Instability of Anisotropic Inflationary Expansion Supported by a Fixed Norm Vector Field

COSMO 08

Burak Himmetoglu
University of Minnesota

Work with Marco Peloso and Carlo Contaldi, in completion

Motivations: Anomalies observed in the CMB

- Low quadrupole.
- Quadrupole and octopole are aligned and planar.
- Alignment in $\ell = 2$-5 (axis of evil).
- Asymmetry in the power between the northern and southern ecliptic hemispheres.

+

(An apparent non-gaussian deviation in the southern galactic hemisphere (Cold Spot))

An indication to a need for modifying standard cosmology?
(Or at least the earliest stage of inflation)

An anisotropic stage of inflationary expansion could lead to the anomalous alignment of the lowest multipoles.

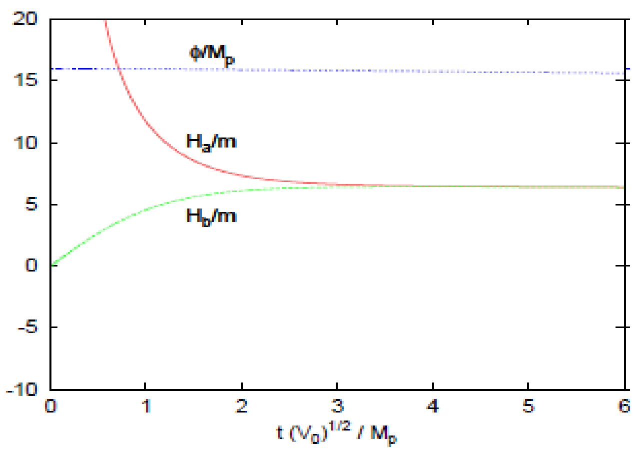
Models so far considered in the literature:

Simplest possibility:

- Anisotropic initial expansion (Bianchi-I) + inflaton.
- Isotropization due to slowly rolling inflaton.

Formalism for cosmological perturbations in this case are developed.

- Gumrukcuoglu-Contaldi-Peloso (astro-ph/0608405 and 0707.4179)
- Pitrou-Pereira-Uzan (0707.0736 and 0801.3596)



- Isotropization occurs after a Kasner singularity (strong gravity solutions).
- No controllable anisotropy (cannot be arbitrary small).
- No firm control over initial conditions.
- One of the metric fluctuations is unstable during the Kasner stage.

Models so far considered in the literature...

Vector Field Driven Models:

Extended anisotropic stage, controllable anisotropy.

- Ford (Phys. Rev. D40: 967,1989)

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A^2)$$

- Golonev-Mukhanov-Vanchurin (0802.2068) (isotropic)
- Kanno-Kimra-Soda-Yokoyama (0806.2422), Yokoyama-Soda (0805.4265) Chiba (0805.4660), Kovisito-Mota (0805.4229)

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\left(m^2 - \frac{R}{6}\right) A_\mu A^\mu$$

- Ackerman-Carroll-Wise (astro-ph/0701357)*

$$\mathcal{L}_A = -\beta_1 \nabla^\mu A^\sigma \nabla_\mu A_\sigma - \beta_2 (\nabla_\mu A^\mu)^2 - \beta_3 \nabla^\mu A^\sigma \nabla_\sigma A_\mu + \lambda (A_\mu A^\mu - m^2)$$

Anisotropic inflation with fixed norm vector fields:

action: $$S = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2} R + \mathcal{L}_A - V_0 \right)$$

constant vacuum energy

$$\mathcal{L}_A = -\beta_1 \nabla^\mu A^\sigma \nabla_\mu A_\sigma - \beta_2 \left(\nabla_\mu A^\mu\right)^2 - \beta_3 \nabla^\mu A^\sigma \nabla_\sigma A_\mu + \lambda \left(A_\mu A^\mu - m^2\right)$$

Lagrange multiplier

Background Solution:

metric: $ds^2 = -dt^2 + a^2(t)\, dx^2 + b^2(t) \left[dy^2 + dz^2\right] \qquad a = e^{H_a t} \;.\; b = e^{H_b t}$

$$H_a = \frac{\sqrt{V_0}/\sqrt{3}M_p}{\sqrt{\left(1 + \beta_1 m^2/M_p^2\right)\left(1 + 4\beta_1 m^2/3M_p^2\right)}} \;, \quad H_b = \left(1 + 2\beta_1 m^2/M_p^2\right) H_a$$

vector field: $A_\mu = (0,\, m\, a,\, 0,\, 0)$

m is a free parameter. $m \to 0$ background -> dS

Linear Perturbations and Stability:

- Ackerman-Carroll-Wise (astro-ph/0701357)

  Perturbations of the test field only $\mathcal{L} \rightarrow \frac{1}{2}\partial_\mu \chi \partial^\mu \chi \rightarrow P_\chi$

  No $\delta g_{\mu\nu}$, δA_μ considered. CMB studies based on this calculation only.

