Update on Brane Inflation from String Theory

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Inflation from String Theory is Not Easy

- String theory is a precise and constrained framework. Can't just do whatever you want.
- This is a good thing—focus on fewer classes of models.
- We get novel/complicated Lagrangians which look strange from field theory point of view. (*e.g.*, $\phi^{2/3}$ chaotic inflation from monodromy, Silverstein *et al.*)
- Also good; this makes stringy inflation potentially distinctive.

Outline

- DBI inflation
- Nonlocal (p-adic string theory) inflation
- Racetrack inflation
- Warped brane-antibrane inflation
- D3-D7 inflation

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- DBI inflation
- Nonlocal (p-adic string theory) inflation
- Racetrack inflation

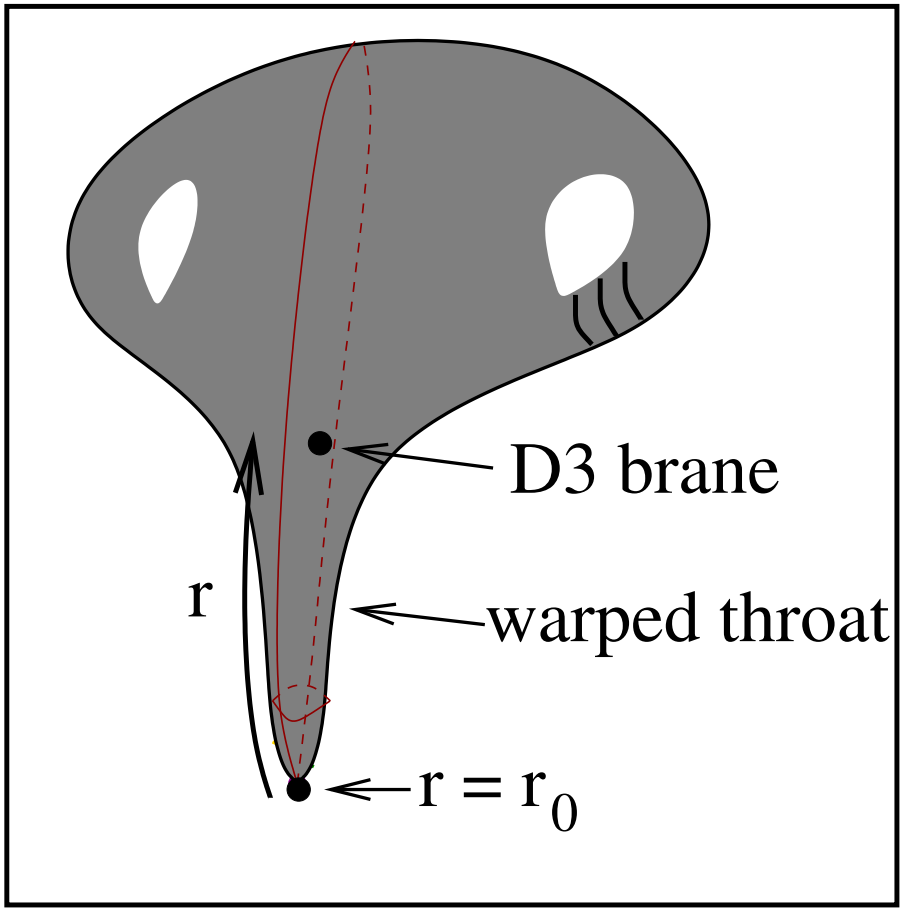
examples of difficulties

- Warped brane-antibrane inflation
- D3-D7 inflation

main subjects

1. DBI inflation

(Alishahiha, Silverstein, Tong, '04) Uses brane motion in type IIB string theory with warped (Klebanov-Strassler) throat:



Metric in throat:

$$ds^2 = h^2(r)(-dt^2 + a^2(t)d\vec{x}^2) + h^{-2}(r)dr^2 + \text{angular}$$

with $h(r) \cong \frac{r}{R}$ (AdS$_5$) and

$$h(r_0) \sim e^{-\text{fluxes}}$$

— exponentially small

D3-brane Dirac-Born-Infeld + SUSY-breaking Lagrangian:

$$\mathcal{L} = -T_3\, h^4(r)\sqrt{1 - h^{-4}(r)(\partial r)^2} - V(r)$$

DBI: dinstinctive features

- Warped space can give slow roll despite steep potential, due to relativistic speed limit:

$$\frac{dr}{dt} \rightarrow h^2 = \frac{r^2}{R^2} \rightarrow \frac{r_0^2}{R^2}$$

- Fluctuations are enhanced by relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - h^{-4}(r)(\partial r)^2}}$$

leading to large nongaussianity, $f_{NL} \sim 100$

Bean, Chen, Peiris, Xu, '07
Chen, Huang, Kachru, Shiu, '07

DBI: theoretical challenges

- Strong warping at bottom of throat, needed for slow roll, is generically killed by back-reaction, from:

  $\Rightarrow$ SUSY-breaking (Maldacena; McAllister, Silverstein '08)

  warp factor gives scale of lowest KK masses, which are large due to SUSY-breaking

  $\Rightarrow$ Inflationary background (X. Chen, '08), *i.e.*,

$$h'(r)^2 = \text{vacuum contributions} + \frac{H^2}{h^2}$$

- Almost all phenomenologically acceptable models violate constraint on field range from size of extra dimensions:

  Bean, Shandera, Tye, Xu '07
  Peiris, Baumann, Friedman, Cooray '07

  Allowed models which have observable f_{NL} predict $n_s > 1$.

2. Nonlocal inflation

N. Barnaby, T. Biswas, JC '06; Barnaby, JC '07 & '08

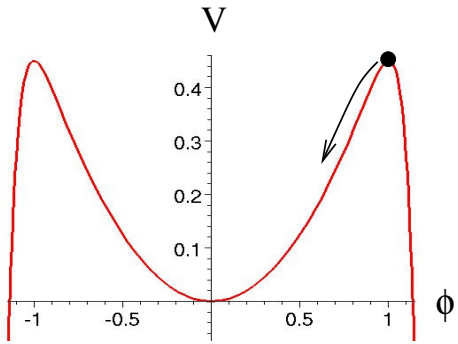
- String theory is UV complete: consistently describes physics above string scale where low energy effective field theory description is inadequate. Can we have inflation above the string scale?

- p-adic string theory: worldsheet restricted to p-adic numbers (Freund, Olson, Witten '87) sums $\alpha' = m_s^{-2}$ corrections to *all orders*, giving nonlocal action:

$$S = \frac{m_s^4}{g_s^2} \frac{p^2}{p-1} \int d^{26}x \left(-\tfrac{1}{2}\phi p^{-\Box/2m_s^2}\phi + \frac{\phi^{p+1}}{p+1} \right)$$

- Theoretical laboratory for studying inflation with $\Box \sim H^2 > m_s^2$ in a theory where higher derivative corrections make sense (*e.g.,* no ghosts: N. Barnaby's talk).

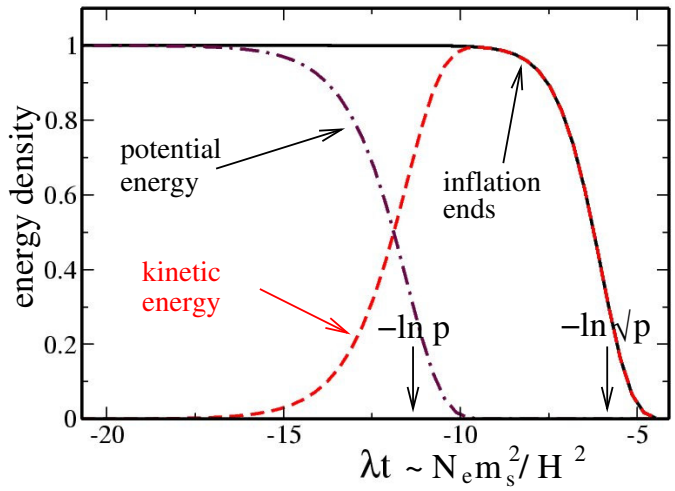
Nonlocal inflation: distinctive features



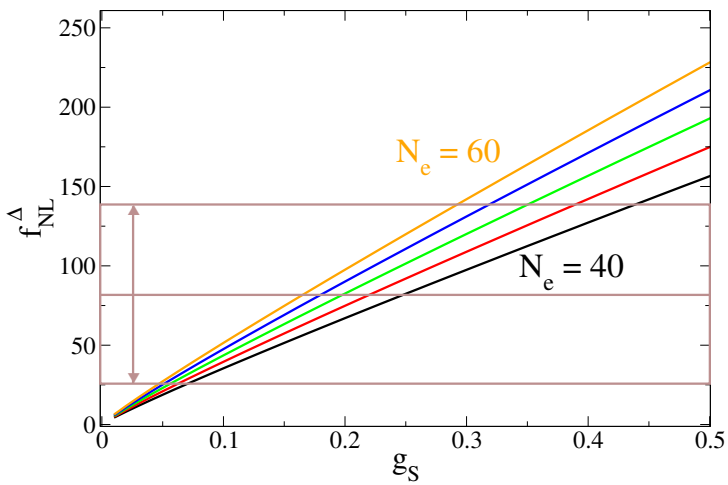
Can get slow roll from potential

$$V \propto \frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1}$$

even if naive η parameter $\gg 1$



Last stage of inflation dominated by inflaton *kinetic* energy — new mechansim for slow roll



