

Gravity Waves

and

Linear Inflation from
Axion Monodromy

E.S., A. Westphal '08

- McAllister, E.S., Westphal '08

Gravity Waves from Monodromies

tensor/scalar

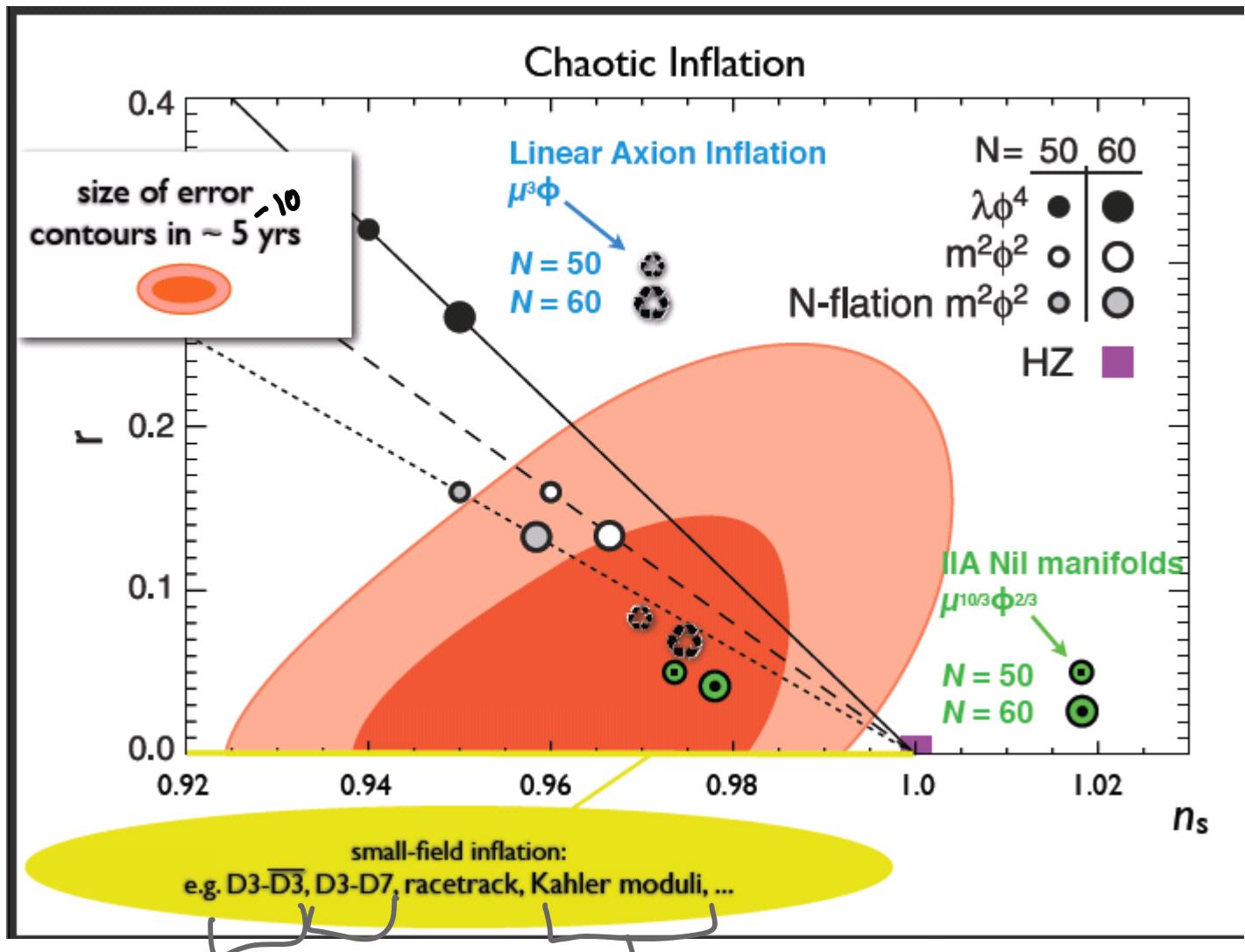
[Silverstein & AW '08]

[McAllister, Silverstein & AW '08]

- Lyth bound: $\frac{\Delta\phi}{M_P} \sim \left(\frac{r}{0.01}\right)^{1/2} \geq 1 \Leftrightarrow \begin{cases} \text{observable!} \\ \text{UV sensitive} \end{cases}$
 chaotic inflation [Linde '83] $\Rightarrow \Delta\phi > M_P$ protected by symmetry:
 natural inflation [Freese et al. '90] Can this arise naturally in string theory?
- Yes: Monodromy $\begin{cases} \text{would-be periodic direction (brane position, axions, ...)} \\ \text{not periodic in presence of wrapped brane:} \end{cases}$
 → kinematically unbounded field range; potential from brane action:

$$\mathcal{L} \sim \begin{cases} u\dot{u}^2 - \sqrt{1+u^2} \Rightarrow V(\phi) \sim \phi^\alpha, \alpha = 2/3, \text{ Nil manifold} \\ f_a^2 \dot{a}^2 - \sqrt{\ell^4 + a^2} \Rightarrow V(\phi) \sim \phi^\alpha, \alpha = 1, \text{ axions in e.g. CYs} \end{cases}$$
- Systematic control:
 shift symmetry weakly broken by $V(\phi)$:
 Nil manifold case: simple & explicit construction, $O(1\%)$ tuning
 Calabi-Yau case: holomorphy & exponential suppression of instantons
 control corrections naturally for axion monodromies

predictive:	$n_s = 0.98$	$r = 0.04$
	$n_s = 0.975$	$r = 0.07$



cf Kachru talk

cf Zagerman talk
(various versions
& r values)

The many ongoing CMB polarization measurements are crucial cf e.g. Church talk ...