  It is assumed that the spectrum of CMB is same as P_χ

- Linearized stability analysis (Wise, Gresham, Dulaney 0801.2950):

  For $\beta_1 + \beta_2 + \beta_3 = 0$, $\beta_1 > 0$

  - Background is consistent and stable. ($p \gg H$ and $p \ll H$)
  - Energy of fluctuations is positive around the background.

- Problems with general models containing fixed norm vector fields (Clayton gr-qc/0104103)

  It is possible to construct configurations with unbounded energy from below (in Minkowski space).

⇨ We find that problems arise when $p \sim H$

The Instability

- We carried out the full perturbative analysis using gauge invariant (GR) combinations of perturbations and for all ranges of momenta.
- We find an instability @ $p \sim H$
- For illustration, we ignore metric perturbations. Results qualitatively agree.

(we take $\beta_1 = -\beta_3 = 1/2, \quad \beta_2 = 0 \qquad \mathcal{L}_A \to -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$)

$$A_\mu = (0, a\,\mu\,M_p,\, 0,\, 0) + \delta a_\mu \qquad \mu \equiv \frac{m}{M_p} \qquad p_L \equiv \frac{k_L}{a} \quad , \quad p_T \equiv \frac{k_T}{b}$$

$$\delta a_\mu = (\delta_0,\, \delta_1,\, \partial_i \delta + v_i) \;\; , \;\; \partial_i v_i = 0 \qquad i = 2, 3$$

$$\delta a_\mu = \int \frac{d^3 k}{(2\pi)^{3/2}}\, \delta a_\mu(k) e^{-i\,k_L\,x - i\,k_{T,2}\,y - i\,k_{T,,3}\,z}$$

Lagrange multiplier forces ⟶ $\delta_1 = 0$

$$\dot{\delta} \;=\; \frac{p_L^2 + p_T^2 - 2\left(1+\mu^2\right) H_a^2}{p_T^2}\, \delta_0$$

$$\dot{\delta}_0 \;=\; -\frac{\left(1+2\,\mu^2\right) p_L^2 - 2\left(1+\mu^2\right)\left(3+2\,\mu^2\right) H_a^2}{p_L^2 - 2\left(1+\mu^2\right) H_a^2}\, H_a\, \delta_0 - p_T^2\, \delta$$

$$p_{L*}^2 = 2(1+\mu^2)\, H_a^2$$

The Instability...

Expanding around the singularity (set to be @ t=0);

$$p_L^2 = p_{L*}^2 \, e^{-2\,H_a\,t} \quad , \quad p_T^2 = p_{T*}^2 \, e^{-2\,H_b\,t}$$

Approximate analytical soln:

$$\ddot{\delta} + \frac{\dot{\delta}}{t} + p_{T*}^2 \, \delta \simeq 0$$

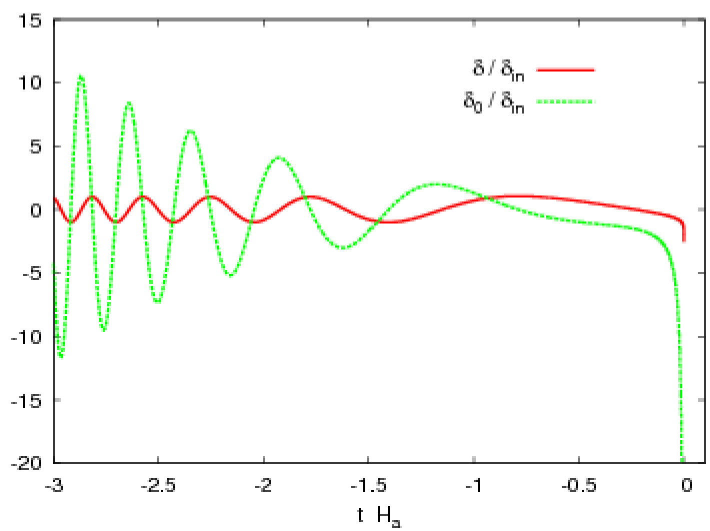
$$\delta \simeq C_1 \, J_0 \left(p_{T*} \, t \right) + C_2 \, Y_0 \left(p_{T*} \, t \right)$$

