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Radiation-driven inflation

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Abstract. A novel, scalar-field-free approach to cosmic inflation is presented. The inflationary Universe and the radiation-dominated Universe are shown, within the framework of unified brane cosmology, to be two different phases governed by one and the same energy density. The phase transition of second order (the Hubble constant exhibits a finite jump) appears naturally and serves as the exit mechanism. No re-heating is needed. The required number of e-folds is achieved without fine tuning.

Keywords: cosmology with extra dimensions, cosmological phase transitions, inflation

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The remarkable idea that our Universe has undergone, in its very earliest stages of evolution, a phase of exponential expansion [1, 2] is widely accepted as the solution to the horizon, flatness and magnetic monopole puzzles. The inflationary Universe scenario [3], which has gained strong experimental support from the detailed observations of the cosmic microwave background radiation [4], is now considered part of the standard hot big bang cosmology. With this in mind, it is highly frustrating that the physical mechanism underlying inflation is essentially obscure. The conventional ad hoc theoretical prescription of the inflationary scenario invokes a scalar field of some sort, in analogy to the Higgs field introduced in particle physics. The accompanying potential is carefully engineered to address certain desired features of inflation. It is not clear what degrees of freedom are collectively represented by this so-called inflaton, and what actually determines the shape of its model-dependent potential. The beginning and end of the inflationary era are theoretical challenges by themselves, with the major goal being the production of a sufficient number $N_{\rm e} \simeq 60$ of e-folds. While the beginning may resemble a spontaneous creation, by means of quantum nucleation [5, 6], the end traditionally occurs once the inflaton starts oscillating around the absolute minimum of the scalar potential.

In this paper, within the framework of unified brane gravity [7], we present a novel approach to cosmic inflation. The inflationary Universe and the subsequent radiation-dominated Universe are shown to be two different phases governed by one and the same brane energy density. It differs from other models of inflation in that it does not involve scalar fields and/or scalar potentials [8]. Alternatively, the model forces the brane energy/momentum tensor, predominantly in the inflationary phase, to consist of radiation and surface tension, which are essential standard cosmological ingredients. Furthermore, the model offers a natural built-in exit mechanism, implemented by means of a second-order phase transition. The Universe exiting the inflationary era is then necessarily radiation-dominated and hot. In turn, no re-heating [2, 9] is needed, neither as a graceful end of Guth's inflation, nor as the arena for recreating matter and filling the Universe with radiation. In addition, the required number of e-folds emerges naturally without fine tuning.

Dirac [10], in his 'Extensible Model of the Electron', has paved the way for a consistent brane variation. He was concerned with the fact that the 'linearity of the variation' may be in jeopardy. Rephrasing Dirac, 'a tiny deformation of the brane corresponding to the brane being pushed a little to the right will not be minus the variation corresponding to the brane being pushed a little (equally) to the left, on account of the left and right bulk sections not being a smooth continuation of each other'. To bypass the problem, a general curvilinear coordinate system has been invoked, such that in the new so-called Dirac frame, *the location of the brane does not change during the variation process.* Imposing Dirac's prescription on modern brane theories based on an action principle, such as the Collins–Holdom [11] model, which unifies the familiar Randall–Sundrum [12] and the Dvali–Gabadadze–Porrati [13] models, and assuming a discrete Z_2 symmetry on simplicity grounds, the corresponding brane field equations (see [7] for the derivation) take the form

$$\frac{1}{4\pi G_5}\left(K_{\mu\nu} - g_{\mu\nu}K\right) = \frac{1}{8\pi G_4}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + T_{\mu\nu} + \lambda_{\mu\nu}, \tag{1}$$

where $G_{5(4)}$ denote the gravitational coupling constants in the bulk (brane), respectively. In addition to the conventional terms (the Israel [14] junction term, the Einstein tensor associated with the scalar curvature R on the brane and the physical energy–momentum tensor of the brane $T_{\mu\nu} = \delta\mathcal{L}_{\text{matter}}/\delta g^{\mu\nu}$), unified brane gravity furthermore gives rise to $\lambda_{\mu\nu}$. The latter tensor consists of Lagrange multipliers associated with the fundamental induced metric constraint $g_{\mu\nu}(x) - g_{MN}(y(x))y^M_{,\mu}y^N_{,\nu} = 0$. It has been proven that $\lambda_{\mu\nu}$ is conserved and that its contraction with the extrinsic curvature vanishes:

$$\lambda^{\mu\nu}_{;\nu} = 0, \qquad \lambda_{\mu\nu}K^{\mu\nu} = 0. \tag{2}$$

As is evident from the above field equations, $\lambda_{\mu\nu}$ serves as a geometrical (embedding originated) contribution to the total energy–momentum tensor of the brane and, as such, may have far-reaching gravitational consequences. It is thus crucial to make sure that, although deviating from the RS approach, the GR limit is still there. Cosmological analysis has been shown [7] to reproduce the GR limit. On top of it, by analyzing the weak field perturbations caused by the presence of matter on the brane, we have already recovered [15] the four-dimensional Newton force law.

