Averaging Robertson-Walker Cosmologies

Iain A. Brown

Institut für Theoretische Physik, Universität Heidelberg

“Backreaction from Perturbations”, J. Behrend, IB and G. Robbers, `JCAP 0801 013`

‘Averaging Robertson-Walker Cosmologies”, IB, G. Robbers and J. Behrend, in preperation

Cosmo 08, Madison, 25th August 2008

Motivation

Standard Cosmology

- Copernican Principle + CMB observations $\Rightarrow$ Universe homogeneous and isotropic.

Standard Cosmology

- Copernican Principle + CMB observations $\Rightarrow$ Universe homogeneous and isotropic.
- Robertson-Walker cosmology: foliate spacetime with maximally-symmetric three-spaces
  - Line element: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
  - Friedmann equation: $(\dot{a}/a)^2 = (8\pi G/3)\overline{\rho} + \Lambda/3$
  - Raychaudhuri equation: $\ddot{a}/a = -(4\pi G/3)(\overline{\rho} + \overline{p}) + \Lambda/3$
  - Perturb metric with $\mathcal{O}(\epsilon) \approx 10^{-5}$

Standard Cosmology

- Copernican Principle + CMB observations $\Rightarrow$ Universe homogeneous and isotropic.
- Robertson-Walker cosmology: foliate spacetime with maximally-symmetric three-spaces
  - Line element: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
  - Friedmann equation: $(\dot{a}/a)^2 = (8\pi G/3)\overline{\rho} + \Lambda/3$
  - Raychaudhuri equation: $\ddot{a}/a = -(4\pi G/3)(\overline{\rho} + \overline{p}) + \Lambda/3$
  - Perturb metric with $\mathcal{O}(\epsilon) \approx 10^{-5}$
- We have assumed the existence of an average and added perturbations

Averaging in Cosmology

- An implicit averaging in cosmology transfers local equations to global cosmology; should be made explicit
- $\langle \partial_t \rho \rangle \neq \partial_t \langle \rho \rangle \Rightarrow$ Naïve EFE for assumed averages does not reflect a true average of small-scale physics.

Averaging in Cosmology

- An implicit averaging in cosmology transfers local equations to global cosmology; should be made explicit
- $\langle \partial_t \rho \rangle \neq \partial_t \langle \rho \rangle \Rightarrow$ Naïve EFE for assumed averages does not reflect a true average of small-scale physics.
- We should be using

$$\langle G_{\mu\nu}(g_{\mu\nu}) \rangle = 8\pi G \langle T_{\mu\nu} \rangle + \Lambda \langle g_{\mu\nu} \rangle$$