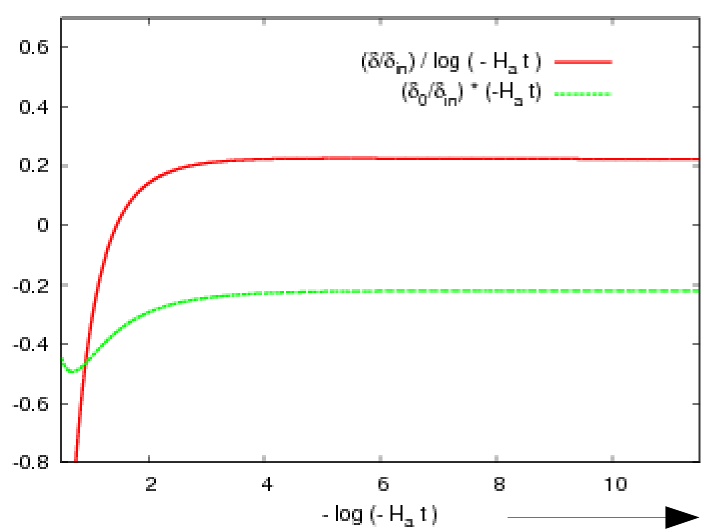
$$\dot{\delta} \simeq \delta_0$$

1/t singularity

log singularity

Numerical Solution to the complete equations:



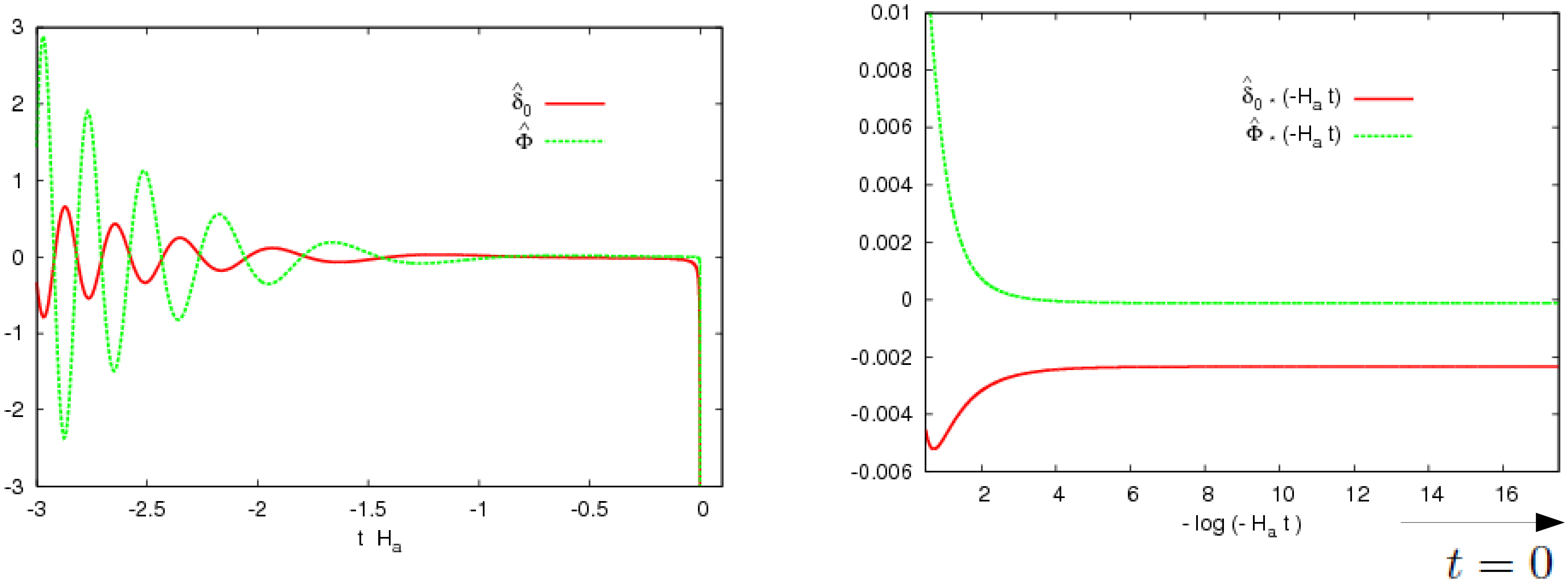


$$t = 0$$

Conclusions

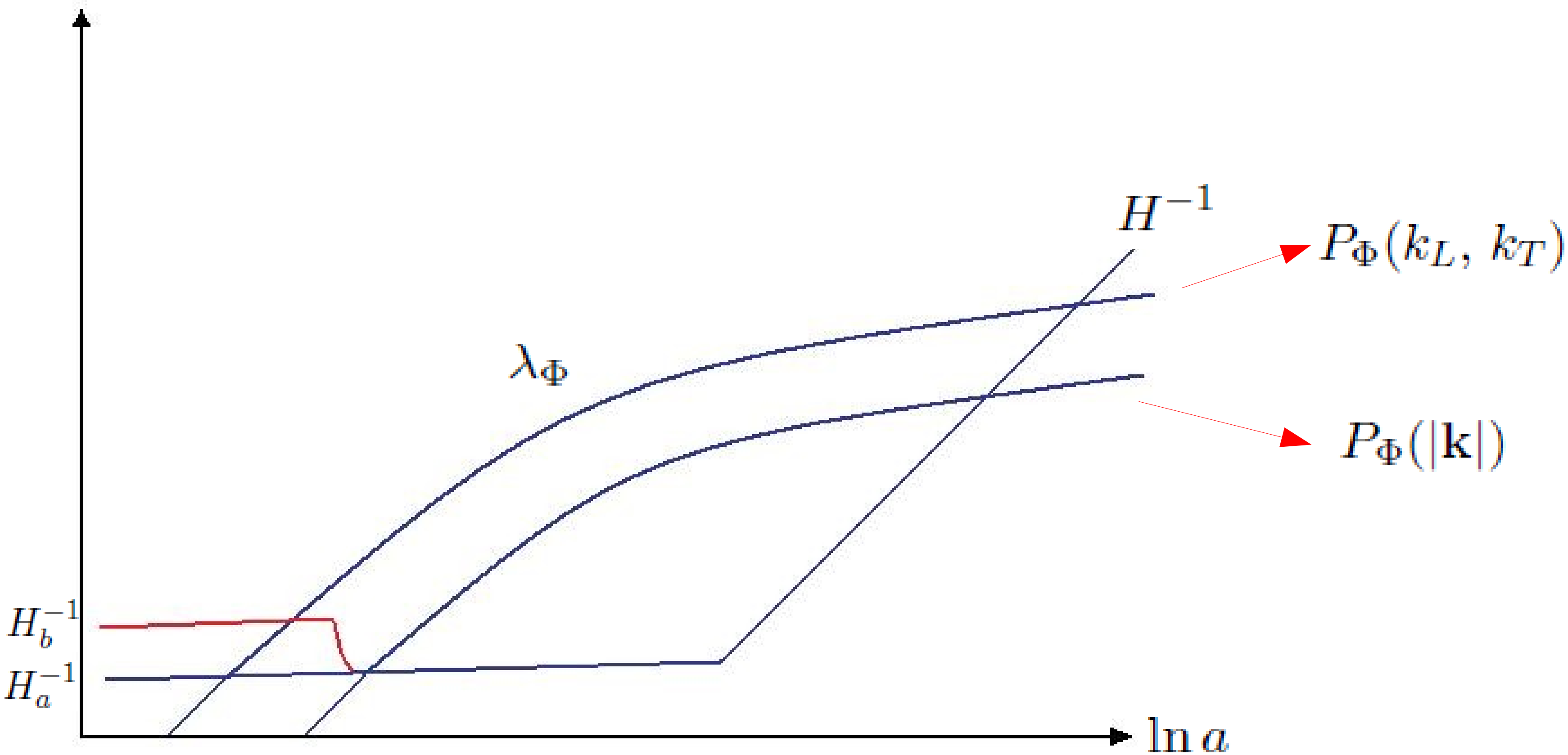
- Linearized study of perturbations indicate an instability of the background. Modes grow beyond perturbative control @ horizon crossing.
- All CMB predictions based on this model cannot be trusted.
- Other alternatives to anisotropic inflation should be investigated.

When metric perturbations are included:



Both the vector field perturbations and the gravitational potential Φ diverge.

How anisotropic inflation works?



