Stars and Solar System constraints in extended gravity theories

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K Kainulainen & D Sunhede – in preparation

Introduction

- 6D brane-world with large extra dimensions $r \sim \mu\mathrm{m}$, true scale of gravity $M_\mathrm{b} \sim 10\ \mathrm{TeV}$
  ~~Hierarchy problem~~

Albrecht et al
Phys Rev D65 (2002) 123507

Burgess et al (SLED)
Nucl Phys B706 (2005) 71

- Presence of renormalizable bulk scalar fields induce a Casimir potential for the radion as well as logarithmic corrections to both kinetic terms and the potential

- Effective 4D action, $\varphi \equiv \log M_\mathrm{b} r$:

$$S_\mathrm{LED} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_\mathrm{b}^2 e^{2\varphi}}{2}\left(AR_g + 2B(\partial\varphi)^2\right) - CM_\mathrm{b}^4 U_0 e^{-4\varphi} + \mathcal{L}_\mathrm{m}(g_{\mu\nu}, \psi_\mathrm{m})\right]$$

$$A(\varphi) = 1 + a\varphi\,, \quad B(\varphi) = 1 + b\varphi\,, \quad C(\varphi) = 1 + c\varphi\,, \qquad U_0 \sim 1\,, \quad a, b, c \ll 1$$

- Induced potential is of a form suitable for addressing the Dark Energy problem. No small mass scale needed!

Introduction

- In general somewhat difficult to stabilize extra dimensions However if $A(\varphi) = 1 + a\varphi \ll 1$ Einstein frame potential become very steep and easily confines the radius:

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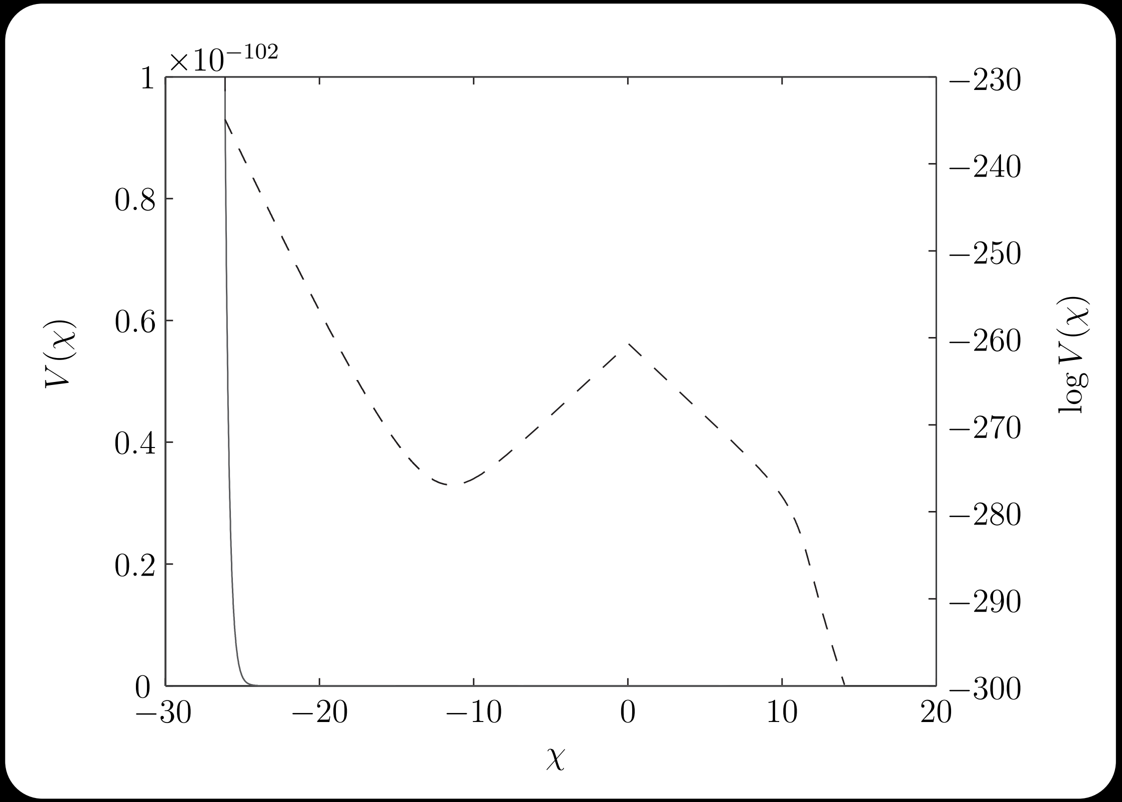
$$V(\chi) \propto \frac{U(\phi)}{A^2(\varphi)}$$