Nongaussianity can be observably large; large cubic term in V need not spoil slow roll

Nonlocal inflation: problems

- Hard to keep extra dimensions from inflating/backreacting if
$$H > m_s > \text{(size of extra dimensions)}^{-1}$$
(Spectral index $n_s = 0.96$ requires $H \cong 4m_s$ due to prediction $n_s = 1 - \frac{4}{3}\left(\frac{m_s}{H}\right)^2$)

- p-adic string theory doesn't include α' corrections to gravity; we just coupled it to general relativity

- We would need fully string theoretic methods — not just effective field theory — to investigate realistic string theories; don't know p-adic version of superstring

3. Racetrack Inflation

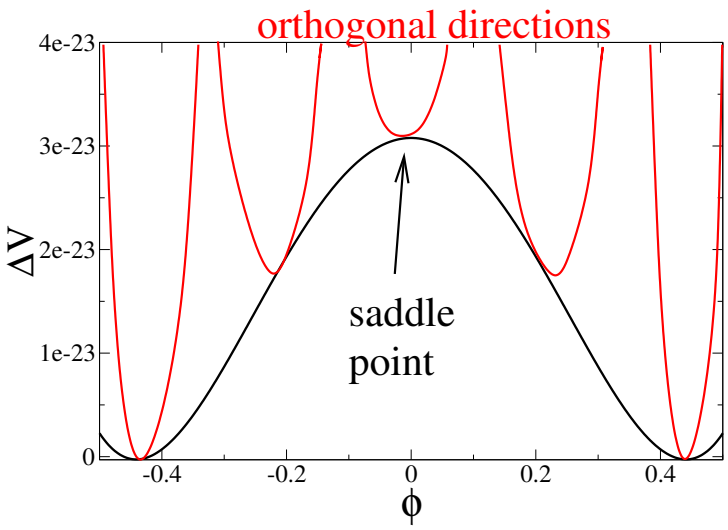
Blanco-Pillado, Burgess, JC, Escoda, Gomez-Reino, Kallosh, Linde, Quevedo, '04 & '06

Kähler moduli can be inflatons in standard compactification with superpotential

$$W = W_0 + Ae^{-aT} + Be^{-bT}$$

or (more realistic "better racetrack")

$$W = W_0 + Ae^{-aT_1} + Be^{-bT_2}$$



Tuning potential gives flat saddle point along some axionic direction,

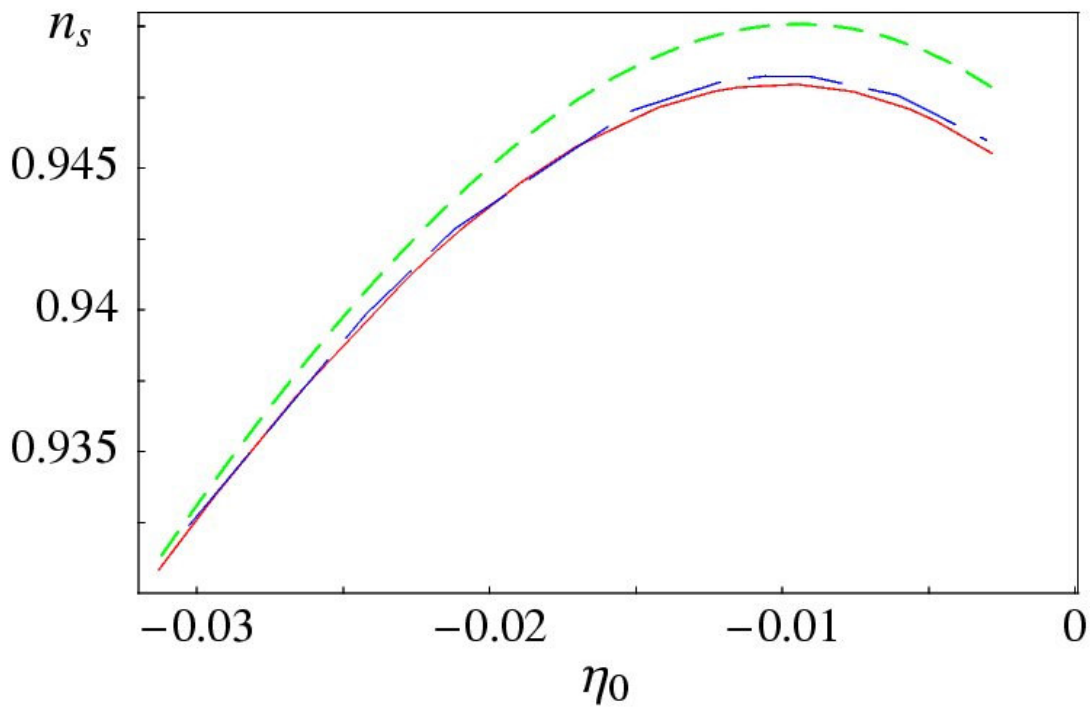
$\phi = c_1 \mathrm{Im} T_1 + c_2 \mathrm{Im} T_2$

— hilltop inflation

Racetrack: positive features

- Based on an explicit string compactification, $\mathbb{P}_{[1,1,1,6,9]}$ (Denef, Douglas, Florea, '04)

- Makes strong prediction for spectral index, $n_s \leq 0.95$ (Brax, Davis, Postma, '07)



Varying η_0 at the hilltop allows limited range of measured n_s at COBE scale

- Eternal topological inflation; needs no tuning of initial conditions

Racetrack: drawbacks

Not particularly fine-tuned, but unattractive parameter values seem to be required in superpotential ($W = W_0 + Ae^{-aT_1} + Be^{-bT_2}$):

$$W_0 = 5.227 \times 10^{-6}, \quad A = 0.56, \quad B = 7.47 \times 10^{-5}$$

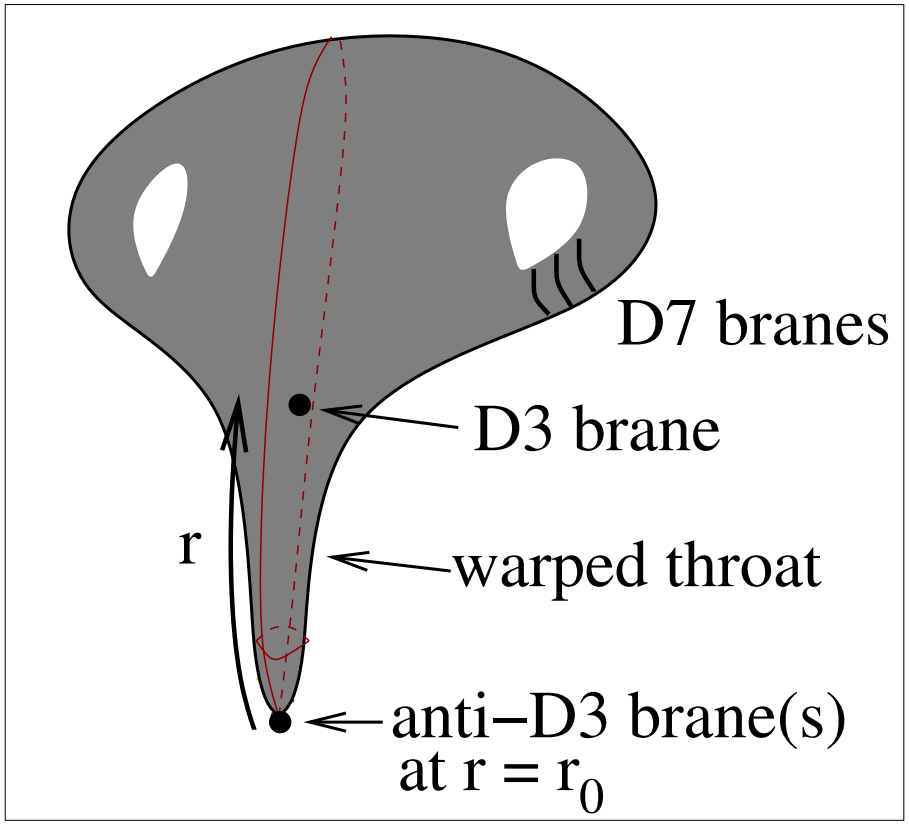
$$a = \frac{2\pi}{40}, \quad b = \frac{2\pi}{258}$$

(Note $a,\ b = \frac{2\pi}{N}$ where $SU(N)$ is condensing gauge group)

4. Warped brane-antibrane inflation

Kachru, Klebanov, Linde, Maldacena, McAllister, Trivedi, '03

Same type IIB setup as DBI; antibranes attract D3 brane:



Coulomb potential is flattened by small warp factor $a_0 = r_0/R$:

$$V(r) \sim a_0^4 V_0 - \frac{a_0^8 T_3 R^4}{2(r-r_0)^4}$$

η parameter suppressed,

$$\eta \sim a_0^4 \ll 1$$

But there is still an η problem!