Within a cosmological framework, equation (1) can be integrated out [7], giving rise to a novel constant of integration ω, the fingerprint of unified brane cosmology. The corresponding FRW equation which governs the evolution of the four-dimensional brane can be conveniently written in the form

$$\frac{8\pi G_N}{3}\rho_{\text{eff}}(a) \equiv \frac{\dot{a}^2 + k}{a^2} = \frac{\Lambda_5}{6} + \xi^2(a). \tag{3}$$

If the only energy/momentum source in the bulk is a negative cosmological constant $\Lambda_5 < 0$, then $\xi(a)$ has been shown [7] to be a solution of the cubic equation

$$P(\xi) \equiv \frac{3\xi^2}{8\pi G_4} + \frac{3\xi}{4\pi G_5} + \frac{\Lambda_5}{16\pi G_4} - \rho(a) + \frac{\omega}{\sqrt{3}\xi a^4} = 0. \tag{4}$$

Equations (3) and (4) thus generalize the familiar RS, DGP and CH cosmologies, which are recovered at the $\omega \to 0$ limit. The Regge–Teitelboim [16] Cordero–Vilenkin [17] stealth Universe models are manifest at the $G_5 \to \infty$ limit. Like in all brane cosmologies [18, 19], it is crucial to notice that $\rho_{\text{eff}}(a)$ defined in equation (3) is no longer the physical energy density on the brane, but rather a function of it. It is $\rho(a) = T^0_0$ which serves as the physical energy density on the brane.

Let us first focus attention on the special case of *eternal* inflation, which clearly requires equation (4) to admit an *exact* a-independent solution. Such a solution, conveniently written as $\xi(a) = \omega/\sqrt{3}A$, with A being a constant, as depicted by the straight horizontal (dashed) asymptote in figure 1, corresponding to a (positive by assumption) cosmological constant

$$\Lambda_0 = \frac{1}{2}\Lambda_5 + \frac{\omega^2}{A^2}, \tag{5}$$

may exist for some conventional energy density $\rho(a)$. The serendipitous observation now being that $\rho(a)$ must solely consist (as hinted first in [20]) of radiation accompanied by a particular amount of surface tension:

$$\rho(a) = \frac{A}{a^4} + \sigma_0, \qquad \sigma_0 = \frac{1}{8\pi}\left(\frac{\omega^2}{G_4 A^2} + \frac{2\sqrt{3}\omega}{G_5 A} + \frac{\Lambda_5}{2G_4}\right). \tag{6}$$

The ‘no-ghost’ condition $A > 0$ is then naturally adopted.

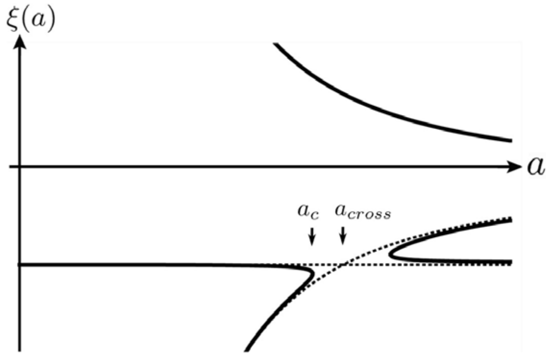

Figure 1. Given $\rho(a) = \sigma + A/a^4$, two roots of cubic equation (4) exhibit a hyperbolic structure. The intersection point of the two asymptotes (the straight horizontal one associated with eternal inflation) sets a natural scale for terminating inflation.

Counter-intuitively, however, even 'eternal' inflation cannot last forever in our model. The slightest deviation of the physical surface tension σ from the particular σ_0 value will expose, as shown in figure 1, the hyperbolic structure of the $\xi(a)$ roots. The amount of finite inflation, however, is insensitive to the value of σ. The intersection point of the two asymptotes (the straight one is associated with eternal inflation), as determined directly from equation (4), with equation (6) imposed, sets a natural FRW scale $a_{\rm cross}$ for terminating inflation, namely

$$a^4_{\rm cross} = \frac{4\pi G_4 A^2}{\sqrt{3}\omega\left(G_4/G_5 + \omega/\sqrt{3}A\right)}. \tag{7}$$

The early Universe cosmological constant Λ_0 is also not sensitive to the value of the surface tension σ. In fact, on realistic grounds (to be revealed soon), σ_0 will be traded for the Randall–Sundrum surface tension

$$\sigma_{\rm RS} = \frac{3}{4\pi G_5}\sqrt{-\frac{\Lambda_5}{6}}, \tag{8}$$

with the price being a finite, yet sufficient, amount of radiation-driven inflation.