- Second order terms remain small if corrections come not from one, but several weakly coupled bulk scalar fields

- Cosmological evolution OK, close to ΛCDM

Solar System constraints

- Scalar field coupling to matter must remain small:

$$\widehat{\Box}\chi = V'(\chi) - \alpha(\chi)\widehat{T}$$

$$\gamma_{\rm PPN} = 1 - \frac{2\alpha^2}{1+\alpha^2}$$

$$\alpha^2 \lesssim 10^{-5}$$

Cassini time delay
Bertotti et al
Nature 425 (2003) 374

- Almost automatically provided by the new radius stabilization mechanism where $A(\varphi) \ll 1$, since

$$\alpha \propto (2A + a)$$

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- Nevertheless some fine-tuning needed...

1. Could evolution of the field in the Solar System spoil the above prediction?

Solar System constraints

- Solar System constraints / Post-Newtonian parameter

$$\mathrm{d}s^2 \equiv g_{\mu\nu} x^\mu x^\nu = -e^{\mathcal{A}(r)}\mathrm{d}t^2 + e^{\mathcal{B}(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\mathrm{d}s^2 = -(1 - \frac{2GM}{r})\mathrm{d}t^2 + (1 + \frac{2\gamma_{\mathrm{PPN}} GM}{r})\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\gamma_{\mathrm{PPN}} - 1 \approx -\frac{\mathcal{B}}{\mathcal{A}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

Cassini time delay
Bertotti et al
Nature 425 (2003) 374

- In GR – exterior vacuum solution is always Schwarzschild and $\gamma_{\mathrm{PPN}} = 1$ regardless of interior

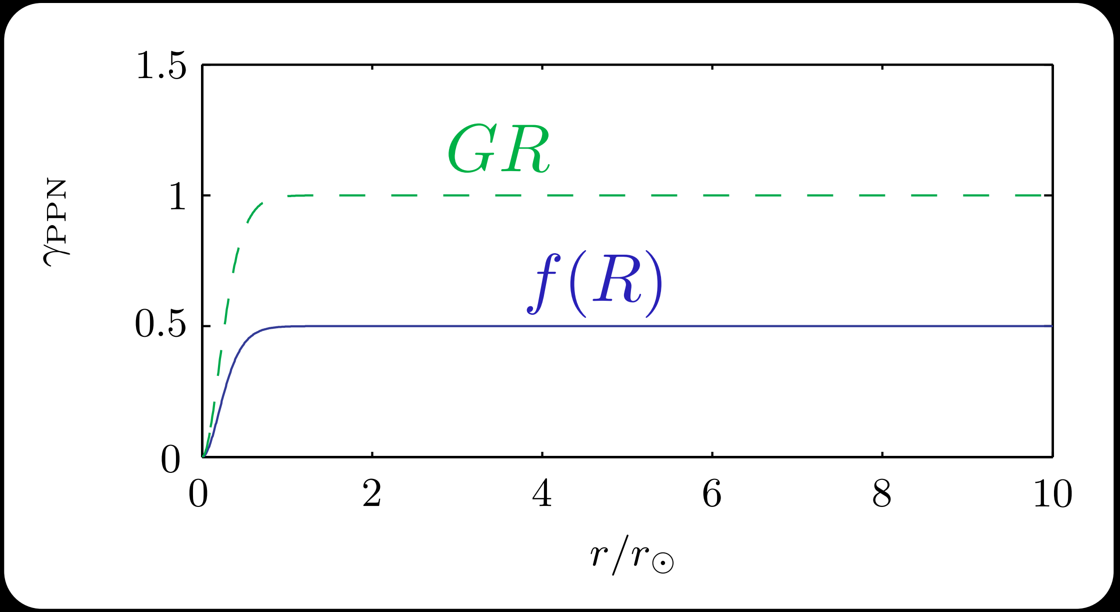
- Extended gravity – lack of unique vacuum solution requires that we compute the full metric to obtain γ_{PPN}

$f(R)$ gravity

- Previous work: Solar System constraints in metric $f(R)$ gravity – $\gamma_{\rm PPN}$ obtained by deriving and solving generalized Tolman-Oppenheimer-Volkoff equations

$$\gamma_{\rm PPN} = \frac{1}{2}$$

Sunhede et al
Phys Rev D76 (2007) 024020
arXiv:0803.0867 (accepted in PRD)