(KKLMMT '03; DeWolfe, Giddings '02)

Complexify D3 coordinates, $r_{a=1,\ldots,6} \;\rightarrow\; \phi_{i=1,\ldots,3}$

Kähler potential for volume modulus T is

$$K = -3\ln(T + \overline{T} - c\,\phi_i\bar{\phi}_i)$$

In SUGRA, F-term potential $V_F \sim e^K$, thus

$$V \;\sim\; e^K V_0 \;\sim\; \tilde{V}_0\left(1 + \tilde{c}|\phi|^2\right)$$

$\Longrightarrow \eta$ parameter is $O(1)$.

Must fine-tune $\eta \rightarrow -0.015$ (WMAP value) using some other correction, *e.g.*, ϕ dependence of nonperturbative superpotential,

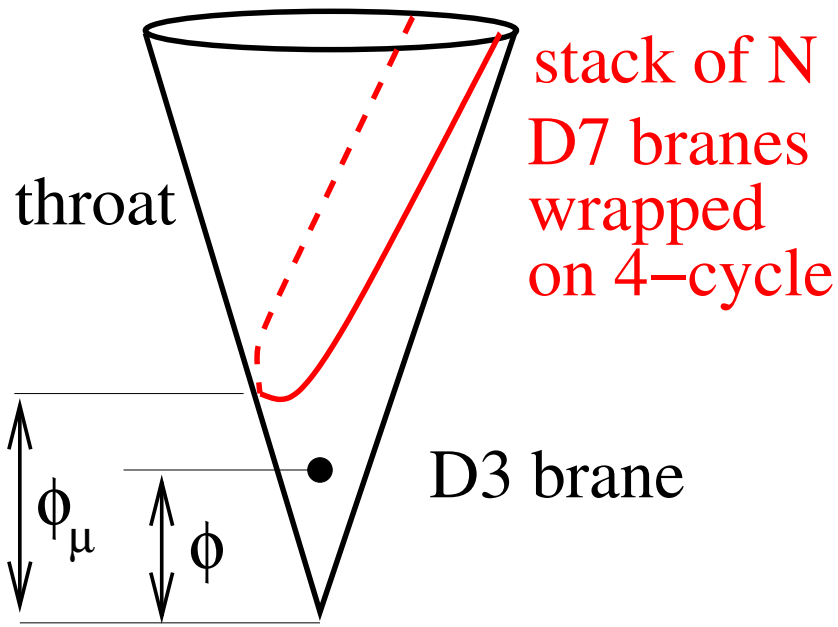
$$W = W_0 + A(\phi)e^{-aT}$$

Superpotential corrections

But is such fine-tuning possible? $A(\phi)$ is computable within string theory

(Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan, '06)

$A(\phi)$ depends on how stack of N D7 branes is embedded in the warped throat:



Roughly,

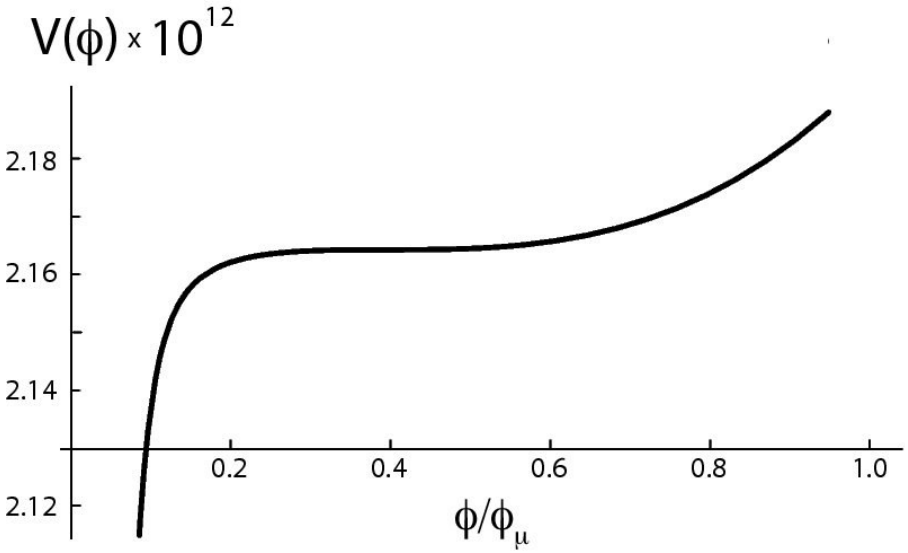
$$A(\phi) \sim \left(1 - \left(\frac{\phi}{\phi_\mu}\right)^p\right)^{1/N}$$

(angular dependences not shown here)

Burgess, JC, Firouzjahi, Dasgupta '06: Inflation not possible in popular Ouyang '04 embedding—corrections go in wrong direction

“Delicate” brane-antibrane inflation

Baumann, Dymarsky, Klebanov, McAllister, Steinhardt ‘07 find embedding (Kuperstein ‘05) where inflation works, with inflection point potential (note intricate form!)



$$V(\phi,\sigma) = \frac{a|A_0|^2}{3}\frac{e^{-2a\sigma}}{U^2(\phi,\sigma)}g(\phi)^{2/n}\Bigg[2a\sigma+6 - 6e^{a\sigma}\frac{|W_0|}{|A_0|}\frac{1}{g(\phi)^{1/n}} + \frac{3c}{n}\frac{\phi}{\phi_\mu}\frac{1}{g(\phi)^2} - \frac{3}{n}\frac{1}{g(\phi)}\frac{\phi^{3/2}}{\phi_\mu^{3/2}}\Bigg] + \frac{D(\phi)}{U^2(\phi,\sigma)}\,.$$

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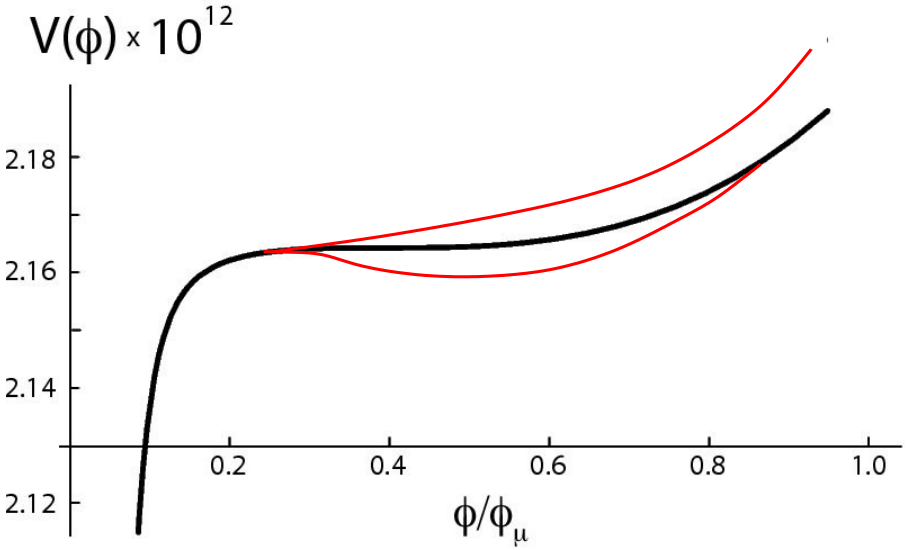


$$V(\phi,\sigma) = \frac{a|A_0|^2}{3}\frac{e^{-2a\sigma}}{U^2(\phi,\sigma)}g(\phi)^{2/n}\Bigg[2a\sigma+6 - 6e^{a\sigma}\frac{|W_0|}{|A_0|}\frac{1}{g(\phi)^{1/n}} + \frac{3c}{n}\frac{\phi}{\phi_\mu}\frac{1}{g(\phi)^2} - \frac{3}{n}\frac{1}{g(\phi)}\frac{\phi^{3/2}}{\phi_\mu^{3/2}}\Bigg] + \frac{D(\phi)}{U^2(\phi,\sigma)}\,.$$

“Delicate” because small change in parameters destroys inflection point.

<u>Problems</u>: Baumann *et al.* tuned uplifting term D/U^2 to get flat potential. But D/U^2 is fixed by need to have $V=0$ at end of inflation.