At very short scale factors, the situation may appear to be confusing. While the energy density $\rho(a)$ clearly explodes as $a \to 0$, the effective energy density $\rho_{\rm eff}(a)$ stays finite in this limit. What this means is that, in the absence of an ultraviolet cutoff (included in particular is the $k = 0$ flat case), the inflationary era can, in fact, start at an arbitrarily small scale. In turn, the total number of e-folds can become arbitrarily large. In such a case, $a_{\rm enter}$ will mark the edge of validity (set by the Planck scale) of our classical field equations. If $k > 0$, on the other hand, the cosmological scale factor marking the entrance of inflation gets fixed by $a_{\rm enter} = \sqrt{3k/\Lambda_0}$. Interpreted as spontaneous creation of a closed baby Universe, such an entrance is presumably governed by the Hawking–Hartle [5] no-boundary proposal, or alternatively by Davidson–Karasik–Lederer [6] brane nucleation.

A great number of physical processes may occur during the later stages of cosmic evolution. One of which, for example, reflecting the fact that the massive particles have eventually cooled down, is the appearance of a dust term B/a^3 in the energy–momentum tensor. Our interest is focused, however, on the early Universe exiting the inflationary era, and not on the very late Universe. In this respect, it remains to be seen whether radiation-driven inflation, based on $\rho(a) = \sigma + A/a^4$, is capable of spontaneously inducing an exit mechanism, and whether the emerging Universe happens to correspond to the physical one we know of. If we insist on $\sigma > 0$, to ensure a positive Newton's constant at the general relativistic limit (as well as brane stability), the cubic equation (4) admits three real roots such that $\xi_-(\infty) < \xi_0(\infty) = 0 < \xi_+(\infty)$ as $a \to \infty$. Only the largest of these roots, namely

$$\xi_+(\infty) = -\frac{G_4}{G_5} + \sqrt{\frac{8\pi G_4}{3}\sigma + \frac{G_4^2}{G_5^2} - \frac{\Lambda_5}{6}}, \tag{9}$$

is capable of supporting an adjustably tiny cosmological constant $\Lambda_\infty \simeq 0$, as required by the present Universe (the other two roots lead to a negative and a positive definite Λ_∞, respectively). This naturally calls for the familiar Randall–Sundrum fine tuning of the surface tension, given by equation (8), with the subsequent Collins–Holdom identification [11] of Newton's constant:

$$\frac{1}{G_N} = \frac{1}{G_4} + \frac{1}{G_5}\sqrt{-\frac{6}{\Lambda_5}}. \tag{10}$$

To probe the nature of the Universe at the post-inflationary era, we expand $\xi_+(a)$ to learn that it is radiation-dominated:

$$\frac{\dot{a}^2 + k}{a^2} \simeq \frac{8\pi G_N}{3}\left(1 - \frac{\omega}{A}\sqrt{-\frac{2}{\Lambda_5}}\right)\frac{A}{a^4}, \tag{11}$$

provided the enhancement factor (in parentheses) is positive.

At this stage, an apparent contradiction is encountered. On the one hand, inflation has been shown to single out the $\xi_0(a)$ branch at the small-a regime, whereas it is the $\xi_+(a)$ branch which is required for the large-a regime. One must thus closely follow the time evolution of the three roots, with the FRW scale factor serving as the evolution parameter. Notice that the role of a finite a is to add the linear piece $(-A\xi + (1/\sqrt{3})\omega)/a^4$ to the asymptotic ($a \to \infty$) expression of $P(\xi)$. There are four cases to examine, corresponding to the different regions on the ξ axis where $\omega/\sqrt{3}A$ can be located. A careful analysis reveals that (i) there is no ω for which $\xi(a)$ would analytically evolve from $\xi_0(a)$ at small a to $\xi_+(a)$ at large a, (ii) there exists a range of parameters, namely

$$\frac{\omega}{\sqrt{3}A} < \xi_-(\infty) = -\frac{2G_4}{G_5} - \sqrt{-\frac{\Lambda_5}{6}}, \tag{12}$$

for which $\xi_0(a)$ at small a *must* connect with $\xi_+(a)$ at large a. The connection is established by means of a second-order phase transition, and (iii) within the above range, the radiation enhancement factor in equation (11) is positive, in fact > 2. The exiting Universe is thus *necessarily* radiation-dominated.