Inflation Guth '81
 Linde '82
 AS '82

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2 + \dots$$

$$H = \frac{\dot{a}}{a} \approx \text{const}$$

provides a solution

to standard cosmological problems

within effective field theory,

diluting high-scale relics, but

it is nonetheless UV-sensitive:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \tilde{\eta} \equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \quad s = \frac{\dot{c}_s}{c_s H}$$

must be $\leq 10^{-2}$ to inflate, but

get $\mathcal{O}(1)$ contributions from

dimension 6 Planck-suppressed

operators: In slow-roll inflation,

$$\eta = M_p^2 \frac{V''}{V} \sim 1 \quad \text{from} \quad V(\phi) \frac{(\phi - \phi_0)^2}{M_p^2}$$

\Rightarrow Useful to formulate inflation
in UV-complete theory of gravity

This UV-sensitivity is especially strong
in any inflation model with observable
tensor modes in the power spectrum:

{Lyth: observable tensor modes $\Rightarrow \Delta Q > M_p$ during inflation}.

$$\frac{\Delta Q}{M_p} \propto r^{\frac{1}{2}} N_e \quad (c_s = 1)$$

\Rightarrow Must ensure $\epsilon \ll 1, \eta \ll 1$

over range $\Delta Q \gg M_p$ in

any model with observable inflationary
gravity waves.

UV Sensitivity of Inflation

① Terms of order

$$V \cdot \frac{(Q - Q_0)^2}{M_p^2} \quad (\text{dimension 6})$$

in the effective action can ruin inflation.

② $\frac{\Delta Q}{M_p} \simeq r^{\frac{1}{2}} \frac{N_e}{\sqrt{24}} \quad (\text{Lyth})$

GUT-scale inflation (with observable tensor modes) $\Leftrightarrow \Delta Q > M_p$

③ General Single-field inflation involves higher derivative terms which affect solution & perturbations

cf Non-Gaussianity

→ Model Inflation in a "UV complete" Theory e.g. String Theory

From a different point of view,
large-field "chaotic" inflation

A.Linde '83 seems very simple.
cf symmetries

e.g. $V(\phi) \sim \mu^{\frac{4-\alpha}{\alpha}} \phi^\alpha$

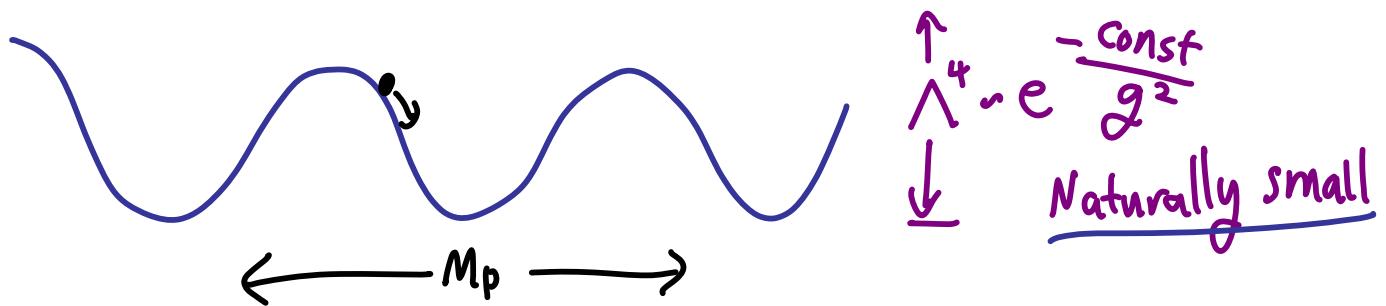
↓ inflation

If this is the leading effect
breaking $\phi \rightarrow \phi + c$ symmetry, it's
radiatively stable. → Does the
UV completion respect this symmetry?

Freese, Frieman, Olinto '90; + Adams, Bond '93

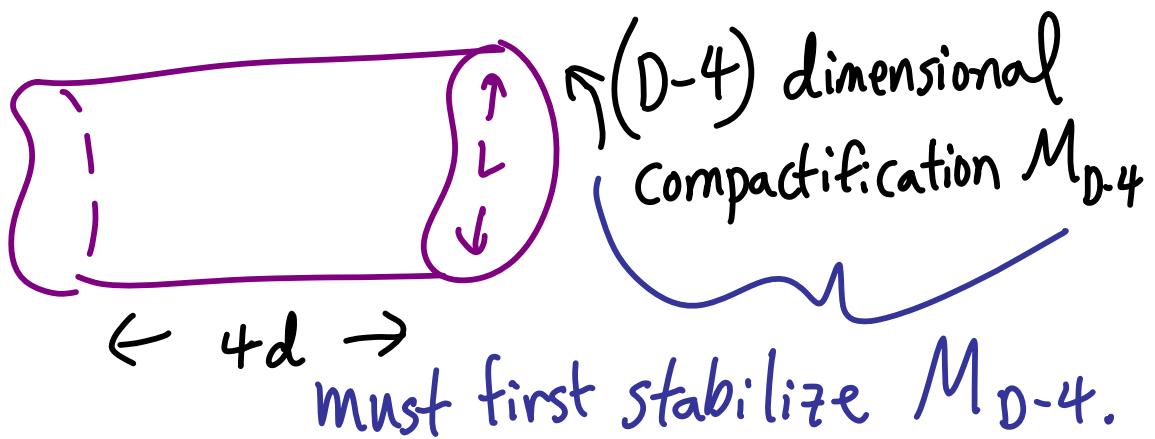
Axions naturally respect an
(approximate) shift symmetry

→ Natural Inflation



A blue circle with a clockwise arrow inside it is shown. To its right, the equation $a \equiv a + (2\pi)^2$ is written above another equation. The second equation, $\dot{\phi}_a = f_a a$, is enclosed in a green oval. To the right of the oval, the text "canonical scalar field" is written in green.