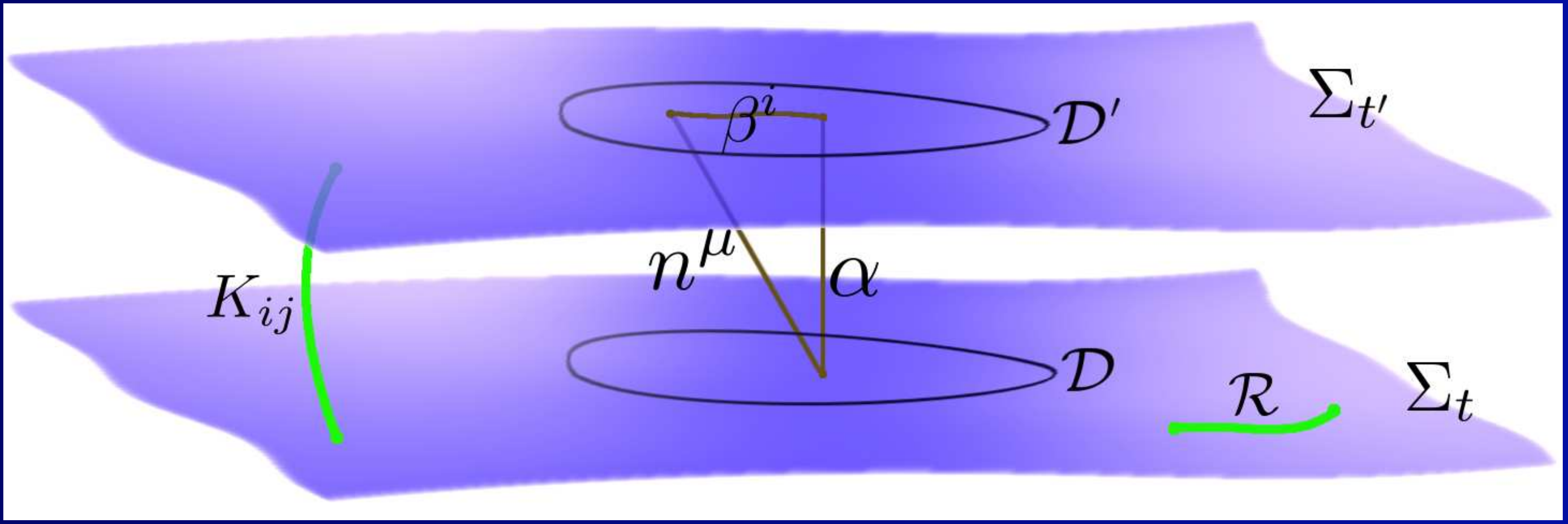
instead of

$$G_{\mu\nu}(\langle g_{\mu\nu} \rangle) = 8\pi G \langle T_{\mu\nu} \rangle + \Lambda \langle g_{\mu\nu} \rangle .$$

- “Backreaction” may not be dark energy, but *all* cosmological models should be properly averaged
- Aim: Express Buchert equations in general form, apply to range of perturbed Robertson-Walker models from radiation domination to present day.

Backreaction

Formalism: 3+1 Split



- Line-element: $ds^2 = -\alpha^2 dt^2 + h_{ij} dx^i dx^j$
- Fluids (Λ, ϕ, b, CDM, γ, ν): $\varrho = n^\mu n^\nu T_{\mu\nu}$, $j_i = -n^\mu T_{i\mu}$, $S_{ij} = T_{ij}$
- Perfect fluids, $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$:

$$\varrho = n^\mu n^\nu T_{\mu\nu} = \rho \left(n^\mu u_\mu\right)^2 + p \left(\left(n^\mu u_\mu\right)^2 - 1\right),$$

$$S = T^i_i = 3p + (\rho + p)\, u^i u_i \,.$$

Formalism: Buchert Averaging

- Select average

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A \sqrt{h} d^3\mathbf{x},$$

Define averaged "scale factor" and Hubble rate by

$$3H_{\mathcal{D}} = 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{\dot{V}}{V} = -\frac{1}{V}\int_{\mathcal{D}} \alpha K \sqrt{h} d^3\mathbf{x} = -\langle \alpha K \rangle = \langle \mathcal{H} \rangle ,$$

- Buchert equations:

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\langle \alpha^2 \varrho \rangle + \frac{\Lambda}{3}\langle \alpha^2 \rangle - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}})$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\langle \alpha^2(\varrho + S) \rangle + \frac{\Lambda}{3}\langle \alpha^2 \rangle + \frac{1}{3}(\mathcal{Q}_{\mathcal{D}} + \mathcal{P}_{\mathcal{D}})$$

Formalism: Modifications to Standard Cosmology

- Kinematical “backreaction”:

$$\mathcal{Q}_{\mathcal{D}} = \left\langle \alpha^2 \left(K^2 - K^i_j K^j_i \right) \right\rangle - \frac{2}{3} \left\langle \alpha K \right\rangle^2$$

- Dynamical “backreaction”: $\mathcal{P}_{\mathcal{D}} = \langle \dot{\alpha} K \rangle + \left\langle \alpha D^i D_i \alpha \right\rangle$
- Curvature contribution: $\mathcal{R}_{\mathcal{D}} = \left\langle \alpha^2 \mathcal{R} \right\rangle$
- Deviation from average density and pressure:

$$\frac{3\mathcal{T}^{(a)}_{\mathcal{D}}}{8\pi G} = \left\langle \alpha^2 \varrho_{(a)} \right\rangle - \overline{\rho}_{(a)}, \quad \frac{3\mathcal{S}^{(a)}_{\mathcal{D}}}{4\pi G} = \left\langle \alpha^2 S_{(a)} \right\rangle - \overline{S}_{(a)}$$

