Calculation of $\frac{dR}{dE_R}$ for O_1 and O_4

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The basic equation

- $\frac{dR}{dE_R} =$
  $\frac{\rho_0}{32\pi {m_x}^3 {m_N}^2} \int_{v>v_{min}} \frac{f\left(\underset{v}{\rightarrow}\right)}{v} \sum_{i,j} \sum_{N,N'=n,p} {c_i}^N {c_j}^{N'} \sum_k a_{ijk} {F_k}^{(N,N')}$
  - k runs over $M, \Sigma'', \Sigma', \Delta, \Phi'', \Phi'$
- $f\left(\underset{v}{\rightarrow}\right)$ is the distribution of incident WIMP velocities on target nuclei.
- ${F_k}^{(N,N')}$ are nuclear form factors
- F_k are functions of q (momentum transferred), and v.
- To look at just the O_1or O_4 operators, take i=j=1 or i=j=4.
- Goal is to find (or set limits on) $c_{i,}$ c_j

Find a_{ijk}

- Detailed in **arXiv:1203.3542**
- $F_{1,1}{}^{(N,N')} = F_M{}^{(N,N')}$ so a_{11k} = 1 for k=M, =0 for k!=M
- $F_{4,4}{}^{(N,N')} = \frac{1}{16}C(j_x)\left(F_{\Sigma''}{}^{(N,N')} + F_{\Sigma'}{}^{(N,N')}\right)$ so a_{44k} = 1/16 for k=∑'', ∑' and = 0 for other ks.
- $C(j_x)$ is a constant that depends on the dark matter spin j_x. $C=(4j_x(j_x + 1)/3)$

Find F_k

- F_k are approximated in tables in, again, **arXiv:1203.3542**
- Depend on q, which I don't yet know how to deal with since only integral in the governing equation is over incoming velocity v.
- One is, for instance for Xe 128
  - $F_M^{(p,p)} = e^{-2y}(2900 - 11000y - 15000y^2 = 11000y^3 + 4200y^4 - 950y^5 - 7.8y^7 + 0.20y^8)$
  - Where $y = \left(\frac{qb}{2}\right)^2, b \approx \sqrt{41.467/(45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}})}$

q problems…

- $\frac{dR}{dE_R}$ depends of F_k, F_k depends on y, y depends on |q|, **q** = **p** - **p’**.
- **p** = m**v**, and we integrate over **v**, so fine, we know **p**. We are also doing this as a function of E_R and so we can know |p’| $= m\sqrt{v^2 - \frac{2E_R}{m}p\prime}$ but this leaves me only knowing the magnitude of p’, so I can’t determine q.