Cosmo 08

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Radiation Driven Inflation

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Aharon Davidson and I.G., JCAP06(2008)01

Outline

1. Introduction
   I. Brane gravity
   II. Unified brane gravity
   III. Unified Brane cosmology
   IV. Inflation

2. Radiation Driven Inflation
   I. Permanent inflation
   II. Finite inflation
   III. Fluctuations (current and future research)

3. Summary

Brane Gravity

- Randall-Sundrum II (RS) scenario: GR is recovered at large scales / low energies.

$$G_N = G_5\sqrt{-\Lambda_5/6} \equiv G_{RS}$$

  - L. Randall and R. Sundrum, Phys.Rev.Lett.**83**:4690 (1999).

- Dvali-Gabadadze-Poratti (DGP) scenario: GR recovered at small scales / high energies + vDVZ anomaly.

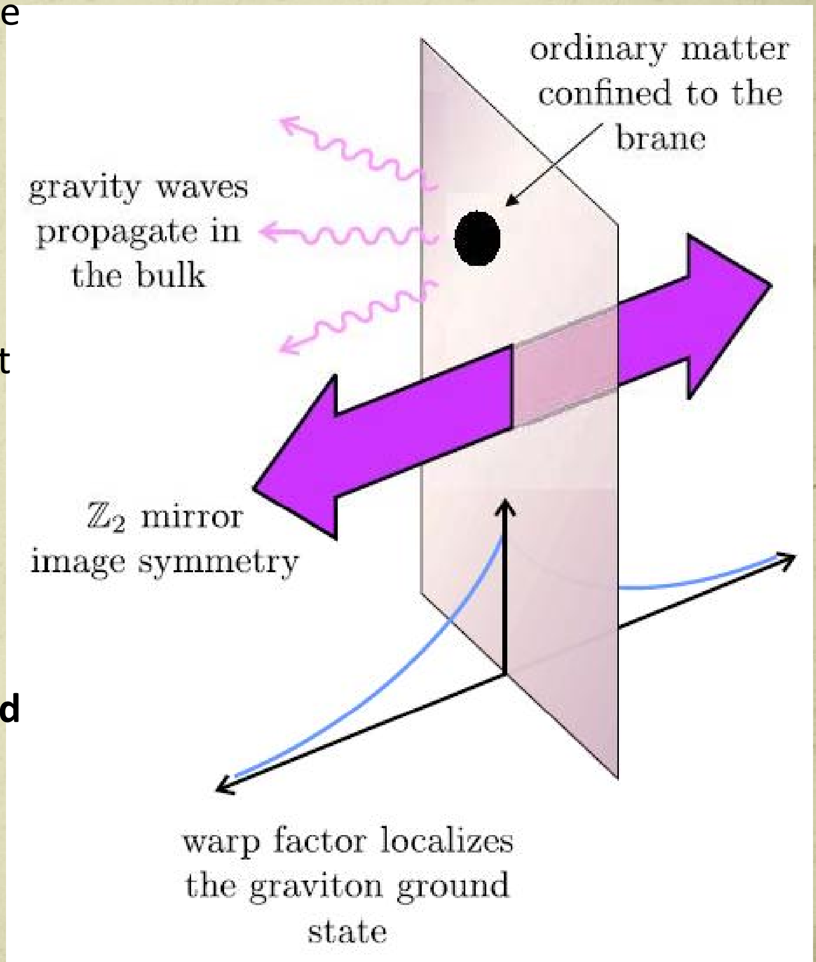
$$G_N = G_4$$

  - G.R. Dvali, G. Gabadadze and M. Porrati, Phys.Lett.**B485**:208 (2000)

- **Collins-Holdom (CH) scenario = RS+DGP: GR is recovered at all scales + no vDVZ anomaly.**

$$1/G_N = 1/G_4 + 1/G_{RS}$$

  - H. Collins and B. Holdom, Phys.Rev.**D62**:105009 (2000).
  - H. Collins and B. Holdom, Phys.Rev.**D62**:124008 (2000).



Dirac's Brane Variation

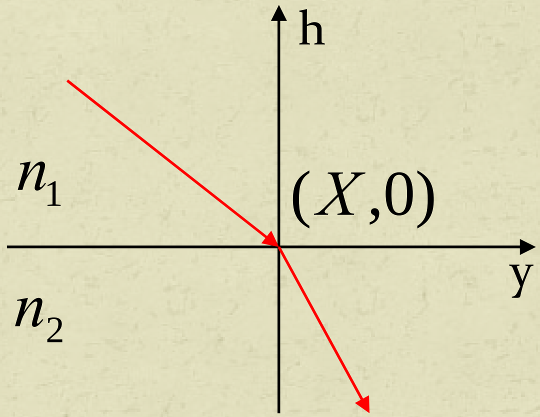
"a tiny deformation of the brane corresponding to the brane being pushed a little to the right will not be minus the variation corresponding to the brane being pushed a little (equally) to the left, on account of the left and right bulk sections not being a smooth continuation of each other"

- P.A.M. Dirac, Proc.Roy.Soc.London **A268**:57 (1962)

Do today's brane theories respect Dirac's prescription for consistent brane variation?