Formalism: Modifications to Standard Cosmology

- The Buchert equations can then be written as

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\sum_a \overline{\rho}_{(a)} + \frac{\Lambda}{3} + \frac{8\pi G}{3}\overline{\rho}_{\rm eff}\,,$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\sum_a \left(\overline{\rho}_{(a)} + \overline{S}_{(a)}\right) + \frac{\Lambda}{3} - \frac{4\pi G}{3}\left(\overline{\rho}_{\rm eff} + \overline{S}_{\rm eff}\right)$$

with effective correction fluid

$$\frac{8\pi G}{3}\overline{\rho}_{\rm eff} = \sum_a \mathcal{T}_{\mathcal{D}}^{(a)} + \left\langle \alpha^2 - 1 \right\rangle \frac{\Lambda}{3} - \frac{1}{6}\left(\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}}\right),$$

$$16\pi G\overline{p}_{\rm eff} = 4\sum_a \mathcal{S}_{\mathcal{D}}^{(a)} - 2\Lambda\left\langle \alpha^2 - 1 \right\rangle + \frac{1}{3}\left(\mathcal{R}_{\mathcal{D}} - 3\mathcal{Q}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}}\right),$$

$$w_{\rm eff} = -\frac{1}{3}\frac{\mathcal{R}_{\mathcal{D}} - 3\mathcal{Q}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}} + 12\sum_a \mathcal{S}_{\mathcal{D}}^{(a)} - 6\Lambda\left\langle \alpha^2 - 1 \right\rangle}{\mathcal{R}_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} - 6\sum_a \mathcal{T}_{\mathcal{D}}^{(a)} - 2\Lambda\left\langle \alpha^2 - 1 \right\rangle}.$$

Link to Perturbation Theory

- Identify ADM and Newtonian co-ordinates (c.f. Mukhanov et. al.)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j = -\alpha^2 dt^2 + h_{ij}dx^i dx^j$$

- $a_{\mathcal{D}}(t)$ is "observational", $a(t)$ is "physical" – drawback of re-averaging assumed average (Kolb, Marra, Matarrese 08; IB, Behrend, Robbers 08)

Link to Perturbation Theory

- Identify ADM and Newtonian co-ordinates (c.f. Mukhanov et. al.)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j = -\alpha^2 dt^2 + h_{ij}dx^i dx^j$$

- $a_{\mathcal{D}}(t)$ is "observational", $a(t)$ is "physical" – drawback of re-averaging assumed average (Kolb, Marra, Matarrese 08; IB, Behrend, Robbers 08)
- Quickly find

$$\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{\dot{a}}{a} - \left\langle \dot{\Phi}\,(1+2\Phi) \right\rangle$$

Link to Perturbation Theory: Backreaction Terms

- Kinematical and dynamical backreactions:

$$
\begin{aligned}
\mathcal{Q}_{\mathcal{D}} &= 6\left(\left\langle \dot{\Phi}^2 \right\rangle - \left\langle \dot{\Phi} \right\rangle^2\right), \\
\mathcal{P}_{\mathcal{D}} &= \frac{1}{a^2}\left\langle \nabla^2\Psi - (\nabla\Psi)^2 + 2\Phi\nabla^2\Psi - (\nabla\Phi)\cdot(\nabla\Psi)\right\rangle \\
&\quad + 3\frac{\dot{a}}{a}\left\langle \dot{\Psi} - 2\Psi\dot{\Psi} \right\rangle - 3\left\langle \dot{\Psi}\dot{\Phi} \right\rangle
\end{aligned}
$$

- Curvature correction:

$$
\mathcal{R}_{\mathcal{D}} = \frac{2}{a^2}\left\langle 2\nabla^2\Phi + 3(\nabla\Phi)^2 + 4(2\Phi+\Psi)\nabla^2\Phi \right\rangle .
$$