→ Does $\frac{\Delta Q}{M_p} \gg 1$, protected by shift symmetry, arise in string theory?



- Many scalar field "moduli" with generically steep potential $U_{\text{mod}}(L, g, \dots)$
size \nearrow \nwarrow coupling
- Many angular moduli θ such as axions, certain brane positions, etc. with more shallow potential at weak coupling \rightarrow Natural candidate inflatons

In string theory, the basic period $f_\theta (2\pi)^2$
 a priori turns out $\ll M_p$ at weak
 curvature + coupling

Banks/Dine
Susskind/Witten

e.g. Axions

$$a = \int \underbrace{A_{i_1 \dots i_p}}_{\begin{array}{l} \sum_p \\ p\text{-dim'l} \\ \text{closed submanifold} \end{array}} dx^{i_1} \dots dx^{i_p}$$

potential field
(higher-dim'l analogue
of Maxwell A_μ)

f_a comes from kinetic term:

$$\int d^D x \sqrt{G_{(D)}} F_{i_1 \dots i_{p+1}} G_{(D)}^{i_1 i_1'} \dots G_{(D)}^{i_p i_p'} F_{i_1' \dots i_{p+1}'} = \int d^4 x \sqrt{g_4} f_a^2 (\partial a)^2 = \int d^4 x \sqrt{g_4} (\partial \varphi_a)^2$$

\Rightarrow for all sizes $\sim R$, this yields

$$f_a \sim M_p \left(\frac{\sqrt{\alpha'}}{R} \right)^p \ll M_p$$

$\sqrt{\alpha'} = \text{string length}$

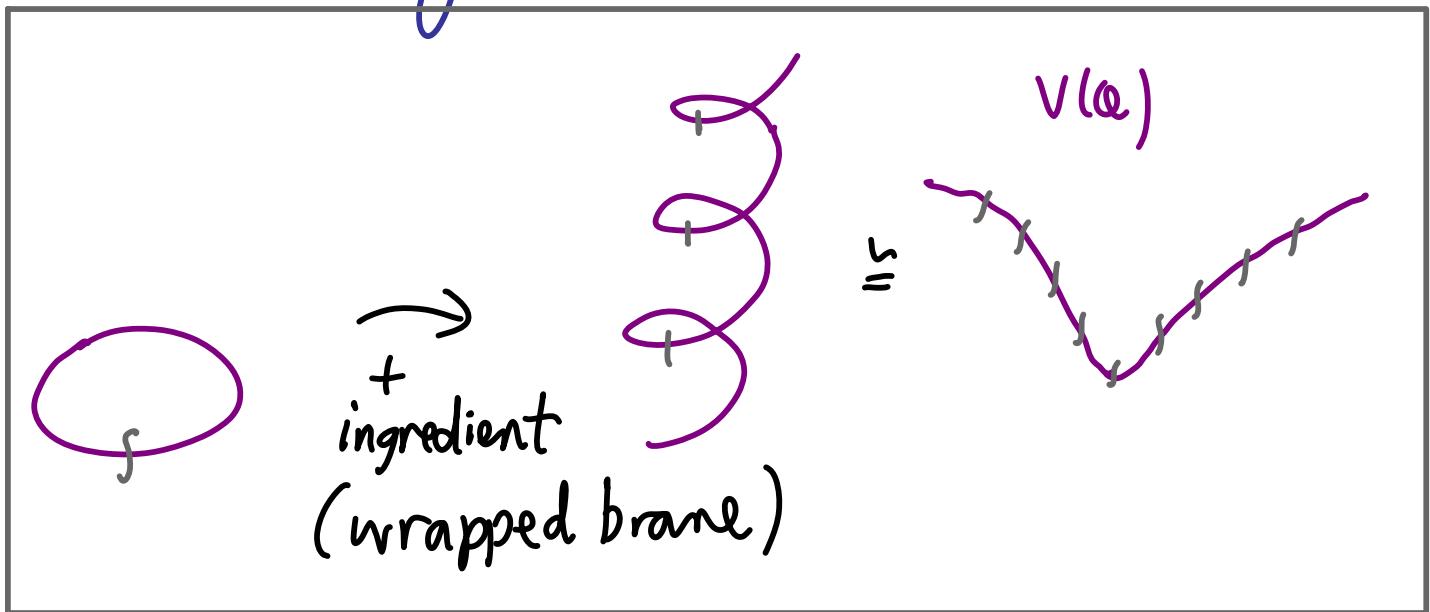
- Similar statement for certain brane collective coordinates
(Bauman, McAllister)
 - Generalization to multiple fields

}	axions	N -flation	Dimopoulos
	branes		

Becker Leblond Shandera
 Greene Witten
 Lindsey-Huston Ward
 Kobayashi et al, ...

kinematically extends range, but yields significant back reaction.
May well be UV-completable upon further work.