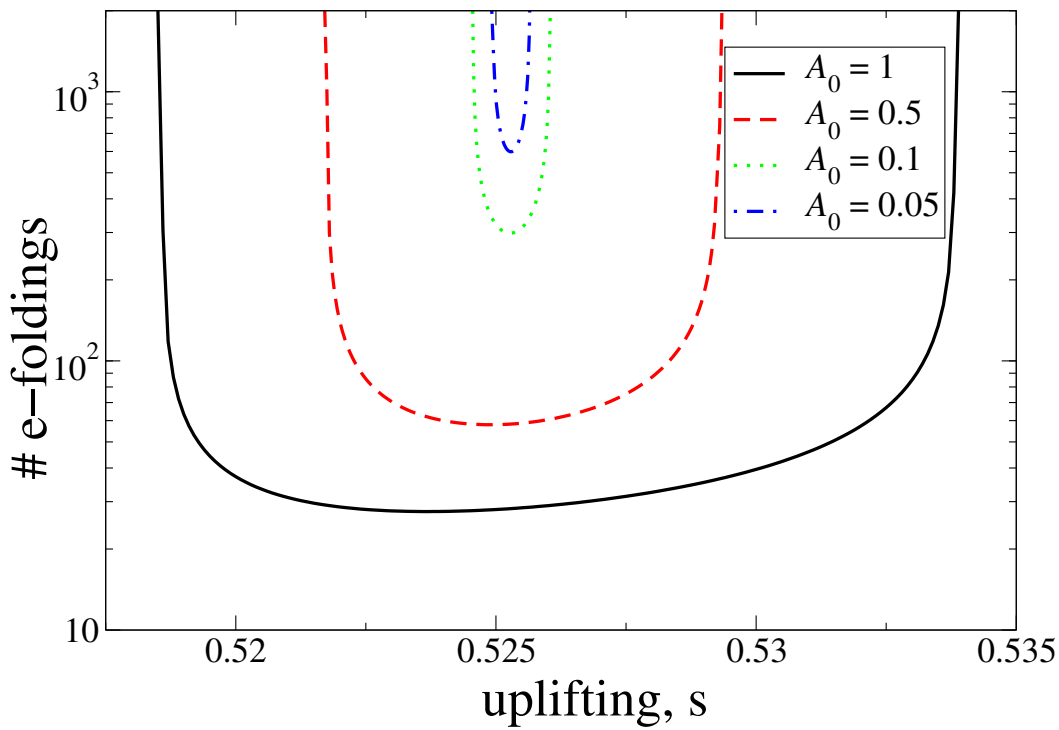
Panda, Sami, Tsujikawa ‘07: Cannot simultaneously satisfy constraints on n_s and amplitude of CMB fluctuations (??)

Underwood, ‘08: Overshoot problem if ϕ starts too large!

Example of "delicateness"

JC, L. Hoi, in progress

Can tune uplifting parameter s within narrow range to get inflection point. Amount of tuning depends on value of A_0 in superpotential $W = W_0 + A_0 e^{-aT}$:



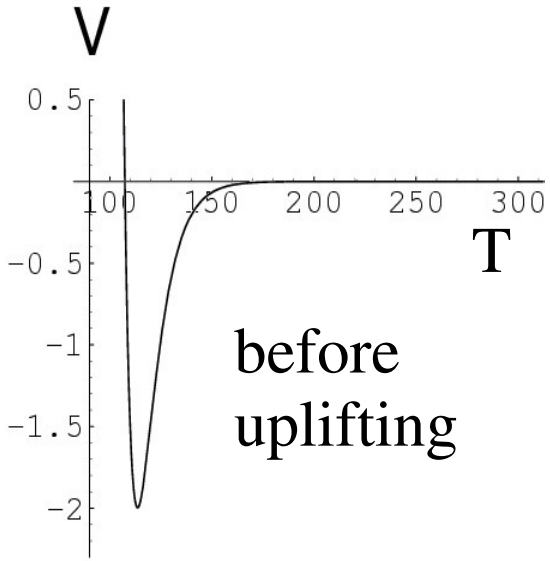
Note minimum number of e-foldings! Potential gets local minimum outside these values.

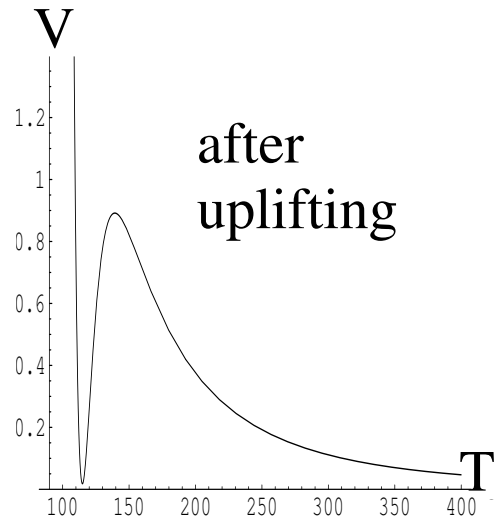
Larger $A_0 \Rightarrow$ less tuning of s, but small A_0 needed for COBE normalization. And s is already fixed by $V(0) = 0$!

Improved treatment of uplifting

Chen, Gong, Shiu '08; JC, L. Hoi, in progress

SUGRA potentials give $V < 0$ at minimum; SUSY-breaking $\overline{\mathrm{D3}}$ at bottom of throat can uplift it (KKLT '03)





For more freedom to tune potential, uplift with extra $\overline{\mathrm{D3}}$'s (with tension D_1) in a distant throat.

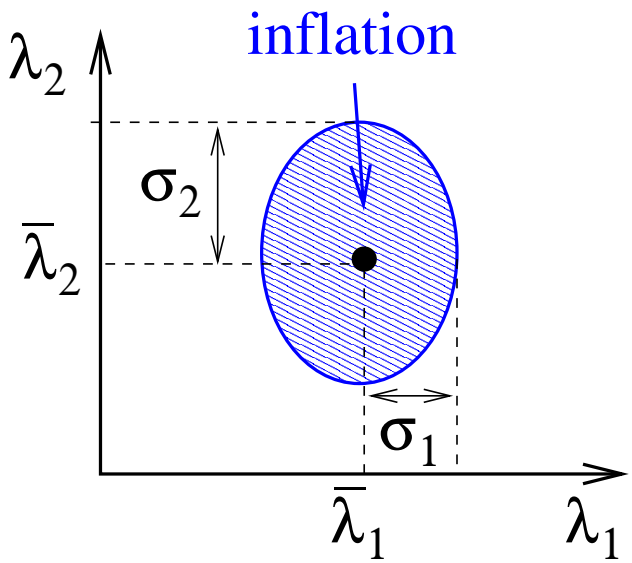
$$V_{\text{uplift}} \to \frac{1}{(T+\bar{T}-c|\phi|^2)^2}\left(\frac{D_0}{1+\tilde{c}D_0/\phi^4}+D_1\right)$$

Can tune D_0/D_1 in place of total uplifting s. (Other relevant parameters: A_0, ϕ_μ, T.)

How fine tuned is the model?

JC, L. Hoi, in progress

We did Monte Carlo search of parameter space to find least fine-tuned, phenomenologically acceptable parameters.



Let σ_i define range of parameter λ_i around value $\bar{\lambda}_i$ which is compatible with inflation:

$$\bar{\lambda}_i + \sigma_i < \lambda_i < \bar{\lambda}_i - \sigma_i$$

Define local relative volume of N model parameters,

$$V_N(\bar{\lambda}_i) = \prod_{i=1}^{N} \frac{2\sigma_i}{\bar{\lambda}_i}$$

Large $V_N \leftrightarrow$ less tuning. Average λ_i is tuned to 1 part in $V_N^{-1/N}$.

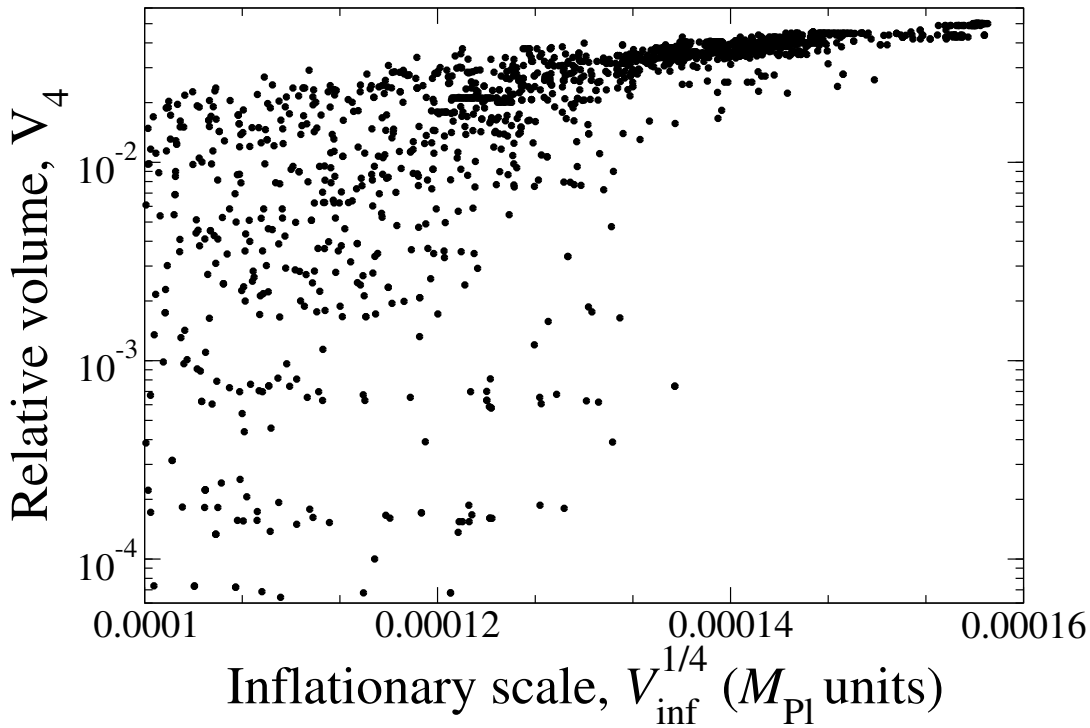
We find that best region has $V_4 = 0.05$, $V_4^{1/4} \sim 0.5$. This is not fine-tuned at all!