Link to Perturbation Theory: Backreaction Terms

- Fluid corrections:

$$\mathcal{T}_{\mathcal{D}} = \frac{8\pi G}{3}\overline{\rho}\left\langle \delta + 2\Psi + (1+\overline{w})a^2v^2 + 2\Psi\delta \right\rangle ,$$

$$\mathcal{S}_{\mathcal{D}} = \frac{4\pi G}{3}\overline{\rho}\left\langle 3c_s^2\delta + 6\overline{w}\Psi + (1+\overline{w})a^2v^2 + 6c_s^2\Psi\delta \right\rangle$$

Link to Perturbation Theory: Backreaction Terms

- Fluid corrections:

$$\mathcal{T}_{\mathcal{D}} = \frac{8\pi G}{3}\overline{\rho}\left\langle \delta + 2\Psi + (1+\overline{w})a^2v^2 + 2\Psi\delta \right\rangle ,$$
$$\mathcal{S}_{\mathcal{D}} = \frac{4\pi G}{3}\overline{\rho}\left\langle 3c_s^2\delta + 6\overline{w}\Psi + (1+\overline{w})a^2v^2 + 6c_s^2\Psi\delta \right\rangle$$

- Note: alternative gauges – uniform density to simplify $\mathcal{T}_{\mathcal{D}}$ and $\mathcal{S}_{\mathcal{D}}$, uniform curvature to remove $\mathcal{R}_{\mathcal{D}}$, synchronous gauge to remove $\mathcal{P}_{\mathcal{D}}$. $\mathcal{Q}_{\mathcal{D}}$ cannot be entirely removed except in EdS matter domination.

