Calculation of $\frac{dR}{dE_R}$ for O_1 and O_4

Shaun Alsum

The basic equation

- $\frac{dR}{dE_R} = \frac{\rho_0}{32\pi {m_x}^3 {m_N}^2} \int_{v>v_{min}} \frac{f\left(\underset{v}{\rightarrow}\right)}{v} \sum_{i,j} \sum_{N,N'=n,p} {c_i}^N \, {c_j}^{N'} \sum_k a_{ijk} {F_k}^{(N,N')} \, \mathrm{d}^3 v$
  - k runs over $M, \Sigma'', \Sigma', \Delta, \Phi'', \Phi'$
- $f\left(\underset{v}{\rightarrow}\right)$ is the distribution of incident WIMP velocities on target nuclei.
- ${F_k}^{(N,N')}$ are nuclear form factors
- F_k are functions of q (momentum transferred), and v.
- To look at just the O_1or O_4 operators, take i=j=1 or i=j=4.
- Goal is to find (or set limits on) c_i^N

Find a_{ijk}

- Detailed in **arXiv:1203.3542**
- $F_{1,1}{}^{(N,N')} = F_M{}^{(N,N')}$ so a_{11k} = 1 for k=M, =0 for k!=M
- $F_{4,4}{}^{(N,N')} = \frac{1}{16} C(j_x) \left(F_{\Sigma''}{}^{(N,N')} + F_{\Sigma'}{}^{(N,N')} \right)$ so a_{44k} = 1/16 for k=∑'', ∑' and = 0 for other ks.
- $C(j_x)$ is a constant that depends on the dark matter spin j_x. $C=(4j_x(j_x+1)/3)$
- Note that a_{ijk} can depend on *v*.

Find F_k

- F_k are approximated in tables in, again, **arXiv:1203.3542**
- Depend on q, which I don't yet know how to deal with since only integral in the governing equation is over incoming velocity v.
- One is, for instance for Xe 128
  - ${F_M}^{(p,p)} = e^{-2y}(2900 - 11000y - 15000y^2 = 11000y^3 + 4200y^4 - 950y^5 - 7.8y^7 + 0.20y^8)$
  - Where $y = \left(\frac{qb}{2}\right)^2, b \approx \sqrt{41.467/(45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}})}$

q

- $\frac{dR}{dE_R}$ depends of F_k, F_k depends on y, y depends on |q|, **q** = **p** - **p'**.
- **q** = -$\mathbf{p}_{Xe}{}'$ = $\sqrt{2m_{Xe}E_R}$ (in the non-relativistic limit)

f(v)

- $f\left(\underset{v}{\rightarrow}\right)\mathrm{d}^3v = v^2 f(v)\mathrm{d}v = v^2 e^{-(v+v_{earth})^2/{v_0}^2}\mathrm{d}v$
  - $v_{earth} \approx 244\ \frac{km}{s}$
  - $v_0 \approx 230\ \frac{km}{s}$
- v_{min} is the slowest speed that could result in a recoil of energy E_R. Given by $v_{min} = \left(\frac{m_X + m_{Xe}}{m_X m_{Xe}}\right)\sqrt{\frac{m_{Xe}E_R}{2}}$ in the non-relativistic limit

Back to the integral

- $\frac{dR}{dE_R} = \frac{\rho_0}{32\pi {m_x}^3 {m_N}^2} \int_{v>v_{min}} \frac{f\left(\vec{v}\right)}{v} \sum_{i,j} \sum_{N,N'=n,p} {c_i}^N {c_j}^{N'} \sum_k a_{ijk} {F_k}^{(N,N')} \, \mathrm{d}^3 v$
- For O_1, a_{11k} independent of v, so
- $\frac{dR}{dE_R} = \frac{\rho_0}{32\pi {m_x}^3 {m_N}^2} \sum_{N,N'=n,p} {c_1}^N {c_1}^{N'} {F_M}^{(N,N')} \int_{v_{min}}^{v_{esc}} v e^{-(v+v_{earth})^2/{v_0}^2} \mathrm{d}v$
- In the 0 momentum transfer limit $\boldsymbol{q} \rightarrow \boldsymbol{0}$ this should replicate the result from Lewin and Smith (http://particleastro.brown.edu/articles/Lewin_Smith_DM_Review.pdf) and does so at least up to a constant factor(I haven't checked the full math yet).