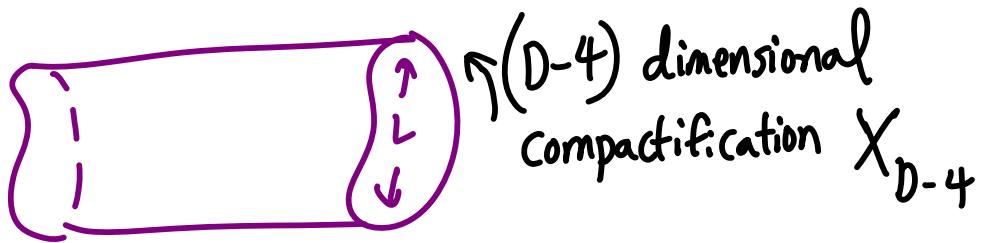
However, a rather generic structure -Monodromy - in string compactifications



unwraps the would-be periodic direction. \rightarrow Large field range with distinctive potential with

$$V(\varphi > M_p) \sim \begin{cases} \varphi^{2/3} & \text{twisted torus } \stackrel{\text{ES, AW}}{'} 08 \\ \varphi & \text{axions } \stackrel{\text{LMcA, ES, AW}}{'} 08 \end{cases}$$

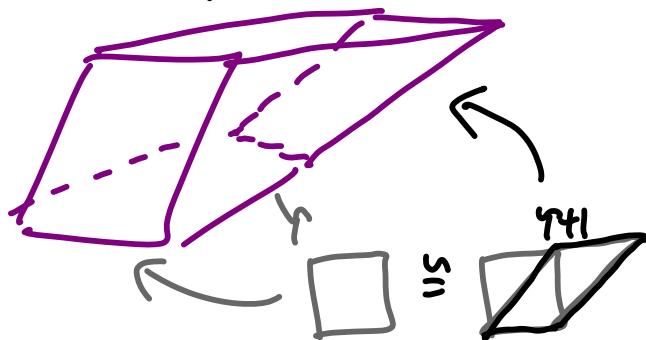
the so far fully worked out examples.



The Simplest compactification
is a torus
(periodic boundary conditions)



The next simplest* is perhaps a "twisted
torus" \cong nilmanifold



cf also

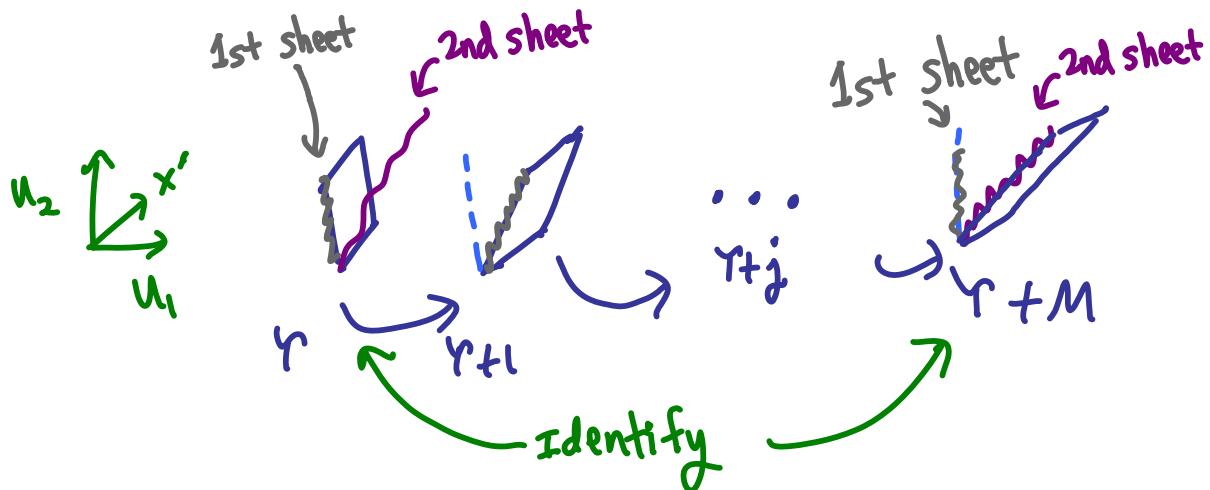
$$K3 \times T^2/\mathbb{Z}_2$$

Haack, Kallosh, Krause,
Linde, Lust, Zagerman

Nil manifold (Twisted Torus) :

$$ds^2 = L_u^2 \left(\frac{du_1^2}{\beta} + du_2^2 \right) + L_x^2 (dx' + M u_1 du_2)^2$$

$$u_1=0 \quad u_1=\frac{1}{m} \quad u_1=\frac{j}{m} \dots \quad u_1=1$$



D4 brane wrapped on u_2 direction:
Scalar fields arise from transverse motion

- As move in u_1 direction away from $u_1=0$, the brane wraps a larger cycle \Rightarrow heavier
- As move around u_1 circle, the D4 does not come back to itself : Monodromy
 \Rightarrow Scalar field range is unbounded geometrically (at fixed $M_p^2 \sim \text{Vol}/g_s^2$)

- Brane action

$$V(\varrho > M_p) \sim \varrho^{\frac{2}{3}} \mu^{\frac{10}{3}} + \text{corrections}$$

\ll (curvature-induced moduli potential barrier)