Numerical Study

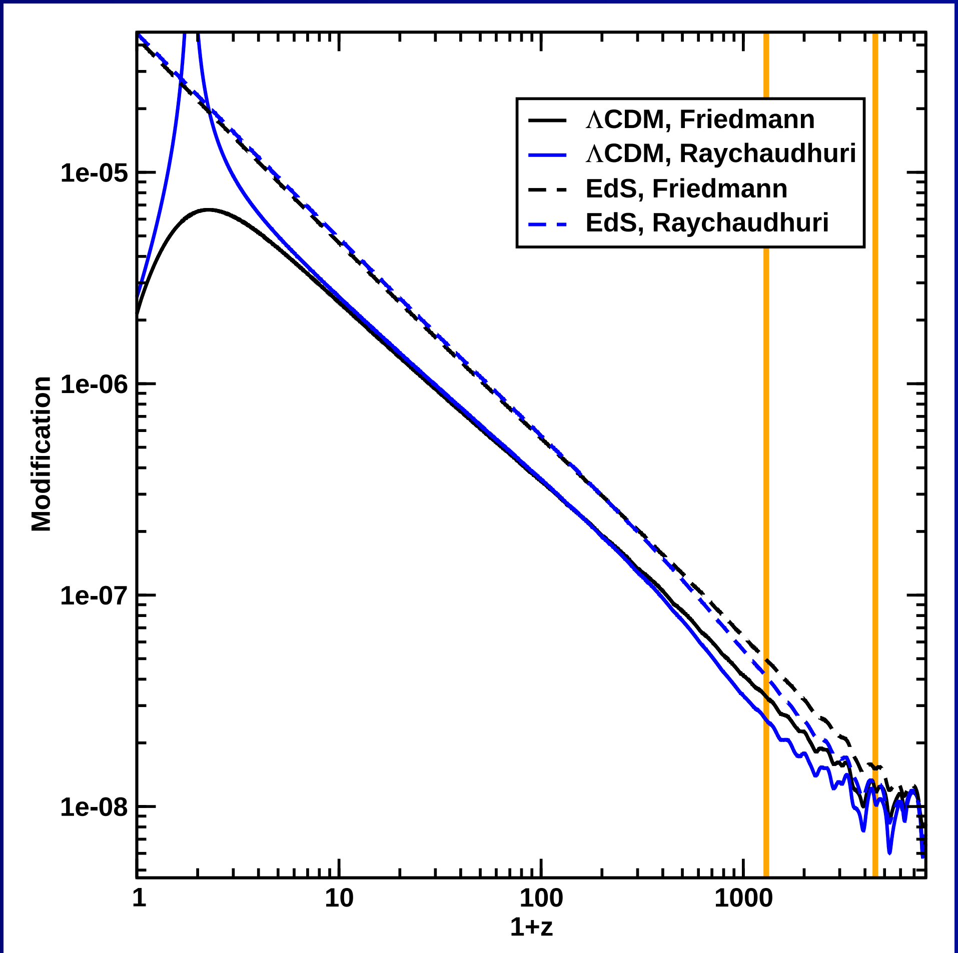
Ergodic Averaging

- Boltzmann codes are 1-d, averages are 3-d, so take $\mathcal{D}$ large enough to employ ergodic principle

Ergodic Averaging

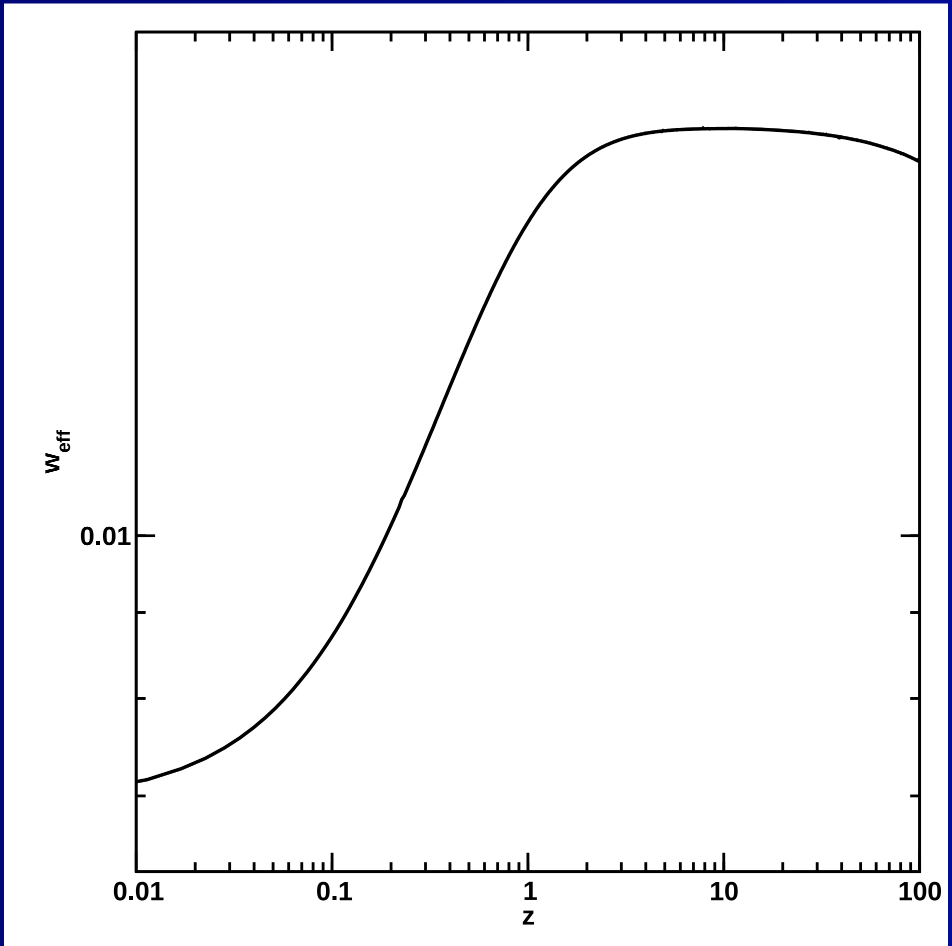
- Boltzmann codes are 1-d, averages are 3-d, so take $\mathcal{D}$ large enough to employ ergodic principle
- Corrections to standard case to be evaluated with cmbeasy: $\mathcal{Q}_{\mathcal{D}} = 6 \int \mathcal{P}_{\psi}(k) \left| \dot{\Phi} \right|^2 (dk/k)$ etc.
- Integration domain $k \in (1/\eta, 100\mathrm{Mpc}^{-1})$