Dirac's Variation: Example

A simple example – Snell's law.



$$I = \int_0^X dy n_1 \sqrt{1+\left(\frac{dh}{dy}\right)^2} + \int_X^L dy n_2 \sqrt{1+\left(\frac{dh}{dy}\right)^2} \Longrightarrow n_1 \sin\theta_2 = n_2 \sin\theta_1$$

$$I = \int_0^{\hat{X}} dz n_1 \sqrt{y'^2 + h'^2} + \int_{\hat{X}}^L dz n_2 \sqrt{y'^2 + h'^2} \Longrightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- D. Karasik and A. Davidson, Class.Quant.Grav.**21**:1295-1302 (2004)

Geodetic Brane

We can get a clue that there is something wrong with the current formalism by looking at the limit of *no bulk gravity*.

$$\frac{1}{8\pi G_5}\left(K_{\mu\nu} - g_{\mu\nu}K\right) = \frac{\partial L_{brane}}{\partial g_{\mu\nu}} \xrightarrow{G_5\to\infty} \frac{\partial L_{brane}}{\partial g_{\mu\nu}} = 0$$

Under this condition we should get the Regge-Tietelboim / Cordero-Vilenkin (stealth brane) model:

$$\frac{\partial L_{brane}}{\partial g_{\mu\nu}} K^{\mu\nu} = 0$$

- T. Regge and C. Tietelboim, in Proc. Marcel Grossman (Trieste) 77 (1975)
- R. Cordero and A. Vilenkin, Phys.Rev.**D65**:083519 (2002).

Unified Brane Gravity

The Action:

$$I=\sum_{L,R}\int d^5y\left(-\frac{1}{16\pi G_5}R+L_m\right)\sqrt{-G}$$

$$+\sum_{L,R}\int d^4x\frac{1}{8\pi G_5}K\sqrt{-g}+I_{brane}+I_{con}$$

$$I_{con}=\sum_{L,R}\int d^4x\Big[\lambda^{\mu\nu}\left(g_{\mu\nu}-G_{AB}y^A_{,\mu}y^B_{,\nu}\right)$$

$$+\eta^\mu y^A_{,\mu}n_A+\sigma\left(G^{AB}n_An_B-1\right)\Big]\sqrt{-g}$$

Unified Brane Gravity

The Variation:

- Variation with respect to the normal.

$$\eta_\mu = 0 \,,$$

$$\sigma = -\frac{1}{16\pi G_5} K \,.$$

- Variation with respect to the brane metric.

$$\frac{1}{16\pi G_4}\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) + \frac{1}{2} T^{\mu\nu} + \sum_{L,R}\left(\lambda^{\mu\nu} - \frac{1}{8\pi G_5}\left(K^{\mu\nu} - \frac{1}{2} g^{\mu\nu} K\right)\right) = 0$$

- Variation with respect to the 5-dimensional metric.

$$R_{AB} - \frac{1}{2} G_{AB} R = 8\pi G_5 T_{AB}$$

- Variation with respect to the 5-dimensional metric on the brane.

Dirac's variation is implemented:

$$\left[\left(\frac{1}{16\pi G_5}\mathbf{K}^{\mu\nu} - \boldsymbol{\lambda}^{\mu\nu}\right)\mathbf{y}^{A}_{,\mu}\right]_{;\nu} + \boldsymbol{\Gamma}^{A}_{BC}\left(\frac{1}{16\pi G_5}\mathbf{K}^{\mu\nu} - \boldsymbol{\lambda}^{\mu\nu}\right)\mathbf{y}^{B}_{,\mu}\mathbf{y}^{C}_{,\nu} = \mathbf{0}$$

Unified Brane Gravity

The general variation:

$$\delta G_{ab} = \underbrace{\delta_1 G_{ab}}_{\substack{general \\ \text{var}iation}} + \underbrace{G_{AB,C}\, y^A_{,a} y^B_{,b} \delta y^C + 2G_{AB}\, y^A_{,a} \delta y^B_{,b}}_{general\ \ coordinate\ \ transformation}$$

In the usual scenario, the general variation has all the 15 degrees of freedom and thus “consumes” the general coordinate transformation degrees of freedom.

$$\delta_1 G_{ab}\big|_{BRANE} = 0$$

However, on the brane, if general variation is permitted, **<u>the brane's location is changed during the variation, violating Dirac’s linearity of the variation.</u>**

On the brane:

Unified Brane Gravity

$$S_{\mu\nu} \equiv \frac{1}{8\pi G_4}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + T_{\mu\nu} - \frac{1}{8\pi G_5}\left[K_{\mu\nu} - g_{\mu\nu} K\right]_{L+R} = \left[\lambda_{\mu\nu}\right]_{L+R}.$$

- $$\lambda_L^{\mu\nu} K_{\mu\nu}^L = \lambda_R^{\mu\nu} K_{\mu\nu}^R = 0,$$
- $$\lambda_{L;\nu}^{\mu\nu} = \lambda_{R;\nu}^{\mu\nu} = 0,$$

An effective 5D energy-momentum of the brane that includes both matter and gravity. For CH it is 0, however, in the more general case, it has no components in the acceleration direction (Analogue to the geodesic equation) and is conserved.