- Brane action + corrections controlled

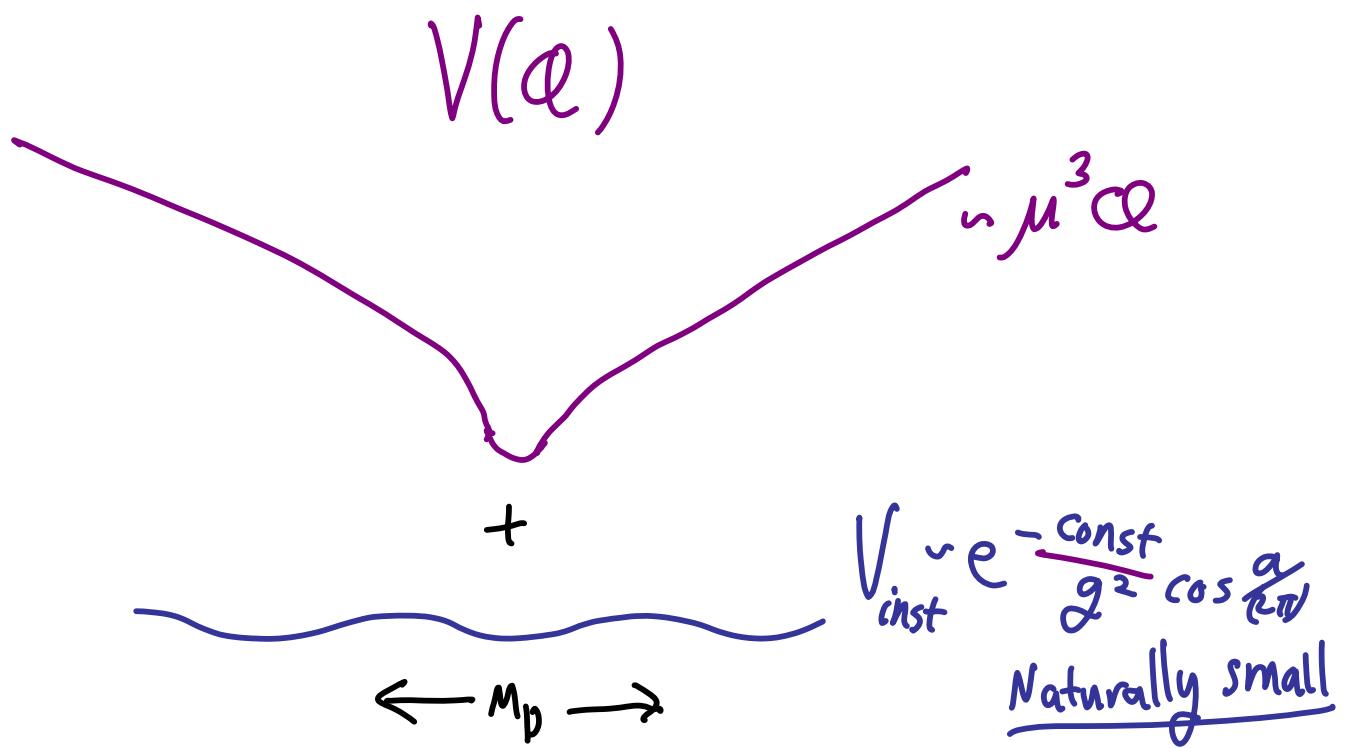
by

- approx. shift symmetry
- weak couplings, curvature
- Given stabilized moduli
 $\sigma = \sigma_*$, negligible shifts
 during inflation
- 1% tune in input discrete parameters.

Axion case : Similar structure

- Monodromy from wrapped branes -

with Naturally suppressed sinusoidal corrections to the brane potential, arising broadly in string compactifications (Calabi-Yau's and Ricci-curved spaces):



$$a = \int A_{ij} dx^i \wedge dx^j$$

$\sum_2 \leftarrow$ wrap 5-brane in
warped region
of C-Y



Start with D5-brane case, with $A_{ij} = B_{ij}$

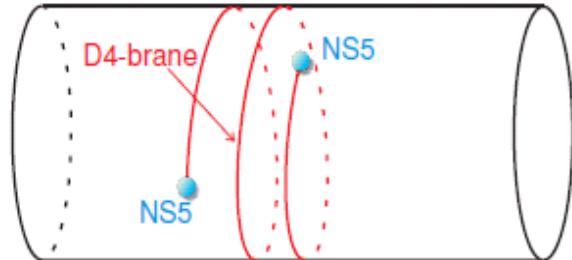
$$S_{DBI} = - \int \frac{d^{p+1}\xi}{(2\pi)^p} \alpha'^{-(p+1)/2} e^{-\Phi} \sqrt{\det(G_{MN} + B_{MN})} \partial_\alpha X^M \partial_\beta X^N$$

$$V(b) = \frac{\epsilon}{g_s(2\pi)^5 \alpha'^2} \sqrt{v_\tau^2 + b^2}$$

↓ ← warp factor
 ↓ size of Σ_2

} Not periodic

A T-dual
description exhibits
the monodromy



Similarly, for NS 5-branes

with $A_{ij} \equiv C_{ij}$, we get

brane action

$$V(c) = \frac{\epsilon}{g_s^2 (2\pi)^5 \alpha'^2} \sqrt{v_f^2 + c^2 g_s^2}$$

These brane potentials provide

candidate* inflaton potentials.

$$V(Q_c) = \mu^3 Q_c \quad \xrightarrow{\quad} \quad \left\{ \begin{array}{l} \mu = 6 \times 10^{-4} M_p \\ \text{(COBE)} \\ \frac{Q_c}{M_p} \leq 11 \quad (N_e = 60) \end{array} \right.$$

* Many conditions must be met
for control & consistency with moduli
stabilization...

... Laid out in 0808.0206[hep-th] ^{L M c A}
^{E S A W}

3 Necessary Conditions for Controlled Inflation

3.1 Axions and the Orientifold Projection

3.2 Conditions on the Potential

3.2.1 Conditions on Flux Couplings

3.2.2 Effects of Instantons

3.3 Constraints from Backreaction on the Geometry

3.4 Constraint from the Number of Light Species

3.5 Consistency with Moduli Stabilization

Some fluxes lift
axions \Rightarrow turn them
off (automatic for
IIB CY no-scale)

Naturally Exponentially
suppressed.