Plotted in figure 2, with a serving as the parameter, is the master cubic polynomial $P(\xi)$. As long as the FRW scale factor is sub-critical, that is $a < a_c$, there exist three real solutions, the central of which, $\xi_0(a)$, is recognized as the inflation-oriented solution.

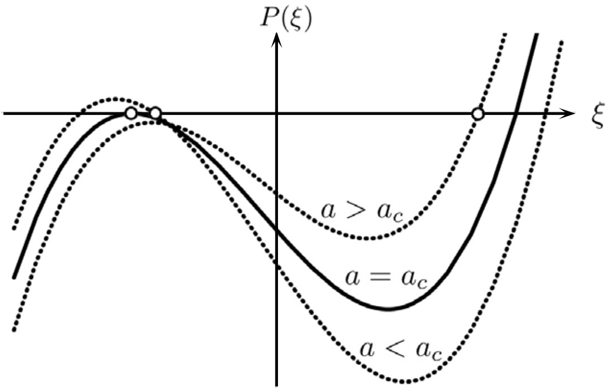

Figure 2. The roots $\xi(a)$ of the cubic equation $P(\xi) = 0$ serve to express the effective energy density $\rho_{\rm eff}(a)$ as a function of the physical energy density $\rho(a)$. Responding to the finite jump $\xi_0(a_c - \epsilon) \to \xi_+(a_c + \epsilon)$, the Universe undergoes an inflation $\to$ radiation-domination phase transition.

Once a approaches criticality, $\xi_0(a)$ and $\xi_-(a)$ merge and are about to mutually disappear (becoming complex) as a crosses the $a_{\rm c}$ barrier. The only leftover real solution is then $\xi_+(a)$. Responding to the finite jump $\xi_0(a_{\rm c}) = \xi_-(a_{\rm c}) \to \xi_+(a_{\rm c})$, the Universe undergoes a second-order phase transition. While the FRW scale factor $a(t)$ remains continuous, the (positive) Hubble constant exhibits a sudden *finite increase* (note that $\ddot{a} \to +\infty$ when nearing a_c from below). As in any phase transition in physics, however, it is expected that fluctuations will smooth the jump ($\ddot{a} < +\infty$). The effective energy–momentum tensor is characterized by a typical critical behavior. Expanding near (below) the critical point, one finds

$$\begin{aligned} \rho_{\rm eff}(a) &\simeq \alpha - \beta(a_{\rm c} - a)^{1/2}, \\ P_{\rm eff}(a) &\simeq -\frac{\beta a_{\rm c}}{6}(a_{\rm c} - a)^{-1/2}, \end{aligned} \tag{13}$$

for some positive constants α, β. $P_{\rm eff}(a)$ is the corresponding effective pressure. The effective energy density $\rho_{\rm eff}(a)$, characterized by a finite jump at the critical scale, is plotted in figure 3. The shape of the graph (not necessarily its physics) greatly reminds us of the specific heat as a function of $1/T$ (which, in fact, is the scale factor) in the λ transition of liquid helium. At any rate, as a keeps growing in the radiation-dominated phase, one will face once again three real roots, but this time for a change, real $\xi_+(a)$ is never lost. It is interesting to remark that, due to the hysteresis-like nature of the evolution, time reversibility is not respected. In other words, a shrinking radiation-dominated Universe will *not* undergo deflation.

To estimate the total number $N_{\rm e}$ of e-folds generated by radiation-driven inflation, one needs to calculate the critical value $a_{\rm c}$ of the FRW scale factor. On simplicity grounds (nothing to do with fine tuning), let us do it in the limit where ω approaches the right edge of the physically allowed range specified by equation (12), that is for $\omega \to \sqrt{3}A\xi_-(\infty)$, which is fully equivalent to setting $\sigma_{\rm RS} \to \sigma_0$. This simple limit is characterized by the fact that the two branches $\xi_-(a)$ and $\xi_0(a)$ intersect, with the intersection point serving as the origin of a local hyperbolic structure generated once ω deviates from the limit value. For $\omega \leq \sqrt{3}A\xi_-(\infty)$, we have $a_{\rm c} \leq a_{\rm cross}$.