- Metric $f(R)$ gravity is equivalent to an $\omega = 0$ JBD theory, indeed giving $\gamma_{\rm PPN} = 1/2$ in the massless limit

- LED scenario at tree level corresponds to an $\omega = -1/2$ JBD theory giving $\gamma_{\rm PPN} = 1/3$. ***II. Is passing Solar System tests really possible? Difference to metric $f(R)$?***

The scalar-tensor TOV:s

- Scalar-tensor gravity

$$S = \int \mathrm{d}^4x\sqrt{-g}\,[\mathcal{L}_\mathrm{G}(g_{\mu\nu},\varphi) + \mathcal{L}_\mathrm{m}(g_{\mu\nu},\psi_\mathrm{m})]$$

$$\mathcal{L}_\mathrm{G} = \frac{1}{2\kappa_*}\left[F(\varphi)R - Z(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi)\right] \qquad \kappa_* = M_b^{-2}$$

- For a static, spherically symmetric metric we obtain

$$\begin{aligned}
\mathcal{A}' &= \frac{-1}{1+\gamma}\left(\frac{1-e^{\mathcal{B}}}{r} - \frac{re^{\mathcal{B}}}{F}\kappa_* p + \frac{re^{\mathcal{B}}}{2F}U - \frac{r}{2F}Z\varphi'^2 + \frac{4\gamma}{r}\right)\\
\mathcal{B}' &= \frac{1-e^{\mathcal{B}}}{r} + \frac{re^{\mathcal{B}}}{F}\kappa_*\left(\rho + \frac{1}{2\varpi}F_{,\varphi}^2 T\right) + \frac{re^{\mathcal{B}}}{2F}\left(1 - \frac{2}{\varpi}F_{,\varphi}^2\right)U + \frac{re^{\mathcal{B}}}{2\varpi}F_{,\varphi}U_{,\varphi}\\
&\quad + \frac{r}{2F}\left(Z + 2F_{,\varphi\varphi} - \frac{1}{\varpi}F_{,\varphi}\varpi_{,\varphi}\right)\varphi'^2 - \gamma\mathcal{A}'\\
2\varpi\Box\varphi &= F_{,\varphi}(\kappa_* T - 2U) + FU_{,\varphi} - \varpi_{,\varphi}(\partial\varphi)^2
\end{aligned}$$

$$2\varpi \equiv 3F_{,\varphi}^2 + 2FZ$$

- Together with the conservation law and a given equation of state, these form a complete generalization of the Tolman Oppenheimer-Volkov equations

$$p' = -\frac{\mathcal{A}'}{2}(\rho + p)\,, \quad p = p(\rho)$$

The LED model

- Reproducing observed strength of gravity: $M_{\mathrm{b}}^2 e^{2\varphi} \approx M_{\mathrm{Pl}}^2$

$$\frac{U}{\kappa_* \rho} \sim \frac{M_{\mathrm{b}} e^{-4\varphi}}{\kappa_* \rho} \sim 10^{-31}$$ Potential terms negligible

- Introduce: $$\frac{\mathrm{d}\Phi}{\mathrm{d}\varphi} \equiv \frac{2\varpi}{F_{,\varphi}}$$

- φ eqn in weak field, Newtonian limit simplifies to

$$(r^2\Phi')' = -\kappa_* \rho r^2 - \cancel{\left(\frac{F_{,\varphi\varphi}}{2\varpi} - \frac{F_{,\varphi}\varpi_{,\varphi}}{(2\varpi)^2}\right)(r\Phi')^2}$$

$$\Phi(r) = -\int_0^r \mathrm{d}r' \frac{2G_* m(r')}{r'^2} + \Phi_0 \qquad m(r) \equiv \int_0^r \mathrm{d}r' 4\pi r'^2 \rho$$

- ***Field stays very close to cosmol background value***

$$\frac{\Delta\Phi(r)}{\Phi_0} \equiv \frac{\Phi(r) - \Phi_0}{\Phi_0} \sim 10^{-35} \quad \Rightarrow \quad \frac{\Delta\varphi(r)}{\varphi_0} \ll 1$$