- A. Davidson and I.G., Phys.Rev.**D74**:044023 (2006)

$\lambda \to 0$ ⟶ CH limit $$\frac{1}{8\pi G_4}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + T_{\mu\nu} - \frac{1}{8\pi G_5}\left[K_{\mu\nu} - g_{\mu\nu} K\right]_{L+R} = 0.$$

$\lambda, G_4^{-1} \to 0$ ⟶ RS limit $$T_{\mu\nu} - \frac{1}{8\pi G_5}\left[K_{\mu\nu} - g_{\mu\nu} K\right]_{L+R} = 0.$$

$\lambda, \Lambda_5 \to 0$ ⟶ DGP limit $$\frac{1}{8\pi G_4}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + T_{\mu\nu} - \frac{1}{8\pi G_5}\left[K_{\mu\nu} - g_{\mu\nu} K\right]_{L+R} = 0.$$

$G_5^{-1} \to 0$ ⟶ RT/CV limit $$\frac{1}{8\pi G_4}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + T_{\mu\nu} = \lambda_{\mu\nu}.$$

Unified Brane Cosmology

General cosmological embedding

General radially symmetric 5-dimensional metric

$$ds_5^2 = -F(R)dT^2 + G(R)dR^2 + R^2 d\Sigma_{3,k}^2$$

On the brane

$$ds_b^2 = -dt^2 + a(t)^2 d\Sigma_{3,k}^2$$

The embedding

$$dt = dT\sqrt{\frac{F(a)}{1+G(a)\dot{a}^2}}, \qquad R = a(t)$$

$$K_{00} = -\frac{\dot{a}^2 G'(a)}{2\sqrt{G(a)}\left(1+\dot{a}^2 G(a)\right)} - \ddot{a}\sqrt{\frac{G(a)}{1+\dot{a}^2 G(a)}} - \frac{F'(a)}{2F(a)}\sqrt{\frac{1+\dot{a}^2 G(a)}{G(a)}}$$

$$K_{ij} = \frac{1}{a}\sqrt{\frac{1+\dot{a}^2 G(a)}{G(a)}}\, g_{ij}$$

Unified Brane Cosmology

$$H^2 + \frac{k}{a^2} = \xi^2 + \frac{\Lambda_5}{6}$$

$$P(\xi) \equiv \frac{3\xi^3}{8\pi G_4} + \frac{3\xi^2}{4\pi G_5} - \left(\frac{\Lambda_5}{16\pi G_4} - \rho(a)\right)\xi - \frac{\omega}{\sqrt{3}a^4} = 0$$

- ω is the "embedding energy" of the brane.
- If ω is 0, we return to the familiar CH (RS and DGP) brane cosmology.

Example: RS Cosmology from UBC

$$\frac{3\xi^2}{4\pi G_5} - \rho(a)\xi = 0 \Rightarrow \frac{3\xi}{4\pi G_5} - \rho(a) = 0 \Rightarrow \xi^2 = \left(\frac{4\pi G_5}{3}\right)^2 \rho(a)^2$$

$$H^2 + \frac{k}{a^2} = \xi^2 + \frac{\Lambda_5}{6} = \left(\frac{4\pi G_5}{3}\right)^2 \rho(a)^2 + \frac{\Lambda_5}{6}$$

$$= \left(\frac{4\pi G_5}{3}\right)^2 \left(\tilde{\rho}(a) + \frac{3}{4\pi G_5}\sqrt{-\frac{\Lambda_5}{6}}\right)^2 + \frac{\Lambda_5}{6}$$

$$= \frac{8\pi G_5}{3}\sqrt{-\frac{\Lambda_5}{6}}\tilde{\rho}(a) + \left(\frac{4\pi G_5}{3}\right)^2 \tilde{\rho}(a)^2$$

$$= \frac{8\pi G_{RS}}{3}\tilde{\rho}(a) + \left(\frac{4\pi G_5}{3}\right)^2 \tilde{\rho}(a)^2$$