Strongest
Constraint

4 Specific Models I: Warped IIB Calabi-Yau Compactifications

4.1 Multiplet Structure, Orientifolds, and Fluxes

4.2 An Eta Problem for B

\Rightarrow use RR axions C_{MN}

4.3 Instantons and the Effective Action for RR axions

4.3.1 Instanton Contributions to the Superpotential

4.3.2 Instanton Contributions to the Kähler potential

4.3.3 Effects of Enhanced Local Supersymmetry

3 Restricted
by holomorphy

3 exponentially
suppressed

4.4 Backreaction Condition

4.4.1 The Large Volume Scenario

4.5 Numerical Toy Examples

4.6 Gravity Waves and Low-Energy Supersymmetry

• • •

Type IIB Calabi-Yau Models

(In this corner, moduli stabilization was outlined)
by KKLT, KL, LARGE Vol. works)

Back reaction:

The B_{ij} or $C_{ij}^{..}$ field on the wrapped 5-brane behaves like 3-brane charge & tension (modulo small binding energy). To avoid backreaction:

$$\frac{g_s C}{(2\pi l)^2} \sim N_{\text{cycles}} \ll \frac{R_+^4}{4\pi g_s \alpha'^2} \quad \begin{matrix} \leftarrow \text{size of} \\ \text{ambient} \\ \text{space} \end{matrix}$$

$$\frac{g_s c}{(2\pi)^2} \sim N_3 \ll \frac{R_\perp^4}{4\pi g_s \alpha'^2} \quad \begin{matrix} \leftarrow \\ \text{size of ambient space} \end{matrix}$$

N_{cycles} is determined by $N_e = 60 \Rightarrow Q_c = 11 M_p$

as $N_{\text{cycles}} = \frac{11 M_p}{f_a (2\pi)^2} \sim 11\sqrt{6} \frac{R^2 / g'}{(2\pi)^2}$
(one scale)

So parametrically, back reaction condition solveable as $\uparrow R$

On the other hand, the volume cannot grow too large: Need

$$V_{\text{Inflation}} < V_{\text{moduli}}$$

$$M_{\text{out}}^4 : \frac{M_{\text{out}}}{M_p} \sim g_s \frac{\alpha'^2}{R^4}$$

The moduli potential is of the form

$$U_{\text{mod}} = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) + U_{D3}$$

with superpotential $(R(t))^\nu \frac{R^+}{g_s^2 g_s}$

$$W = W_0 + A_L e^{-\frac{2\pi T_L}{N_c}} \left(1 + \mathcal{O}(e^{-(T+b/g_s + i c)}) \right)$$

Gaugino condensation on D7's:

$$\Lambda^3 \sim e^{-\frac{8\pi^2}{N_c} \frac{1}{g_h^2}}$$

gauge coupling function:
 $T + 1\text{-loop} + \text{exponentially suppressed}$

$$+ A_+ e^{-\frac{2\pi T_+}{N_+}} \left(1 + \mathcal{O}(e^{-(T+b/g_s + i c)}) \right)$$

and Kähler potential of form

$$K = -3 \log \left[T + \bar{T} + \frac{b^2}{g_s^2} + \mathcal{O}(e^{-2\pi T_+ - b/g_s + i c}) \right]$$

Moduli shifts and $V_R(\alpha)$:

Inflaton potential $V_I(\alpha; L e^{\frac{\sigma}{M_p}}, \dots)$

depends on moduli $\frac{\sigma}{M_p}$ as well as α

$\Rightarrow \alpha$ -dependent shifts of

moduli : $\frac{\sigma}{M_p} \sim \frac{\partial_{\sigma} V_R}{\partial_{\sigma}^2 (U_{\text{mod}} + V_R)} \frac{V_R}{U_{\text{mod}}}$

non-pert.
stabilization

\Rightarrow correction to α -dependence of
full scalar potential $U + V$

$$U_{\text{tot}} \sim U_{\text{mod}}(L) + V_R(\alpha, L) + c_\alpha \frac{V_R(\alpha, L)^2}{U_{\text{mod}}(L)}$$