The LED model

- Weak field, Newtonian limit, dropping negligible non-linear terms

$$(r\mathcal{B})' \approx \left(1 - \alpha^2\right) \frac{\kappa_*}{F} \rho r^2 \qquad \mathcal{A}' \approx \frac{\mathcal{B}}{r} - \frac{2\alpha^2}{F}\Phi' \qquad \alpha^2 = \frac{F_{,\varphi}}{2\varpi}$$

- Tree level, $\omega = -1/2$ JBD $\Rightarrow \alpha^2 = 1/2$

$$B \approx \frac{G_\mathrm{N} M}{r} \qquad \kappa_*/F \approx \kappa_*/F_0 = 8\pi G_\mathrm{N} \qquad \Rightarrow \qquad \gamma_\mathrm{PPN} \approx \frac{1}{3}$$

$$A \approx -\frac{G_\mathrm{N} M}{r} - \frac{\Phi(r)}{F} \approx -\frac{(1+2) G_\mathrm{N} M}{r}$$

- α small via radius stabilization, for $\alpha^2 \lesssim 10^{-6} \Rightarrow \omega(\varphi) \gtrsim 10^3$

$$B \approx \frac{2 G_\mathrm{N} M}{r} \qquad \Rightarrow \qquad \gamma_\mathrm{PPN} \approx 1$$

$$A \approx -\frac{2 G_\mathrm{N} M}{r}$$

Summary

- ***Evolution of scalar field in Solar System negligible compared to cosmological background value*** $\Rightarrow$ α remains small and $\gamma_{\mathrm{PPN}} = 1$

- Field remains light in the LED model, ***Solar System tests fulfilled since for*** $\alpha \ll 1$ ***the theory no longer corresponds to an*** $\omega \ll -1/2$ ***JBD theory. However, metric*** $f(R)$ ***gravity is invariably*** $\omega = 0$

- Successes of the 6D brane-world (S)LED scenario:
  I. Addresses the hierarchy problem
  II. Induced potential naturally yields observed amount of DE
  III. Valid cosmological evolution close to ΛCDM
  IV. Radius stabilization mechanism also provides consistency with Solar System constraints