Example: DGP Cosmology from UBC

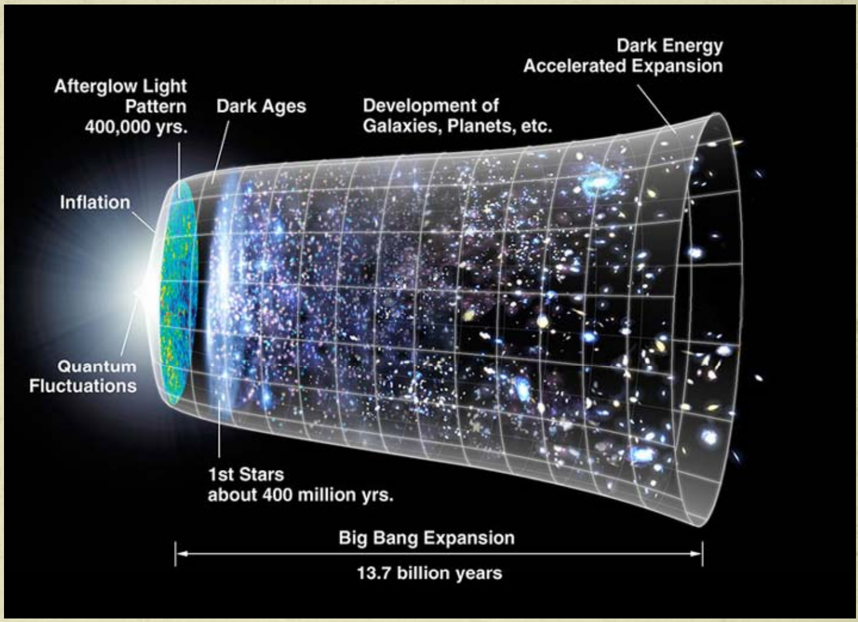
$$\frac{3\xi^3}{8\pi G_4}+\frac{3\xi^2}{4\pi G_5}-\rho(a)\xi=0 \;\Rightarrow\; \xi^2+\frac{2G_4\xi}{G_5}-\frac{8\pi G_4}{3}\rho(a)=0$$

$$\Rightarrow\; \xi_\pm=-\frac{G_4}{G_5}\pm\sqrt{\left(\frac{G_4}{G_5}\right)^2+\frac{8\pi G_4}{3}\rho(a)}$$

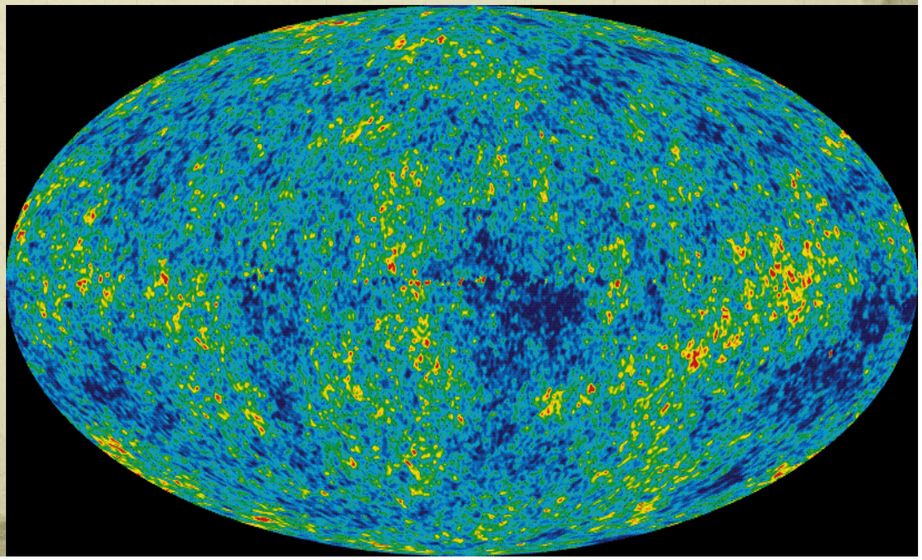
$$\Rightarrow\; \xi_\pm^2=2\left(\frac{G_4}{G_5}\right)^2+\frac{8\pi G_4}{3}\rho(a)\mp 2\frac{G_4}{G_5}\sqrt{\left(\frac{G_4}{G_5}\right)^2+\frac{8\pi G_4}{3}\rho(a)}$$

$$H^2+\frac{k}{a^2}=\xi^2=2\left(\frac{G_4}{G_5}\right)^2+\frac{8\pi G_4}{3}\rho(a)\mp 2\frac{G_4}{G_5}\sqrt{\left(\frac{G_4}{G_5}\right)^2+\frac{8\pi G_4}{3}\rho(a)}$$