Simplest modification, Bianchi-I Universe

$$ds^2 = -dt^2 + a^2(t)\,dx^2 + b^2(t)\,(dy^2 + dz^2)$$

Singularity of the quadratic action

The quadratic action for the vector field perturbations are,

$$S_{\delta,\delta_0} = \frac{1}{2}\int dt\, d^3k\; a\, b^2 \Big\{ p_T^2\,|\dot{\delta}|^2 - p_T^2\left(\delta_0^*\,\dot{\delta} + \delta_0\,\dot{\delta}^*\right) - p_T^2\left[p_L^2 - 2\left(1+\mu^2\right)H_a^2\right]|\delta|^2 + \left[p_L^2 + p_T^2 - 2\left(1+\mu^2\right)H_a^2\right]|\delta_0|^2 \Big\}$$

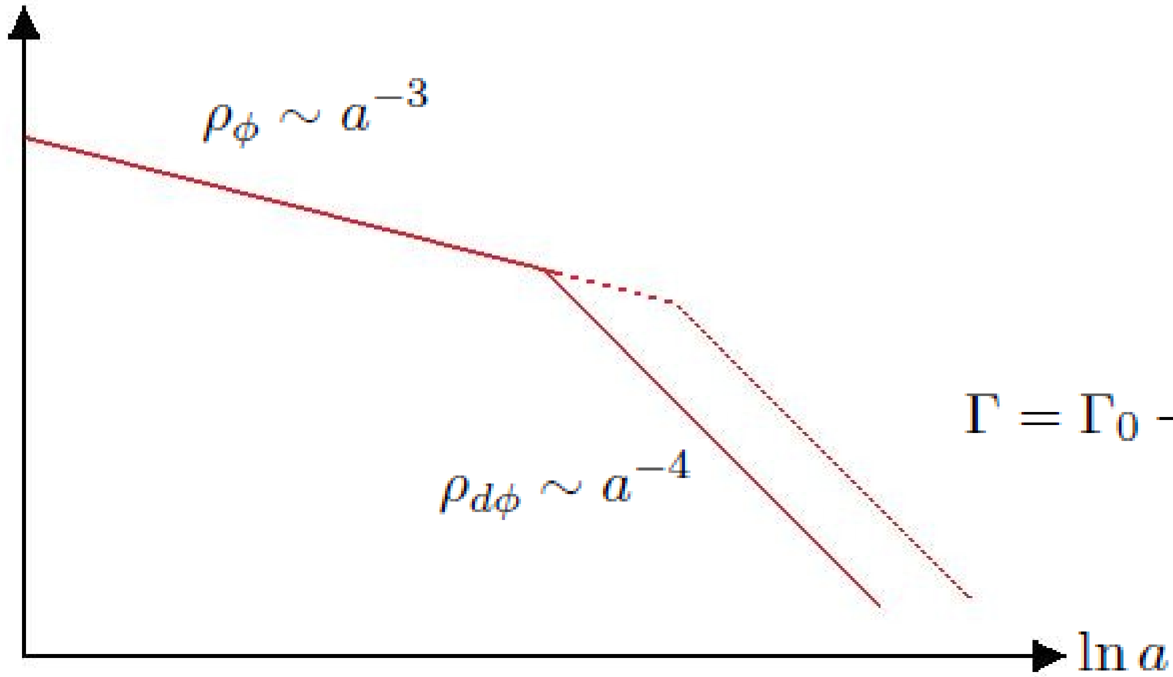
Integrating out δ_0,

$$\delta_0 = \frac{p_T^2}{p_L^2 + p_T^2 - 2\left(1+\mu^2\right)H_a^2}\,\dot{\delta}$$

$$S_\delta = \frac{1}{2}\int dt\, d^3x\; a\, b^2\, p_T^2\left(p_L^2 - 2\left(1+\mu^2\right)H_a^2\right)\left[\frac{|\dot{\delta}|^2}{p_L^2 + p_T^2 - 2\left(1+\mu^2\right)H_a^2} - |\delta|^2\right]$$

The action for the dynamical mode δ vanishes when: $p_{L*}^2 = 2(1+\mu^2)\,H_a^2$

Modulated Perturbations



$$\Gamma = \Gamma_0 + \frac{\delta\Gamma}{\delta\chi}\delta\chi$$

Slightly later decay ⟶ slightly greater energy density

In this way $\delta\chi \to \delta g_{\mu\nu}$

This ignores $\delta g_{\mu\nu}$ that may be produced earlier. We show it blows up.