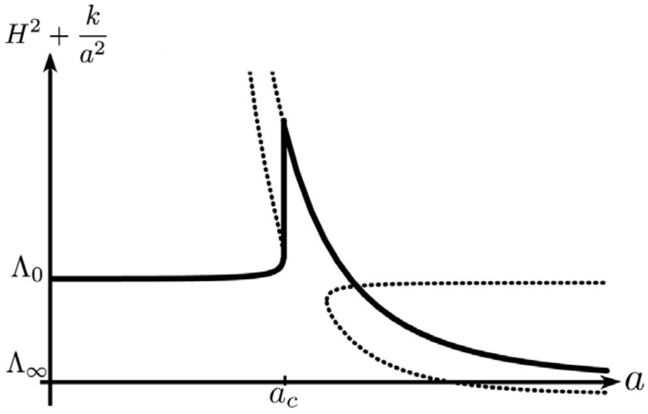

Figure 3. The early Universe phase transition, connecting the inflationary $(a < a_c)$ and the radiation-dominated $(a > a_c)$ eras, is characterized by a finite jump in the Hubble constant H. Remarkably, the two phases share one and the same physical energy density $\rho(a) = \sigma_{\rm RS} + A/a^4$.

We now claim that, up to the $\Lambda_\infty \simeq 0$ fixing, which is a severe fine-tuning problem by itself, our model is free of the conventional inflation-oriented fine tuning. To address the naturalness issue of the various ratios in the theory, we define the dimensionless positive constant

$$\gamma = \frac{G_5}{G_4}\sqrt{-\frac{\Lambda_5}{6}} > 0. \tag{14}$$

Combining the entrance and exit scales, we can estimate the total number of e-folds and find

$$N_{\rm e} \leq \frac{1}{4}\ln\frac{16\pi(\gamma+1)G_N A\Lambda_0}{9\gamma(\gamma+2)k^2}. \tag{15}$$

It is customary to associate the entrance with the Planck scale (the alternative being the GUT scale), that is $\Lambda_0 \simeq 1/t^2_{\rm Planck}$, and further require, taking into account today's relative suppression of radiation versus matter densities, $A \simeq 10^{-4}\rho_{\rm c}t^4_{\rm Hubble}$, where $\rho_{\rm c} = 3/8\pi G_N t^2_{\rm Hubble}$ denotes today's critical energy density. This brings us to the region around

$$N_{\rm e} \leq \frac{1}{2}\ln\frac{10^{-2}t_{\rm Hubble}}{t_{\rm Planck,GUT}} = \begin{cases} 69 & \text{Planck} \\ 65 & \text{GUT.} \end{cases} \tag{16}$$

Although the result is not that sensitive to the value of γ (for example, changing γ by a factor of 100 will only contribute ± 1), it is fully consistent with γ being roughly $\mathcal{O}(1)$, thus emphasizing the naturalness of radiation-driven inflation.

To summarize, the general idea of radiation-driven inflation may seem to be self-contradictory at first glance. After all, it takes a deviation from general relativity to allow the physical energy density $\rho(a)$, stemming from the brane matter Lagrangian, to differ from $\rho_{\rm eff}(a)$, the effective source of the FRW geometry. Within the framework of unified brane gravity, the interplay of these two energy densities is taken one step beyond the Randall–Sundram model, when noticing that $\rho_{\rm eff}(a) \simeq$ const actually dictates $\rho(a) \sim 1/a^4$

at small scales (note that, at such small scales, all particles, massive as well, would be ultra-relativistic and thus radiation-like). This opens the door for the fascinating possibility that the inflationary Universe and the subsequent radiation-dominated Universe are, in fact, two different phases governed by one and the same physical energy–momentum tensor. Furthermore, unlike other models of inflation, brane inflation models included, the present one does not invoke ad hoc scalar fields. As a consequence, the Universe must have been hot before as well as after the phase transition. No re-heating is thus in order. Furthermore, notice that unlike in early models of inflation, $a_{\rm exit}$ and hence the number of e-folds does not depend here on initial conditions. Obviously, since a scalar potential is not a part of our theory, conditions such as 'starting almost at rest at the top of the hill' and 'slow-roll' are irrelevant. Altogether, up to the usual $\Lambda_\infty \simeq 0$, radiation-driven inflation is fine-tuning-free. Needless to say, a number of theoretical questions are still to be addressed in our inflation model, most notably the issue of the phase transition. In particular, it is crucial to understand the behavior of the matter fields under such a transition. The research of fluctuations is also in order. As in any phase transition in physics, one expects the fluctuations to create 'islands' of the second phase, which tend to expand and eventually consume the entire Universe, thereby 'smoothing' the phase transition.

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