Inflation and Brane Inflation
- from a critics view

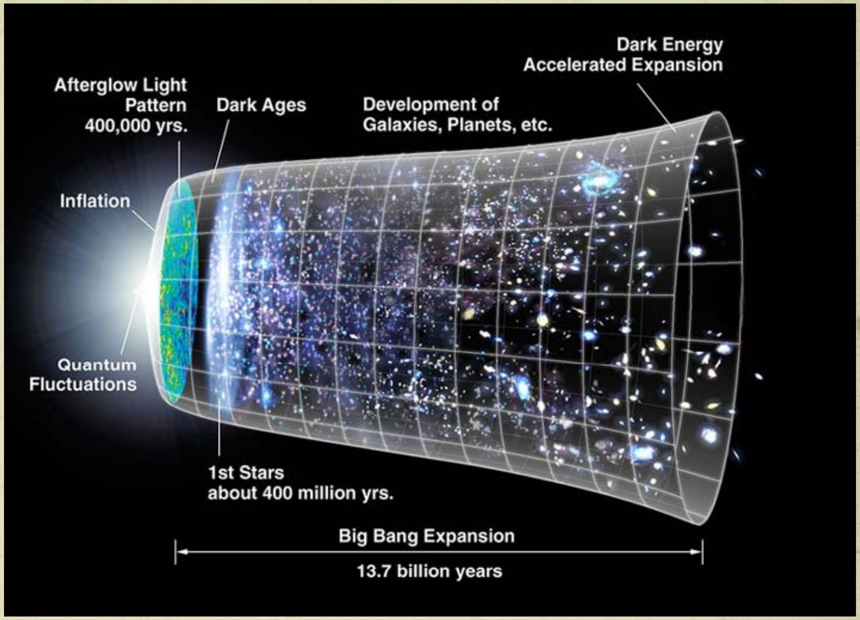


- Inflation is a mandatory element of theoretical cosmology – It resolves multiple problems: horizon, flatness and unwanted relics.



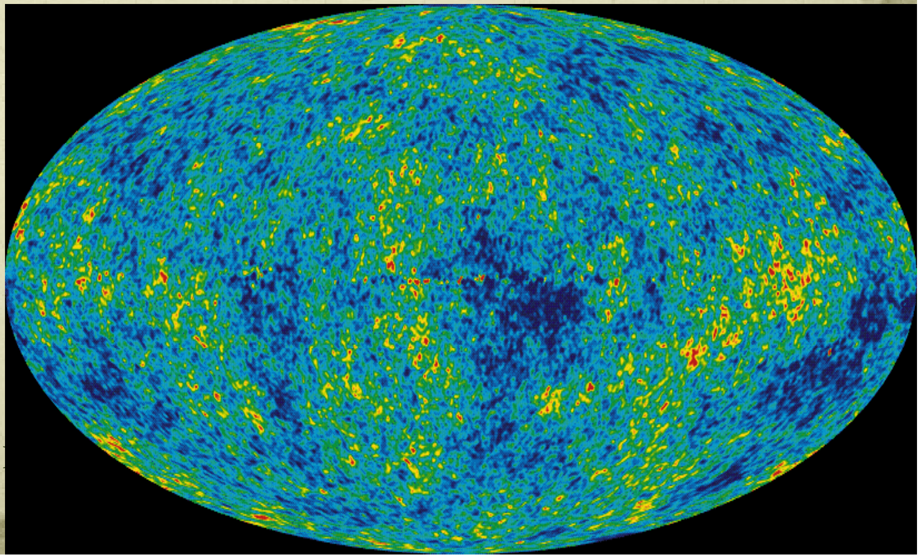
- Following the recent precision cosmology measurements of WMAP, inflation is now an essential component of experimental cosmology as well.

Inflation and Brane Inflation
- from a critics view



<u>However…</u>

- The mechanism for inflation is unknown. Suggested mechanisms often suffer from:
  - sensitivity to initial conditions (this throws away the purpose of inflation).
  - No physical basis (solutions that are "engineered" to generate inflation rather than follow from a physical theory).
  - The scalar fields are non-renormalizable.



Inflation and Brane Inflation
- from a critics view

<u>However…</u>

- The mechanism for inflation is unknown. Suggested mechanisms often suffer from:
  - sensitivity to initial conditions (this throws away the purpose of inflation).
  - No physical basis (solutions that are "engineered" to generate inflation rather than follow from a physical theory).
  - The scalar fields are non-renormalizable.
- The popular scalar field approach is an excellent way to study fluctuations, but is not a viable mechanism. Replacing the arbitrary H(a) in cosmology with the arbitrary V(ϕ) scalar potential is not giving an answer but rather rephrasing the question.

Inflation and Brane Inflation
- from a critics view

Despite its apparent simplicity, inflation is not easy to generate properly. Inflation must provide:

- An exit mechanism.
- Enough e-folds.
- Sufficient reheating (particle creation).

All that without contradicting the known particle physics.
This is very hard to do in the framework of GR.

So, is inflation really generated by an exotic scalar field?

Or, can we find a natural mechanism for inflation outside GR?

Permanent inflation in UBC

Permanent inflation (= de Sitter space) requires ξ=const.