Compare to fiducial point of Baumann *et al.*, $V_4 = 3 \times 10^{-10}$!

Monte Carlo Distributions

JC, L. Hoi, in progress

Example showing relative volume versus inflationary scale:

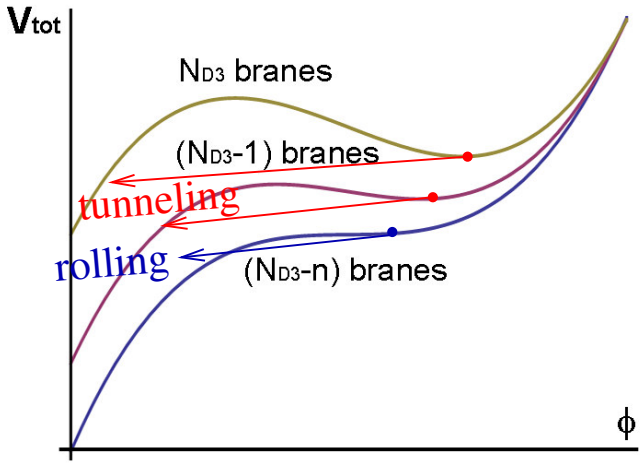


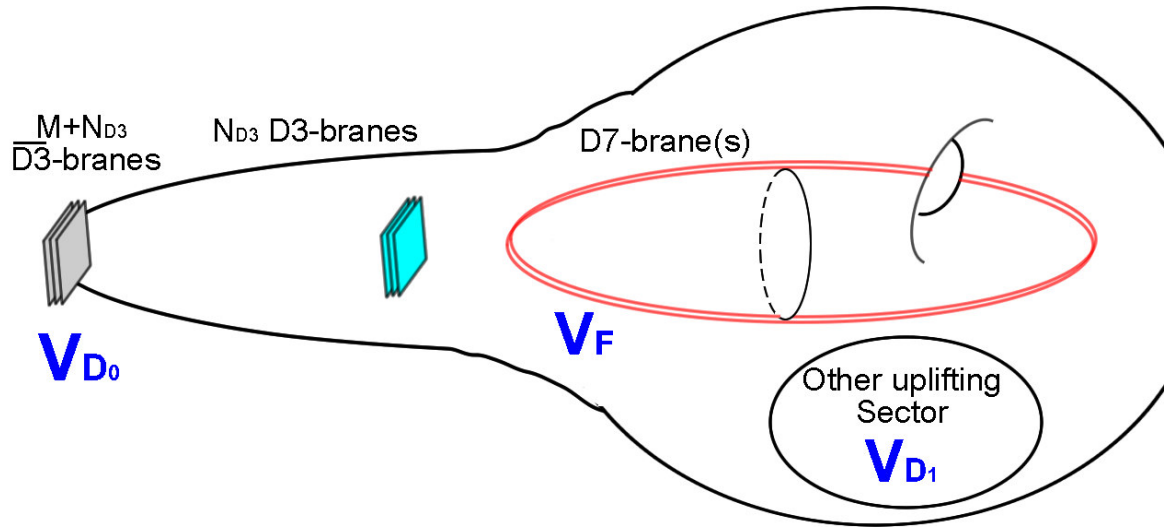
- Less tuned, large V_4 models appear most frequently in Monte Carlo chains.

- High scale of inflation is favored (but still too low to see tensors).

Dynamical self-tuning of potential

JC & H. Stoica '05 noted that if equilibrium position of brane is shifted to $\phi_0 \neq 0$ and mobile $D3$ is a stack of N_{D3} branes, potential can get flatter by brane tunneling:





B. Underwood & JC (in preparation): This idea can work in Baumann *et al.* potential, due to attractive force from D7 branes, and form of uplifting + Coulomb potential,

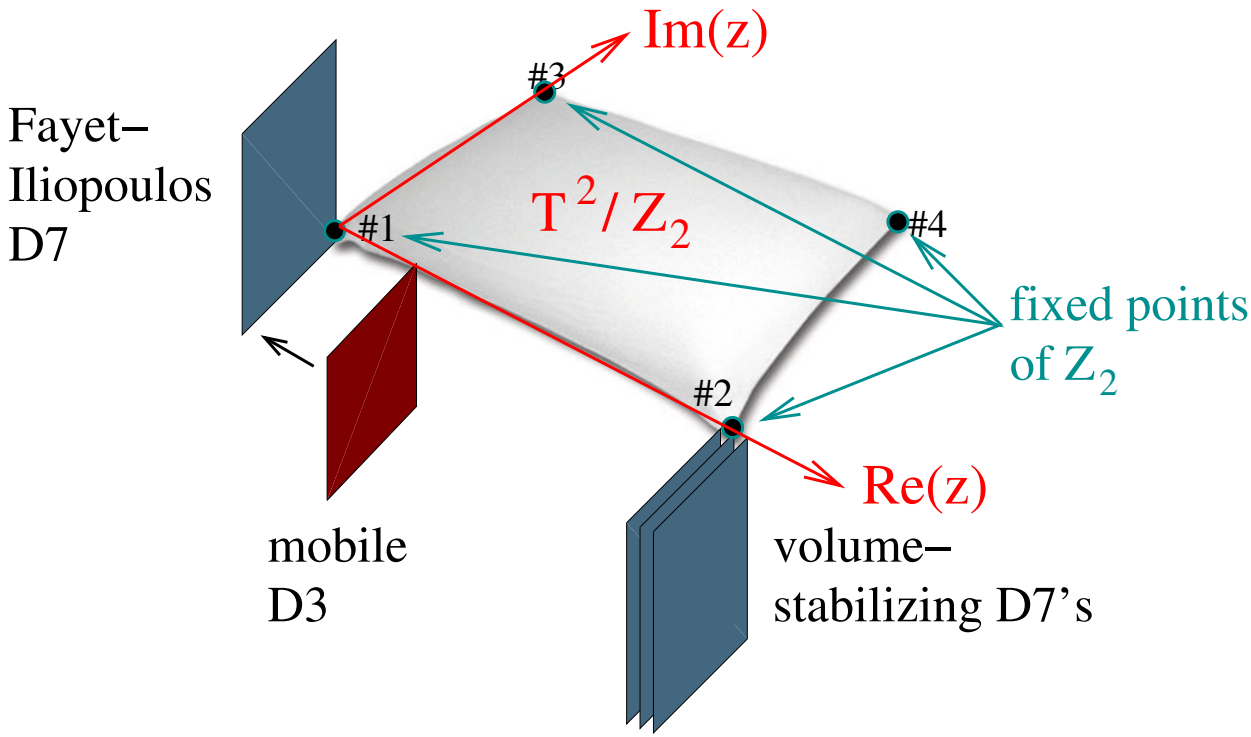
$$V_{\rm uplift} \sim 2\frac{N_{D3}+M}{1+cN_{D3}/r^4}$$

If $N_{D3} \gg 1$, $V_{\rm uplift}$ changes by small steps with each brane tunneling. Potential can naturally achieve flat inflection point. Note: solves initial condition problem; rolling branes start at flat point in potential!

5. D3-D7 inflation

Dasgupta, Herdeiro, Hirano, Kallosh '02; Koyama, Tachikawa, Watari '03; Hsu, Kallosh '04; Dasgupta, Hsu, Kallosh, Linde, Zagermann '04; Haack, Kallosh, Krause, Linde, Lüst, Zagermann '08

Type IIB theory compactified on $K3 \times T^2/Z_2$.
Inflationary D3 brane moves on torus T^2, along Re(z) direction:



Resembles SUSY D-term hybrid inflation (?). Flat potential along Re(z) is protected by approximate shift symmetry Re(z)$\rightarrow$Re(z) $+ c$