ΛCDM and EdS: Friedmann and Raychaudhuri Equations



- Impact at recombination $\sim 10^{-8}$ – potentially observable with Planck?
- Boosts with Halofit not significant

ΛCDM: w



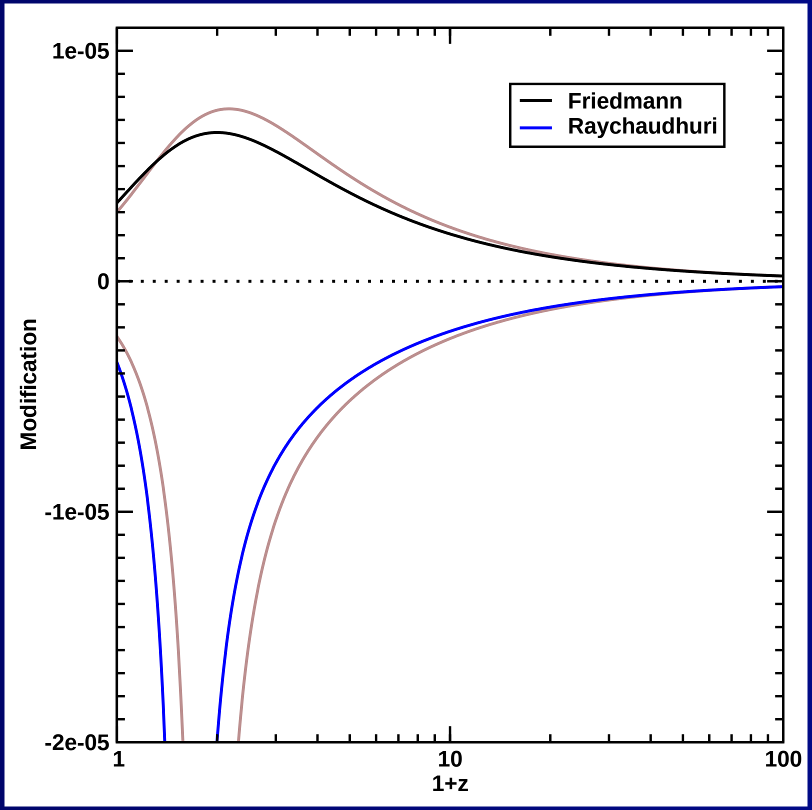
- For $z > \sim 100$ find increase up to $w_{\text{eff}} \approx 0.2$ at $z \approx 8000$