This dictates:

$$\rho(a) = \underbrace{\frac{1}{8\pi}\left(\frac{\omega^2}{G_4 A^2} + \frac{2\sqrt{3}\omega}{G_5 A} + \frac{\Lambda_5}{2G}\right)}_{\sigma_0} + \frac{A}{a^4}$$

$$\underbrace{3\left(H^2 + \frac{k}{a^2}\right)}_{=\xi^2 + \frac{1}{6}\Lambda_5} = \Lambda = \frac{1}{2}\Lambda_5 + \frac{\omega^2}{A^2}$$

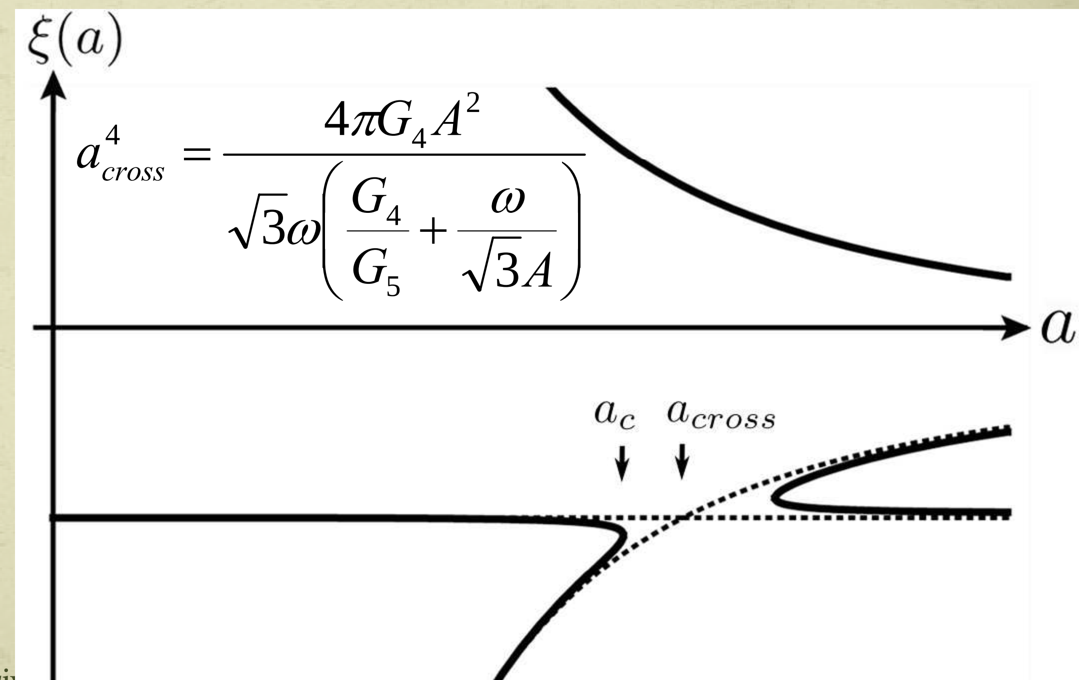
Cosmological Phase Transition

The solution in hand cannot be eternal. The cosmological cubic equation reveals a phase transition.

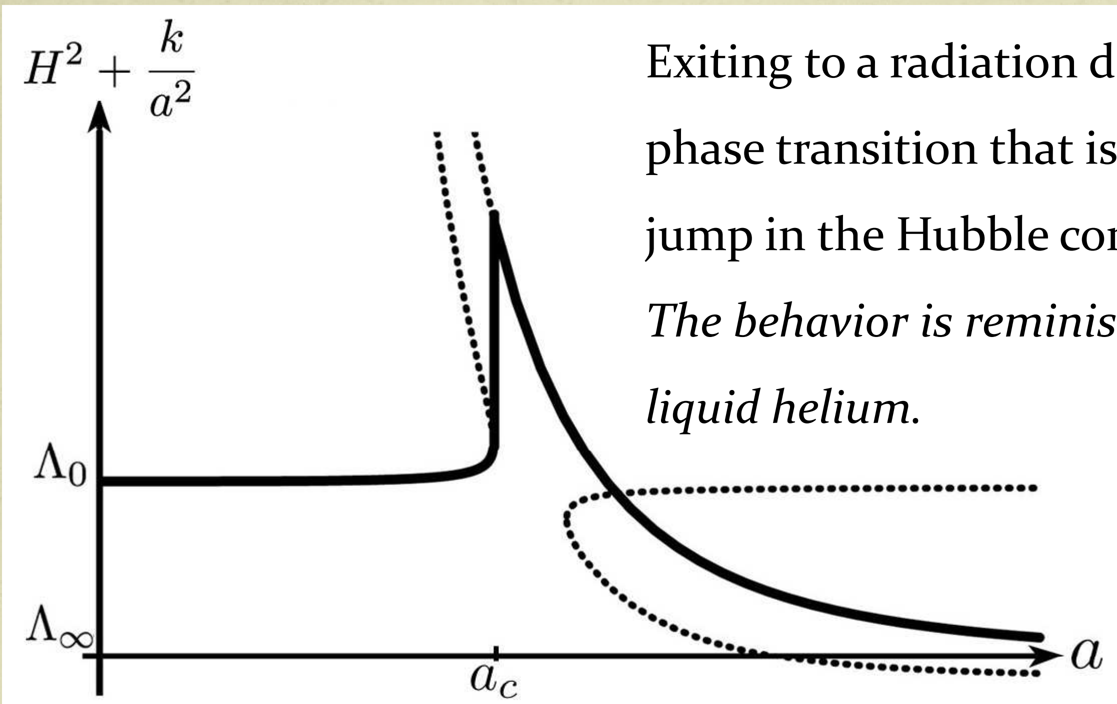
If $\sigma \neq \sigma_0$, inflation will end at a point, almost independent of the amount of deviation.