D3-D7 action; shift symmetry

$K3 \times T^2/Z_2$ has Kähler potential

$$K = -\ln\left(\left[S + \bar{S} - c(z-\bar{z})^2\right]\sum_{i=1}^{19}[T_i + \bar{T}_i]^2\right)$$

volume D3 position K3 moduli

- Has symmetry under shifts of Re(z): gives flat direction in potential at tree level.
- Fayet-Iliopoulos D-term at fixed point $\#1$ gives logarithmic correction to inflaton potential:

$$V_D = \frac{D^2}{2\mathrm{Re}(f_1)}; \quad f_r = S - \frac{1}{a}\left\{\ln\vartheta_1[\pi(z_r - z)|\tau] + \ln\vartheta_1[\pi(z_r + z)|\tau]\right\}$$

gauge kinetic fn. Jacobi theta fn. fixed pt. pos. T^2 modular parameter

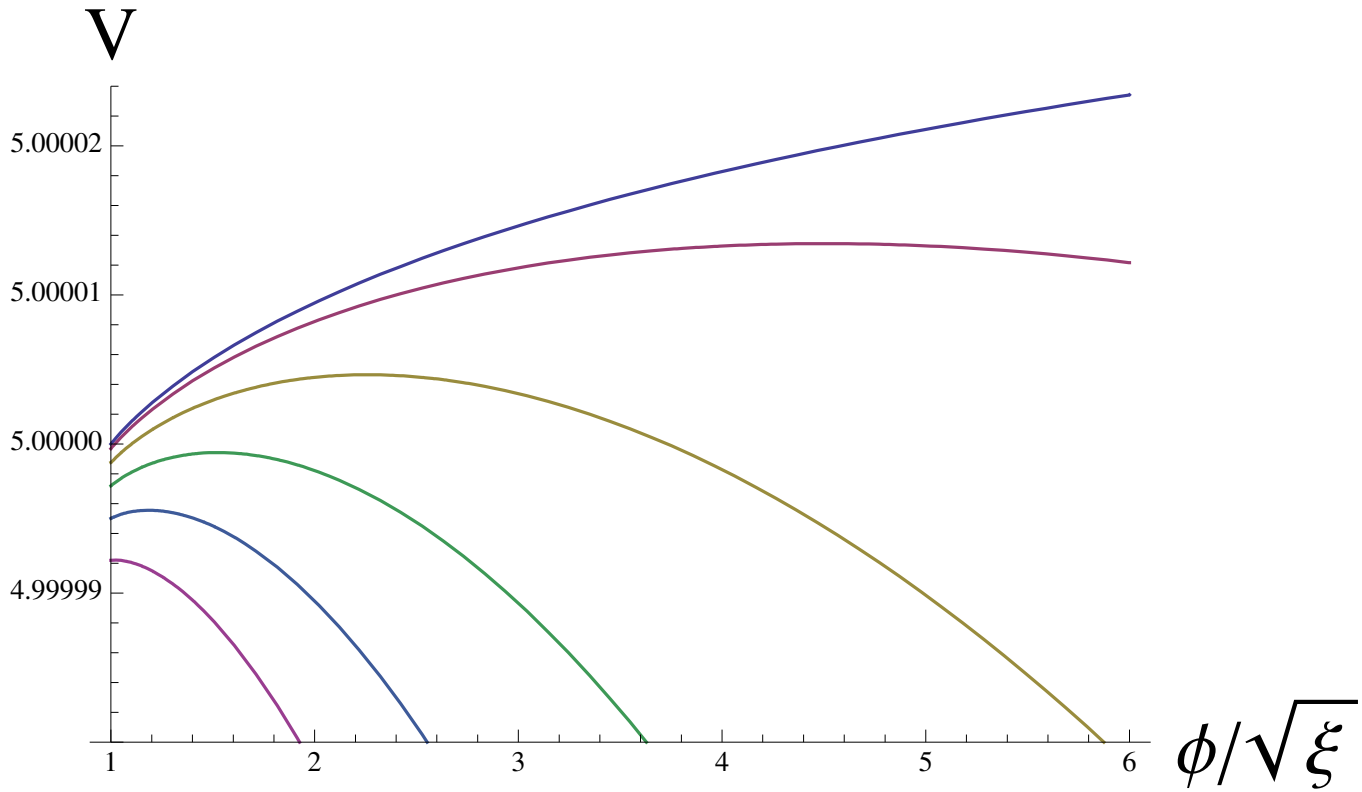
- But superpotential corrections break shift symmetry more strongly:

$$W = W_0 + \sum_r A_r e^{-af_r(S,z)/N_r} + \sum_i B_i e^{-b_i T_i}$$

(Burgess, JC, Dasgupta, Firouzjahi '06)

D3-D7: what kind of inflation?

Recent Haack *et al.* paper finds hilltop inflation by tuning (truncated) superpotential correction against D-term:



But they used only small-z limit of F-term potential.

Using exact potential, we (Burgess, JC, Postma) find different result.

(Exact potential is complicated)