$$\Rightarrow \Delta n \sim n \frac{V_I}{U_{\text{mod}}} \leq O(n) \sqrt{ }$$

All consistency conditions solved for

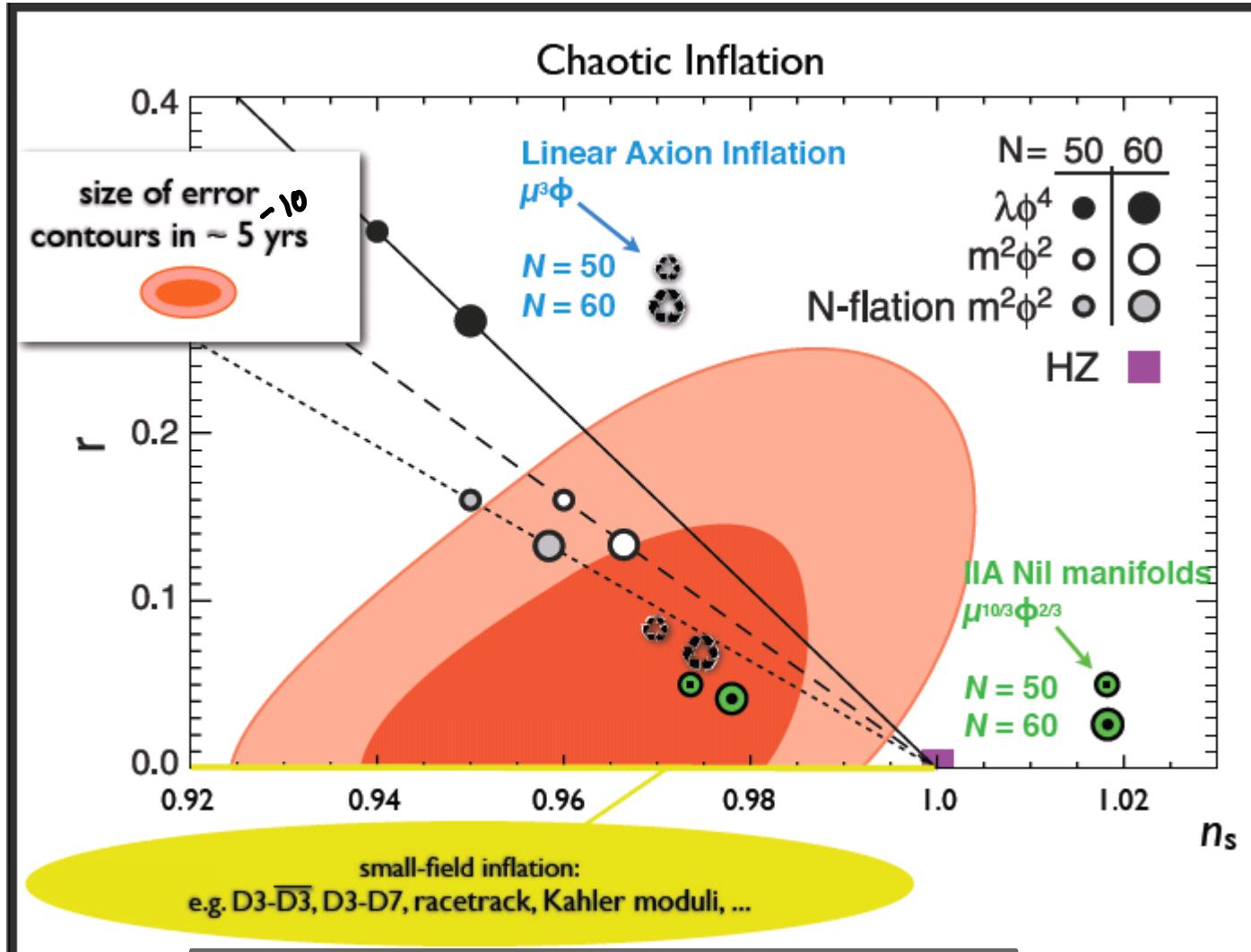
e.g. $A \approx 1$, $N_L \approx 25$, $N_T \approx 3$, $w_0 \approx 10^{-2}$

$\rightarrow T_L \approx 20$, $T_T \approx 4$, $b \approx 0$

\rightarrow About 100 cycles of
the basic axion period

More cycles in other regimes

Result :



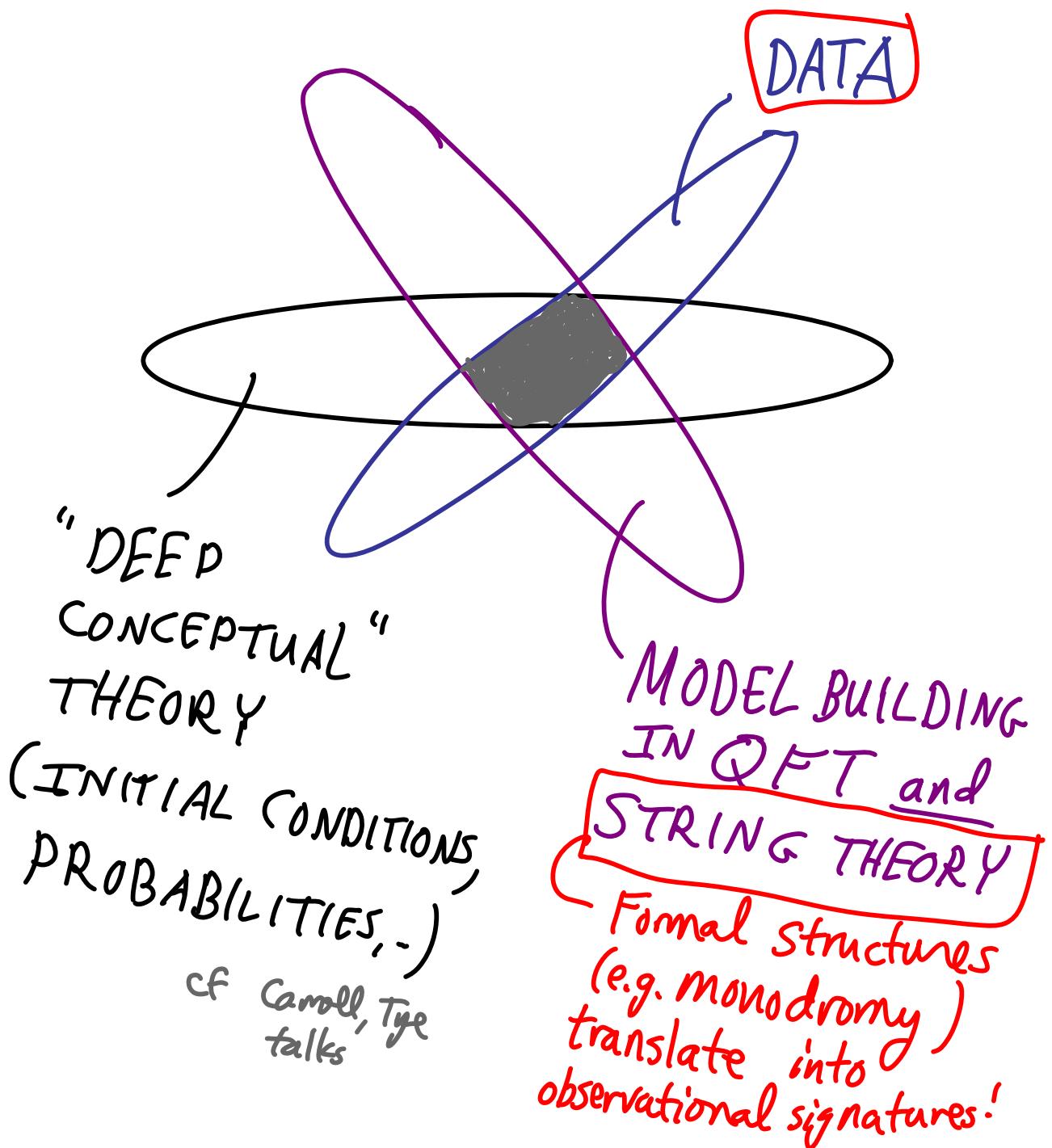
$$r = 0.07$$

$$n_s \approx 0.975$$

$$V(\varrho) \approx \mu^3 \varrho + \lambda^4 \cos\left(\frac{\varrho}{2\pi f}\right)$$