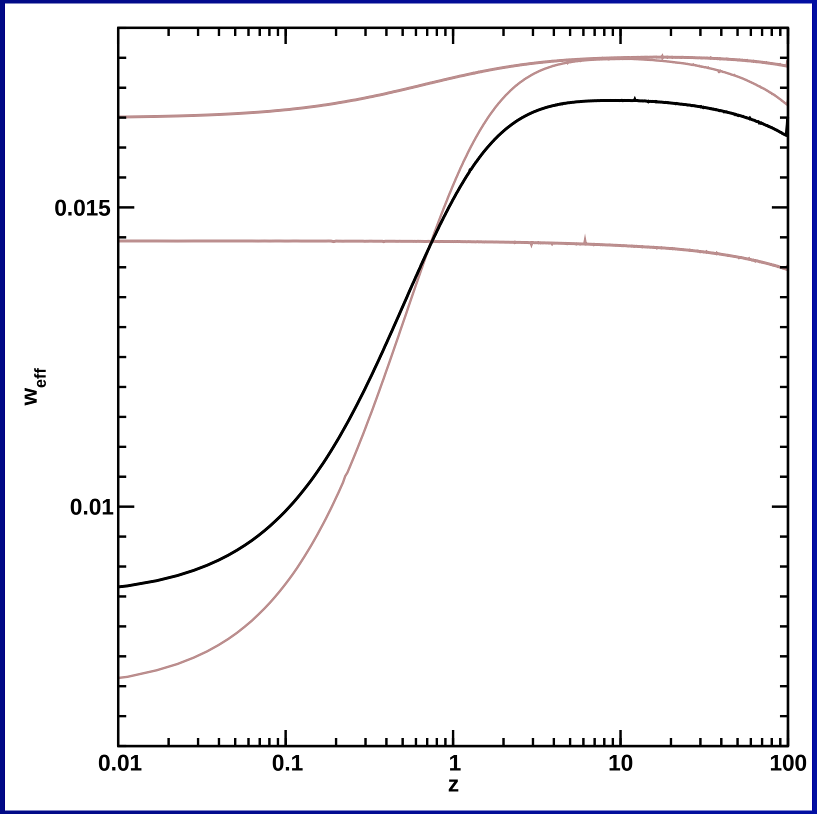
Quintessence Cosmology

- Models tested
  - Early dark energy parameterisation
  - Exponential potential
  - Inverse power-law potential
- Still linear anaylsis $\Rightarrow$ still expect small impacts on the observed evolution
- Expect w_{eff} to increase with dark matter perturbations – so w_{eff} clearest discriminant

Early Dark Energy

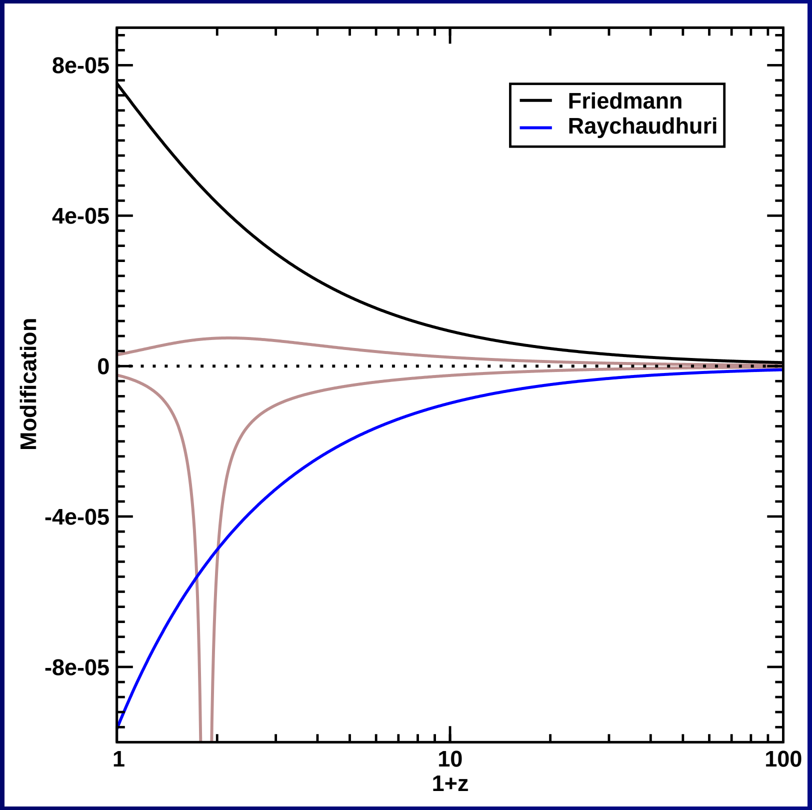
- Rough model of early dark energy; $\Omega_\phi^{z=0} = 0.7$, $\Omega_\phi^{z=\infty} = 0.05$, $w_0 = -0.95$
- Very similar to ΛCDM; larger at present day, smaller at peak

