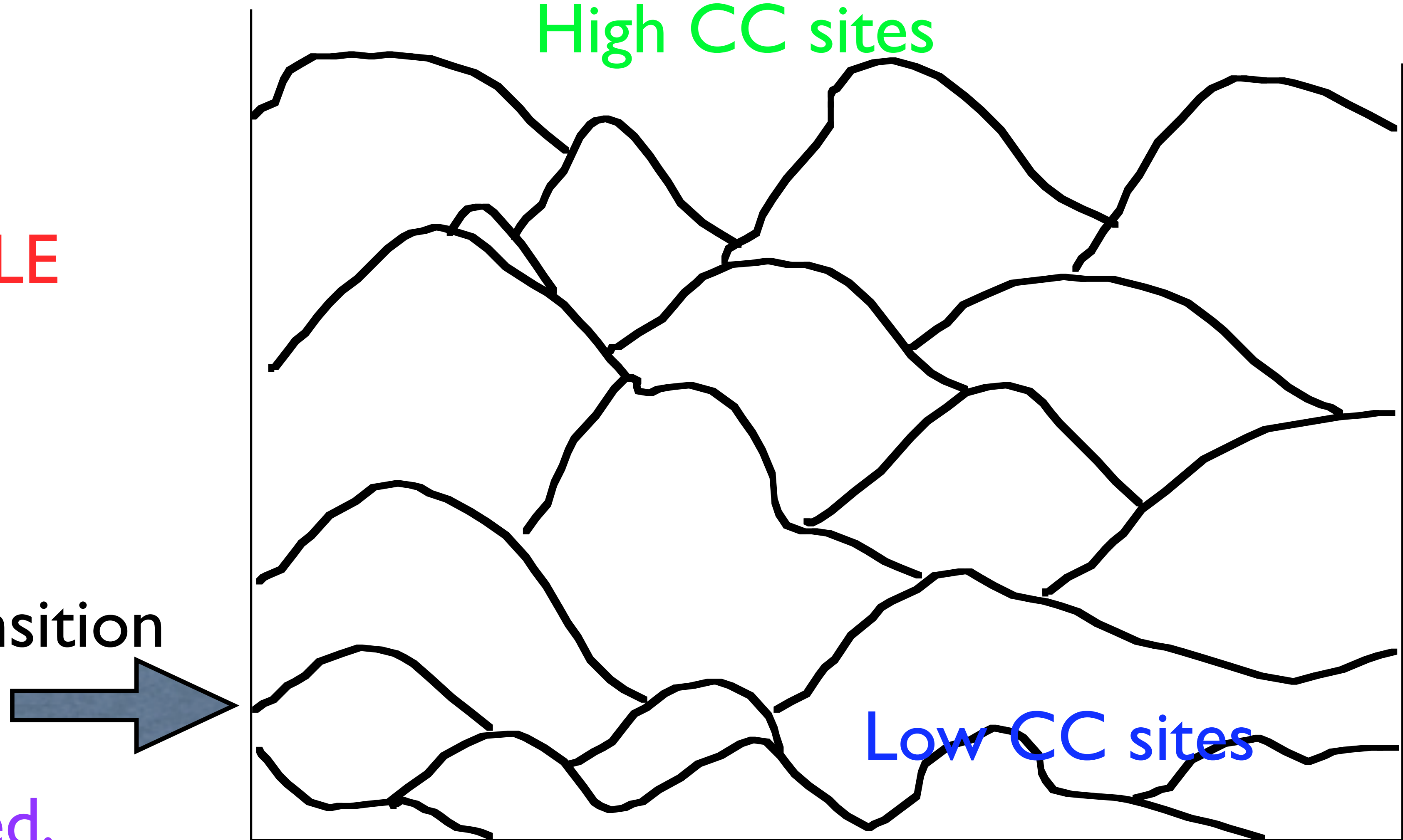


The picture from the scaling theory

MOBILE

Sharp transition

trapped,
exponentially
long lifetimes



$$\Lambda_c \sim d^{-d} M_s^4$$

Summary

- When high up in the landscape, Hawking temperature due to the vacuum energy provides a finite temperature effect to destabilize local classical vacua (minima) in the cosmic landscape.

- The presence of many moduli (tens to hundreds) makes it harder to trap the wavefunction. The wavefunction has many directions to spread.

- The presence of many vacua allows resonance tunneling to come into play, shortening the decay time of any metastable site.

- Treating the cosmic landscape as a random potential, one argues that the wavefunction of universe is mobile in the landscape, so there is no eternal inflation in the landscape until one reaches an exponentially small CC site.

The cosmic landscape is a quantum landscape.

Remark :

We present a strong argument that eternal inflation is improbable in the cosmic landscape.

For those whose work is based on eternal inflation, it is important to look closely at this fundamental assumption that eternal inflation is generic, probable, or even possible in the cosmic stringy landscape.