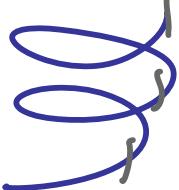
Because of the symmetry, and oscillating nature of the (instanton-suppressed) corrections, these predictions are precise \Rightarrow falsifiable

A Concordance



Ancillary signatures?

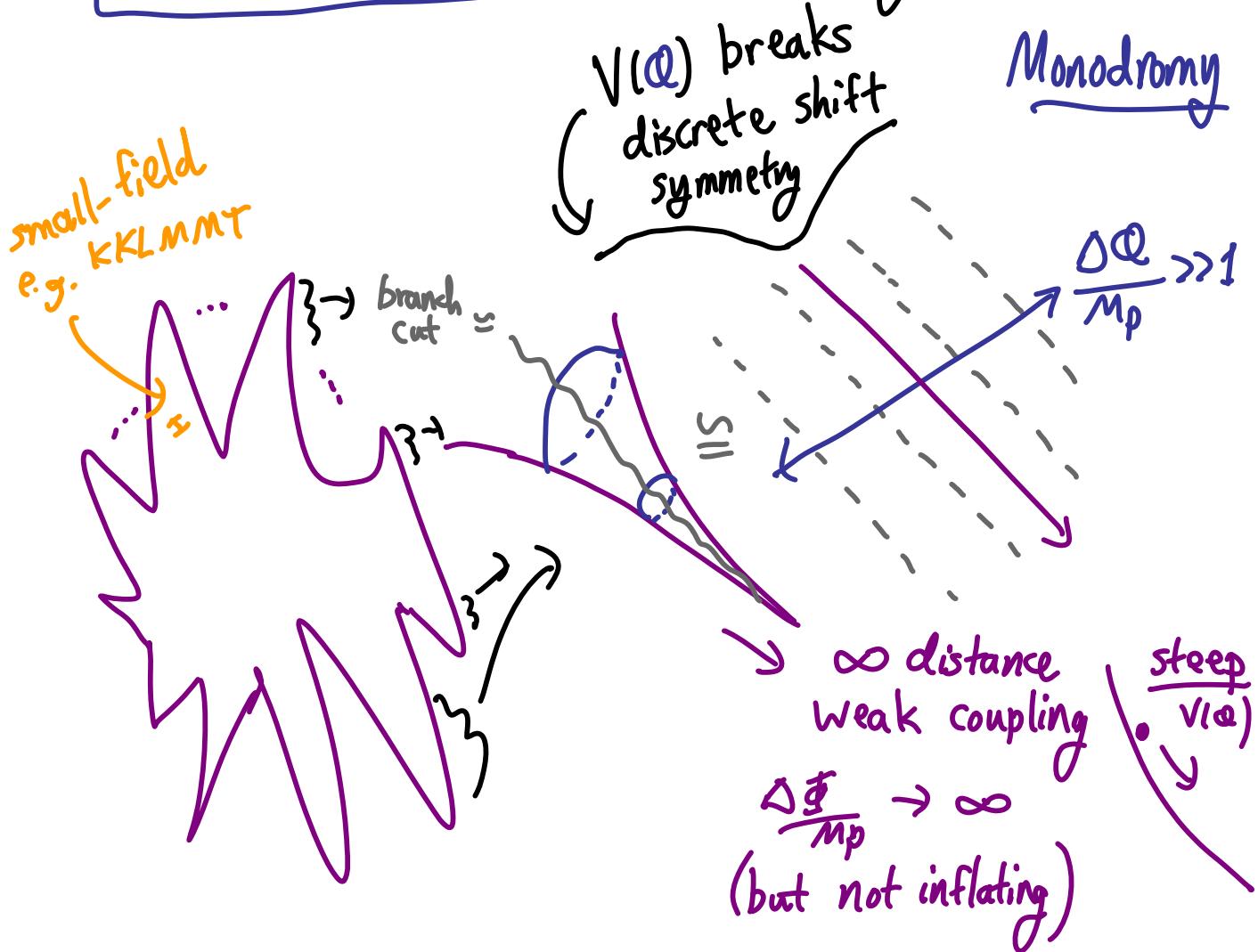
Even given a measurement of r , the "inverse problem" is hard (Look for the simplest UV-complete explanation which fits the facts...).

⇒ For a given mechanism, it is important to look for correlated signals. Monodromy  + reheating

⇒ periodic subleading effects. Also $\Delta \eta$ of $\mathcal{O}(1)$ does not stop oscillating inflation ...

(Work in progress)

Structure of scalar fields ("moduli space") in string theory:



$\Rightarrow *$ No general prediction about detectable r from string theory at the level of model building.

Initial conditions?

- small-field inflation requires landing on a tiny $\Delta\phi$, with somewhat smooth initial patch of size H^{-1}

Genericity?

- Both types of models use common ingredients
- More symmetry required for chaotic inflation, but this is common in "angular" directions.

Anyway, the data will decide...

Conclusions

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tensor/scalar

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