We choose $\sigma = \sigma_{RS} = \frac{3}{4\pi G_5}\sqrt{-\frac{\Lambda_5}{6}}$

To ensure the exiting universe has a zero cosmological constant (a good approximation for the small cosmological constant in our universe).



Radiation Driven Inflation



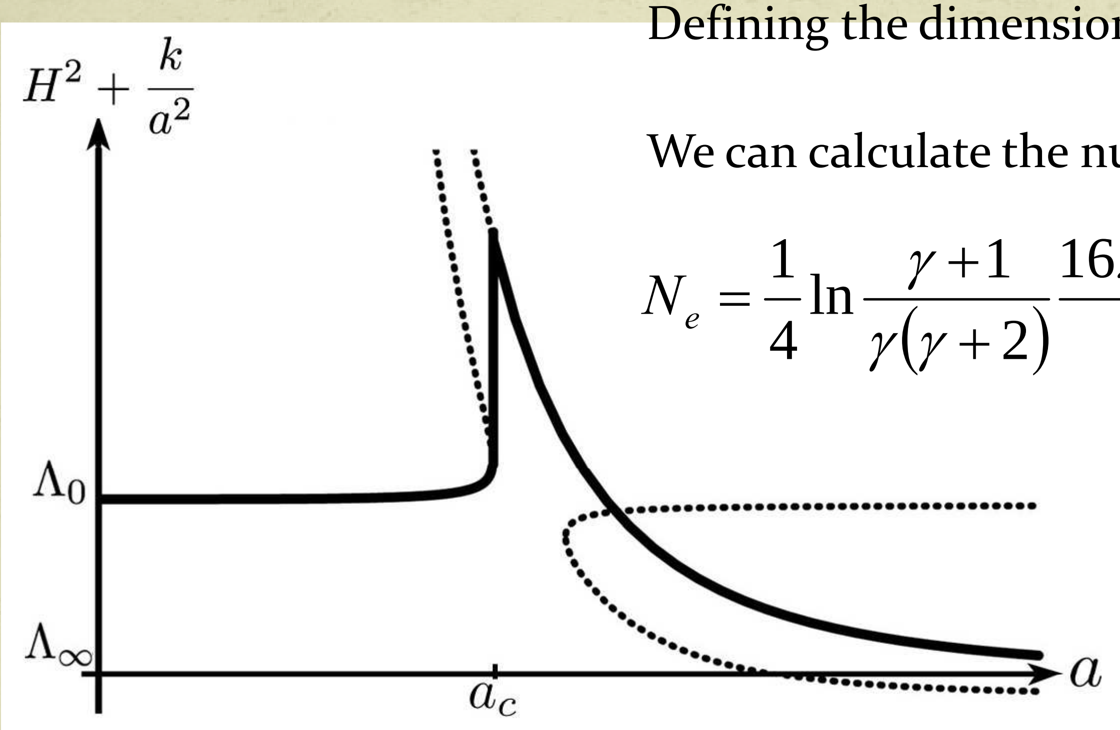
The full solution exhibits a “hysteresis like” behavior.

Exiting to a radiation dominated universe via a phase transition that is accompanied by a finite jump in the Hubble constant.

The behavior is reminiscent of the λ-transition in liquid helium.

Approaching the transition $\ddot{a} \propto \left(a_c - a\right)^{-1/2}$

Radiation Driven Inflation



Defining the dimensionless constant: $\gamma \equiv \frac{G_{RS}}{G_4}$

We can calculate the number of e-folds:

$$N_e = \frac{1}{4}\ln\frac{\gamma+1}{\gamma(\gamma+2)}\frac{16\pi G_N A\Lambda}{9k^2} + \ln\frac{a_c}{a_{cross}} \geq 65$$

Radiation Driven Inflation

- The apparent singularity is smoothened by the quantum fluctuations.
- The RDI model predicts a huge momentary jump in the Hubble constant. This could have an interesting and measurable effect on the CMB fluctuations.
- The affect on the CMB could be model independent and thus have a research appeal beyond the specific model.