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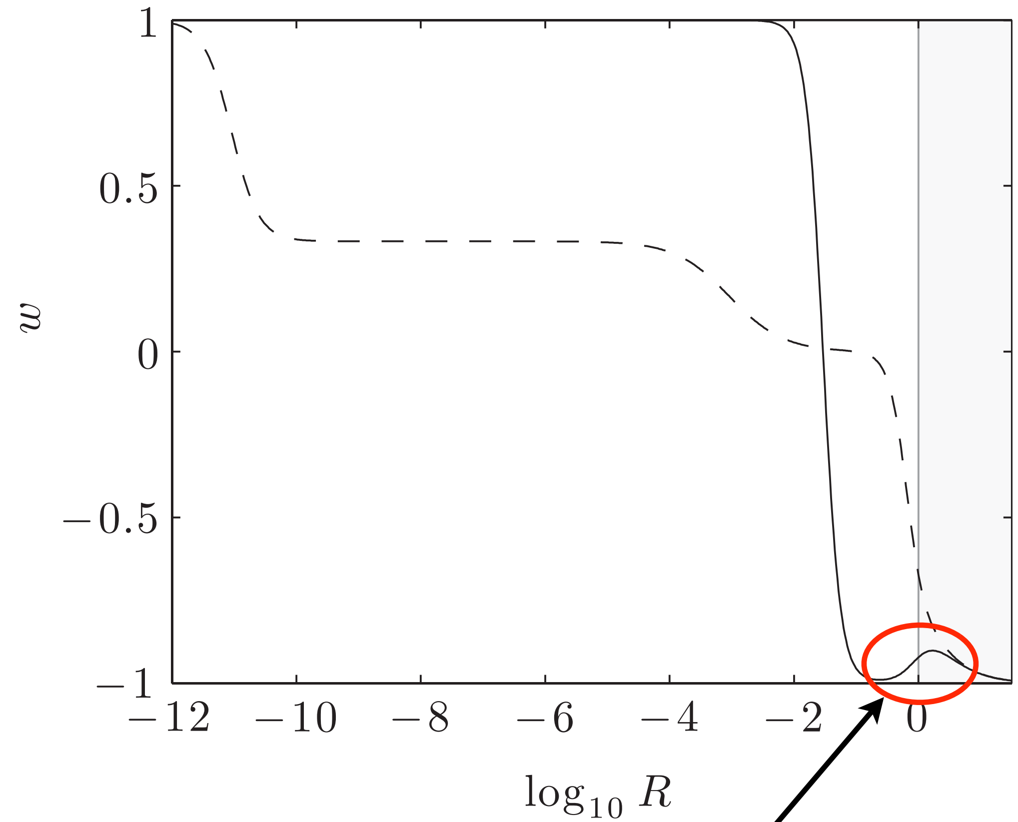
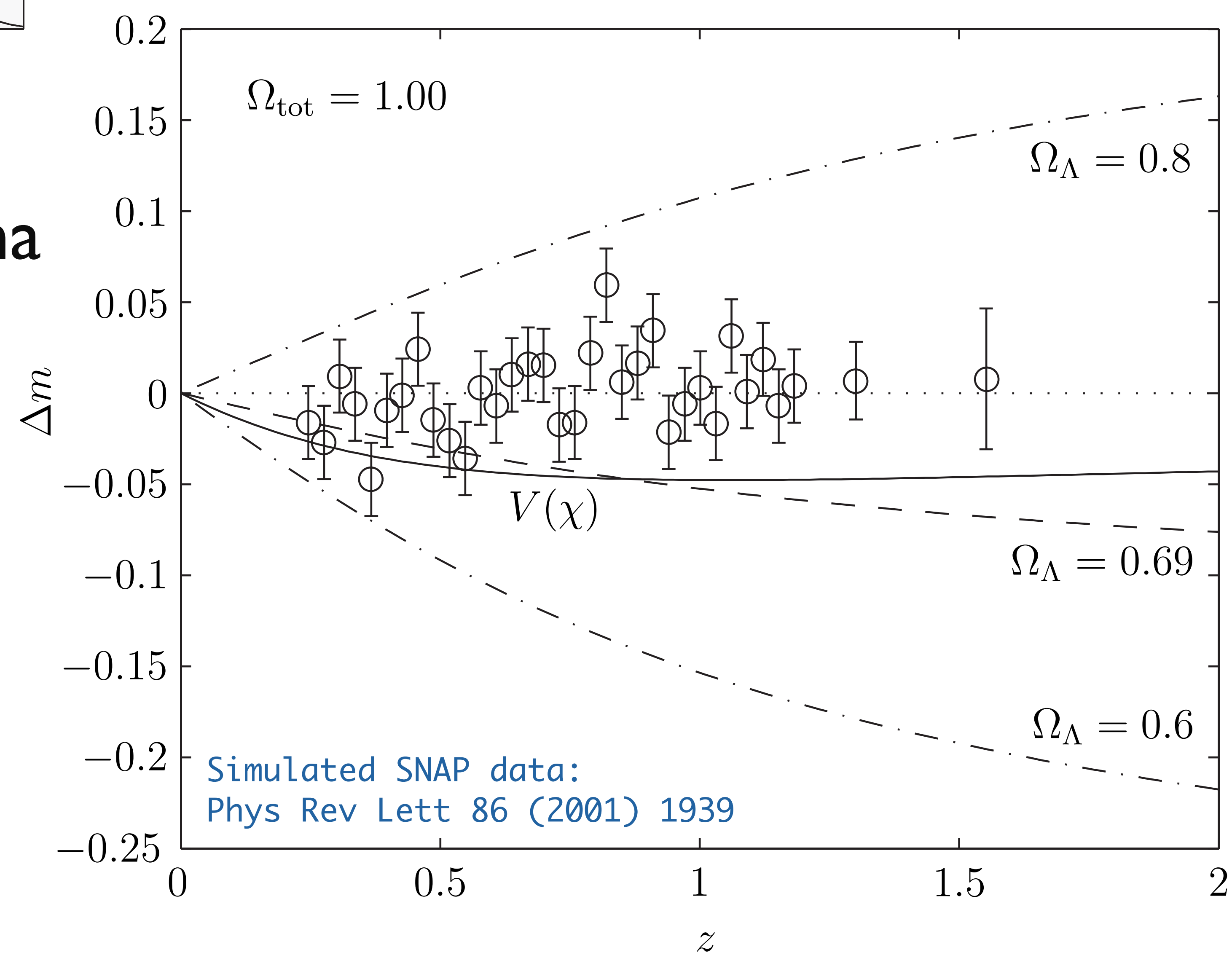
[see also Phys Rev D73 (2006) 083510]

Observational Imprint



Tension between minima of $\alpha(\chi)$ and $V(\chi)$



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