C. Burgess, JC, M. Postma, work in progress

```
In[546]:= Simplify[V1 /. tau2 → τ2 /. tau1 → τ1 /. Np → Np /. EE → D /. Wz → Wz]
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$$
\begin{aligned}
\text{Out[546]=}\ & -D \Big/ \Big(\mathrm{Log}\Big[\mathrm{EllipticTheta}\big[1, -\pi(x - i y), e^{i\pi(\tau_1 + i\tau_2)}\big] \\
& \quad \mathrm{EllipticTheta}\big[1, \pi(x - i y), e^{i\pi(\tau_1 + i\tau_2)}\big]\, \mathrm{EllipticTheta}\big[1, -\pi(x + i y), \\
& \quad e^{i\pi(\tau_1 + i\tau_2)}\big]\, \mathrm{EllipticTheta}\big[1, \pi(x + i y), e^{i\pi(\tau_1 + i\tau_2)}\big]\Big] - 2 s N_p\Big) - \\
& \Big(a\, e^{-2as - 2bt - i(a\alpha + b\beta)} \Big(2 e^{as} \pi N_p^2 \tau_2 W_z[x - i y]\, W_z[x + i y] \\
& \quad \Big(b B e^{a(s + i\alpha)} t \big(B e^{ibb\beta}(2 + bt) + e^{bt}\big(1 + e^{2ibb\beta}\big) W0\big) \tau_2 + A e^{b(t + i\beta)} \\
& \quad \Big(-2\pi\big(B e^{ibb\beta} + e^{bt} W0\big) y^2 + \big(b B e^{ibb\beta} t + a s\big(B e^{ibb\beta} + e^{bt} W0\big)\big)\tau_2\Big) W_z[x + i y]^{\frac{1}{N_p}}\Big) + \\
& \quad A^2 e^{2bt + iaa\alpha + ibb\beta} \tau_2 \big(-2\pi y^2 + a s \tau_2\big) W_z[x - i y]^{\frac{1}{N_p}} W_z[x + i y]^{\frac{1}{N_p}} W_z'[x - i y] \\
& \quad (4 i \pi y N_p W_z[x + i y] + \tau_2 W_z'[x + i y]) + 2 A e^{bt + iaa\alpha} \pi N_p W_z[x - i y]^{1 + \frac{1}{N_p}} \\
& \quad \Big(e^{a(s + i\alpha)} N_p \tau_2 \big(-2\pi\big(B + e^{b(t + i\beta)} W0\big) y^2 + \big(b B t + a s\big(B + e^{b(t + i\beta)} W0\big)\big)\tau_2\Big) \\
& \quad W_z[x + i y] + 2 A e^{b(t + i\beta)} N_p \big(-4\pi^2 y^4 - 2\pi y^2 \tau_2 + a s(1 + a s)\tau_2^2\big) W_z[x + i y]^{1 + \frac{1}{N_p}} + \\
& \quad 2 i A e^{b(t + i\beta)} y \tau_2 \big(2\pi y^2 - a s \tau_2\big) W_z[x + i y]^{\frac{1}{N_p}} W_z'[x + i y]\Big)\Big)\Big) \Big/ \\
& \Big(8\pi t^2 N_p^2 \tau_2 \big(2\pi y^2 - a s \tau_2\big) W_z[x - i y]\, W_z[x + i y]\Big)
\end{aligned}
$$

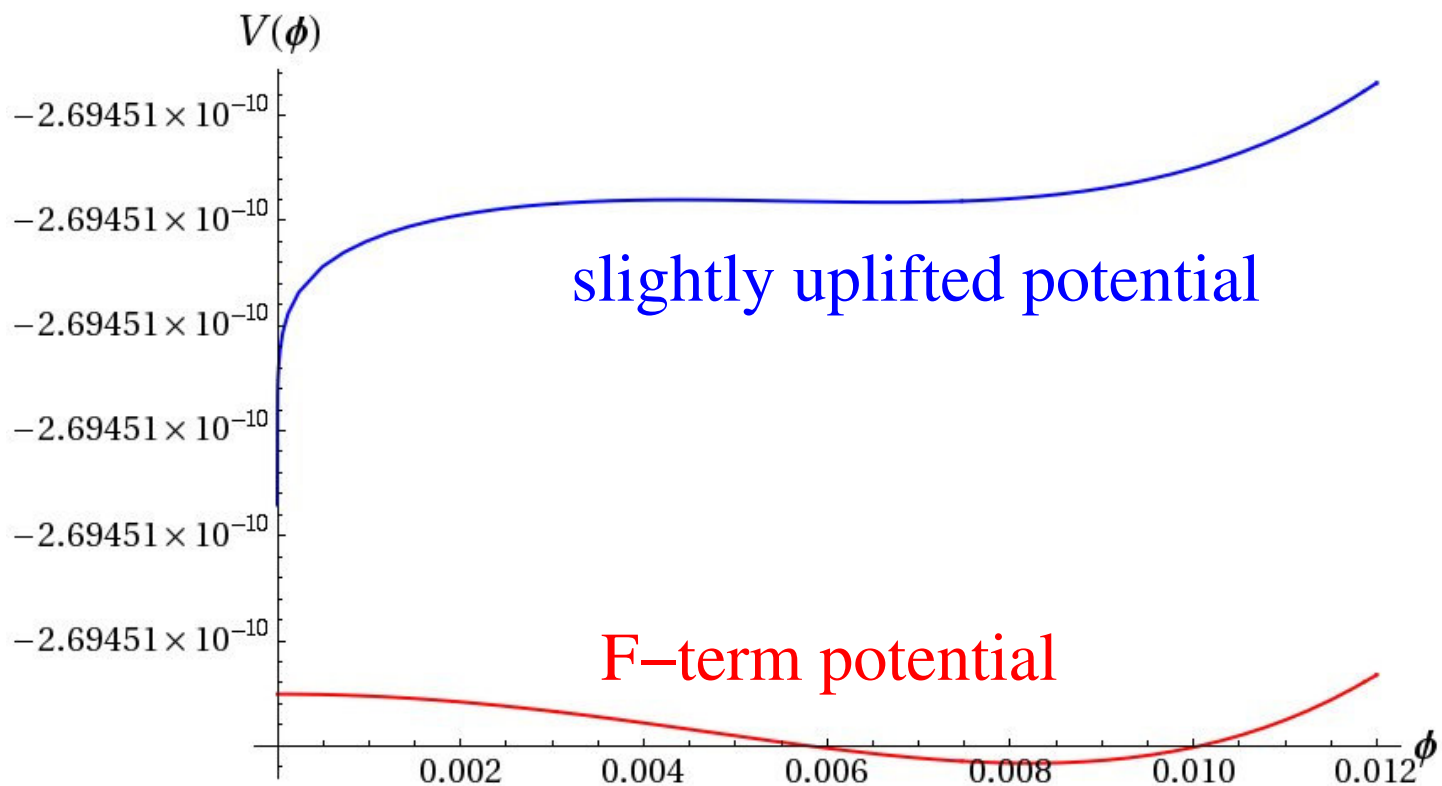
where

$$W_z(z) = \theta_1(\pi(z - \tfrac{1}{2})|\tau)\theta_1(\pi(z + \tfrac{1}{2})|\tau)$$

Why D3-D7 hilltop inflation fails

Potential of form $V = -\frac{1}{2}m^2\phi^2 + A\ln\phi$ has $V'' = -2m^2$ at maximum; must tune m^2 small to get inflation.

But minimum is very shallow for small m^2. Uplifting term destroys minimum even at $10^{-6}\times$ strength needed for uplifting!

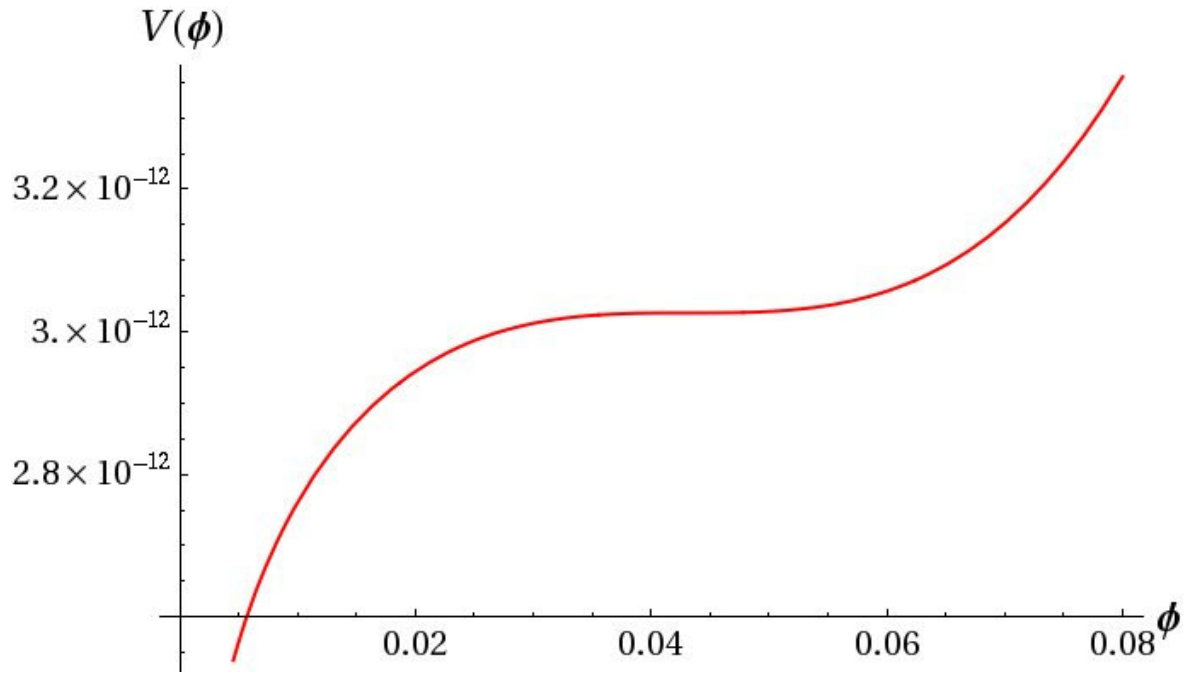


$\lambda\phi^4$ term becomes important when m^2 is small.

D3-D7: inflection point inflation?

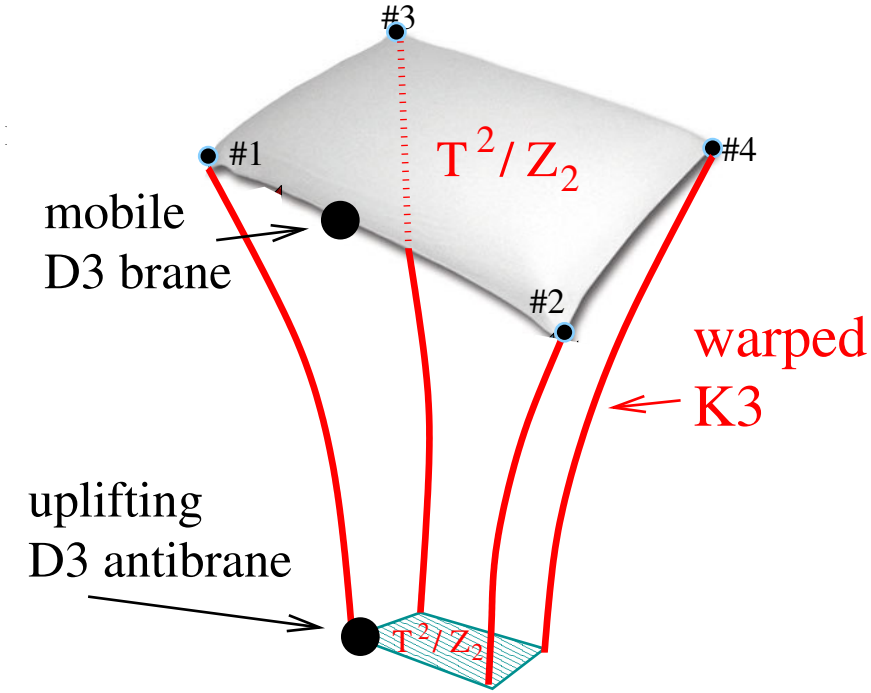