Warning: Preliminary!!!

Warning: Preliminary!!!

Fluctuations of RDI

- In the critical case ($\sigma_{RS} = \sigma_0$) and assuming the radiation consists only of photons (so that $L = F_{\mu\nu}F^{\mu\nu}/4$ exactly), $n_s = 1$. Any deviation will lead to slightly different values.
- The phase transition is smoothened, so that there is no real singularity.
  - In a simplified model the smoothening can be described as a locally "more advanced" cosmological evolution, so that the phase transition starts earlier when nearing a positive perturbation.

$$a_{eff}(t,r) \approx \overline{a}(t)(1+h(r))$$

$$H(t,r) \approx H_{FRW}(a_{eff}) \Rightarrow \overline{H}(t) \approx \int_r H_{FRW}(a_{eff}(t,h(r)))dh$$

Warning: Preliminary!!!

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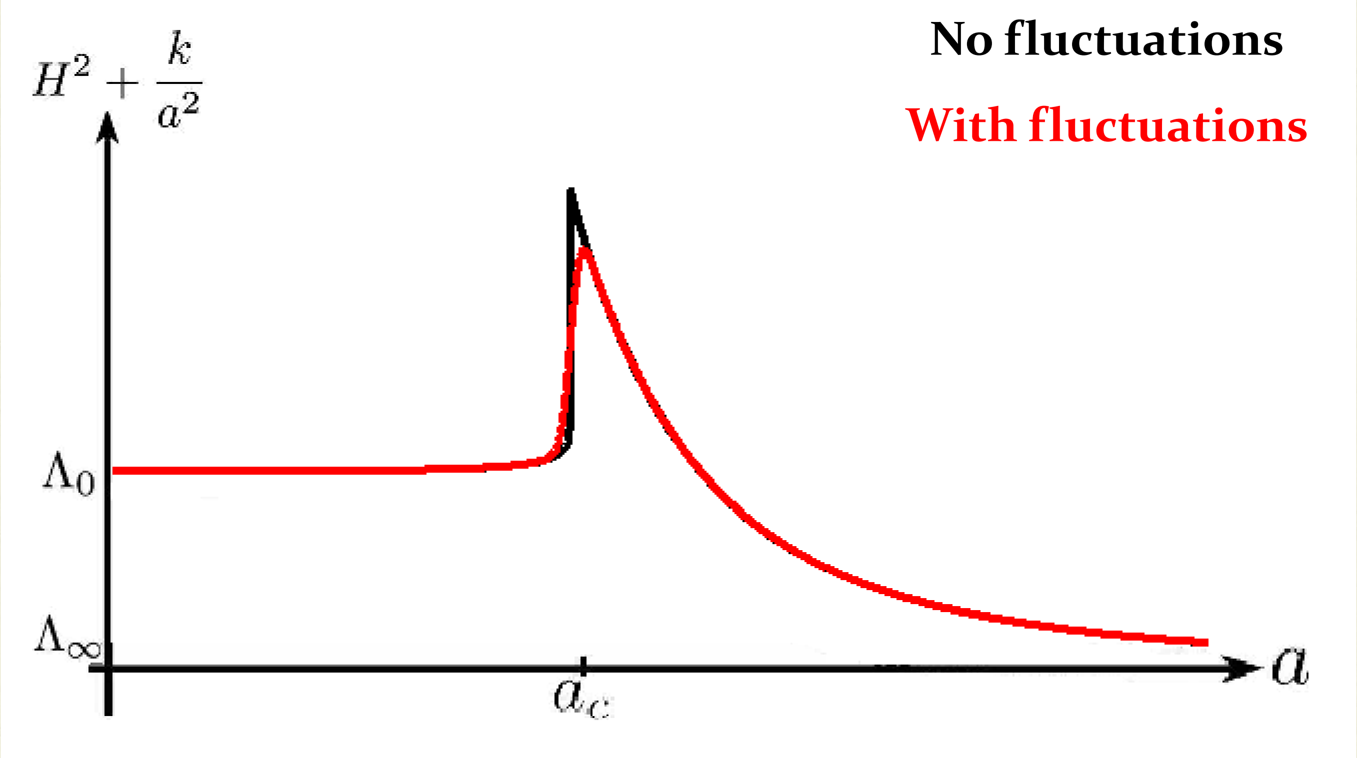
Fluctuations of RDI



The singularity disappears and we are left with a point in time with very high $\dot{H}$.

Summary and Future Directions

- We present a natural, no fine-tuning, mechanism for cosmic inflation.
- Fluctuation are under research.
- The radiation driven inflation predicts a finite jump in the Hubble constant, that can have unique affects on the CMB and maybe interesting even model-independently.
- Possibly, fluctuations can be studied experimentally, using liquid helium (not without precedent)
  - D.I. Bradley et al. Nature Phys.4:46 (2008)

Thank You

End