C. Burgess, JC, M. Postma, work in progress

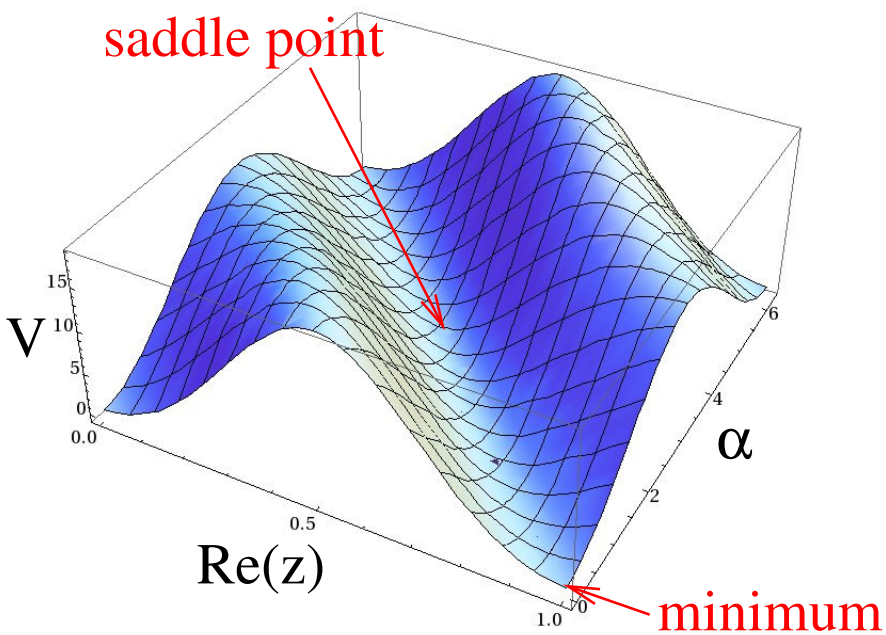
Using exact potential, we instead find inflection point potential:



- However, *extreme* fine tuning of parameters and initial condition is required to get enough inflation.
- Must tune to 7th digit in potential and 3rd digit in initial conditions for 60 e-foldings! Much more delicate than $\mathrm{D3}$-$\overline{\mathrm{D3}}$ inflation.
- D-terms are also theoretically problematic—difficult to keep them nonzero when charged fields relax to minimum.

Better alternative: warped saddle point





(Burgess, JC, Postma, in progress)

Use warped K3 background (Dasgupta, Rajesh, Sethi '99) with anti-D3 uplifting,

$$V_{\rm uplift} \sim e^{\frac{2}{3}K(z,S,T_i)}$$

Assume D3 can be stabilized high in throat, by superpotential corrections of K3 moduli.

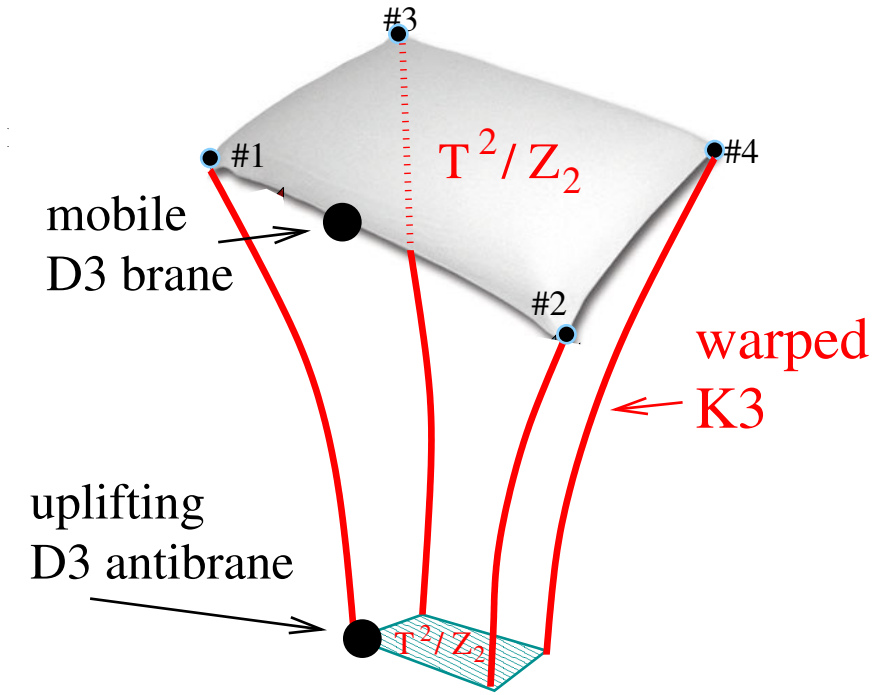
We find saddle point at

$$z_1 \equiv {\rm Re}(z) = \tfrac{1}{2},\ \alpha = \pi$$

$\alpha =$ axion of S (volume) modulus;

Inflaton direction $= 0.99\hat{\alpha} - 0.16\hat{z}_1$

Better alternative: warped saddle point



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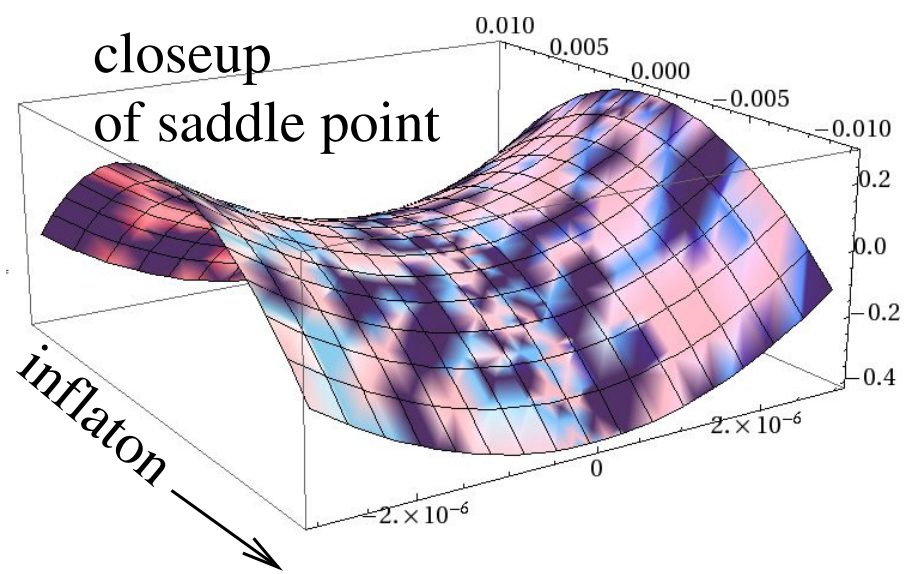

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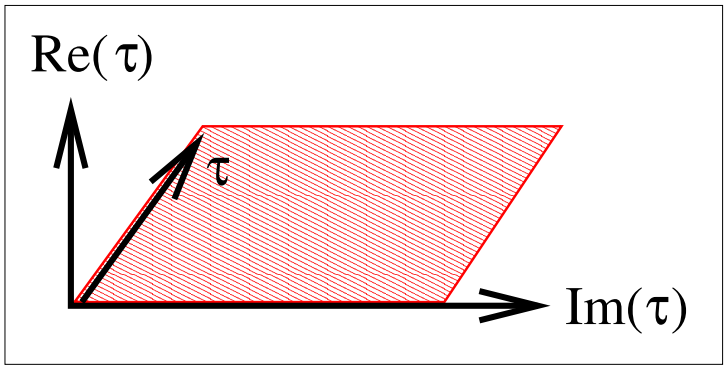
Resembles racetrack inflation

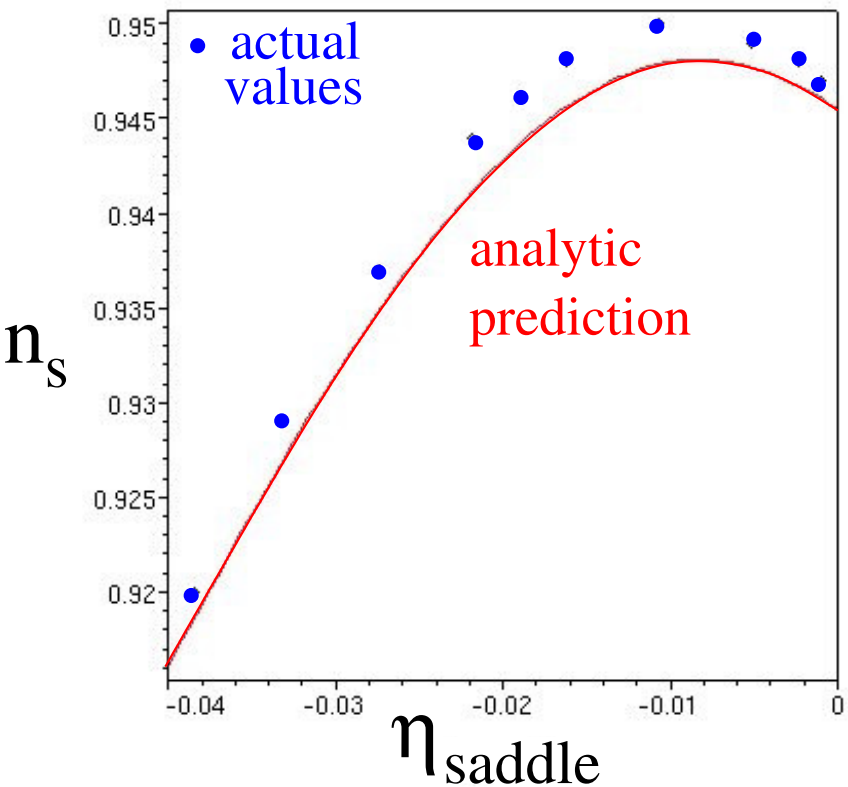
But it is easier to obtain—need only tune T^2 modular parameter τ. A, B, a, b can take any values.



Take Re$(\tau) = 0$ for simplicity.

Need Im$(\tau) \cong 1.00272 - 1.00283$

T^2 is very close to being square.



No initial condition problem, inflation is eternal.

$n_s \leq 0.95$ for all values of $\eta_{\rm saddle}$, just like racetrack model.

We did not find other scenarios for inflation in this model.

Conclusions

- String cosmology has advanced significantly: specific, detailed and constrained string theory constructions are combined with precision data
- Some popular models of string inflation are now disfavored by requirements of internal consistency and data
- Brane inflation scenarios are among most highly developed and studied; seem capable of passing all consistency tests and experimental constraints
- $\mathrm{D3}$-$\overline{\mathrm{D3}}$ inflation not so fine-tuned as initially believed; novel possibility of dynamical self-tuning mechanism by brane tunneling, solving initial condition problem
- $\mathrm{D3}$-$\mathrm{D7}$ with warping and antibrane uplifting may provide a more natural version of racetrack-like inflation