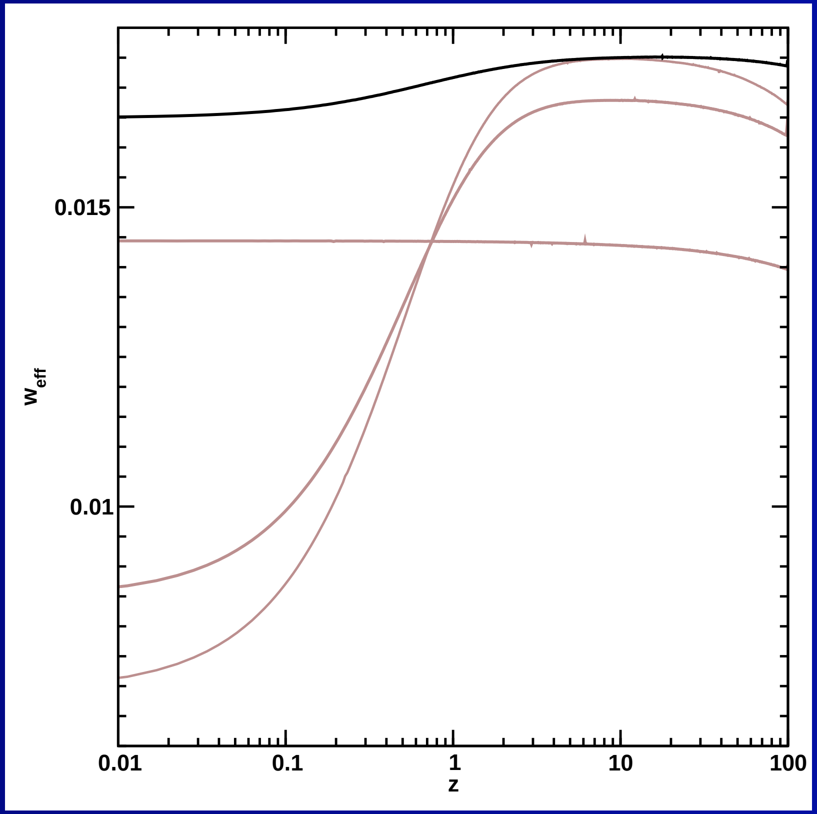




Inverse Power Law Potential

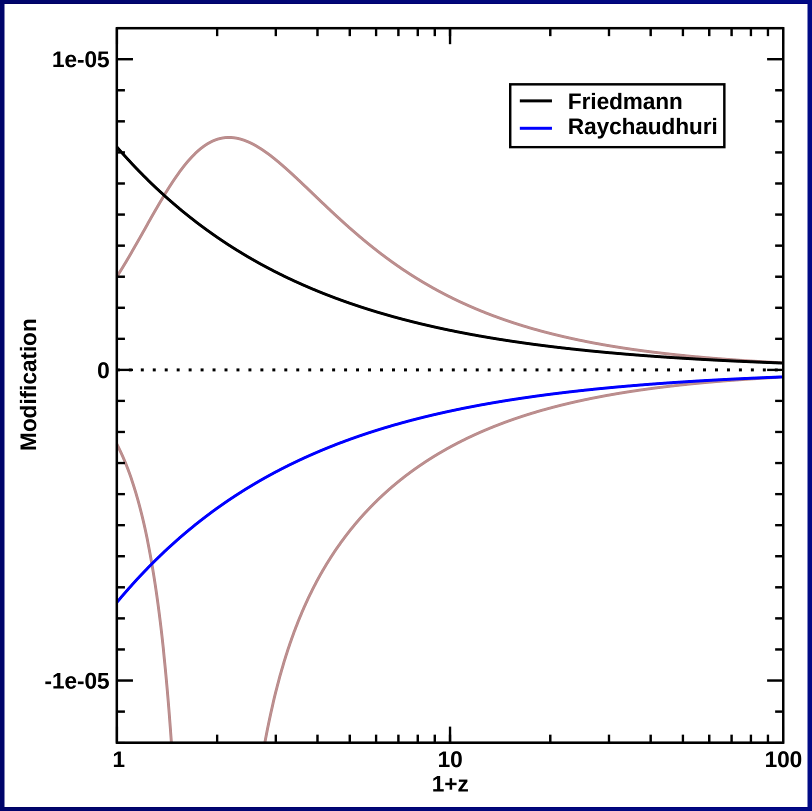
- Inverse-Power Law Potential (Ratra-Peebles); $\Omega_\phi = 0.118$, $\Omega_b = 0.046$, $\Omega_c = 0.837$, $n = -2$
- Similar to but smaller than EdS for these parameters

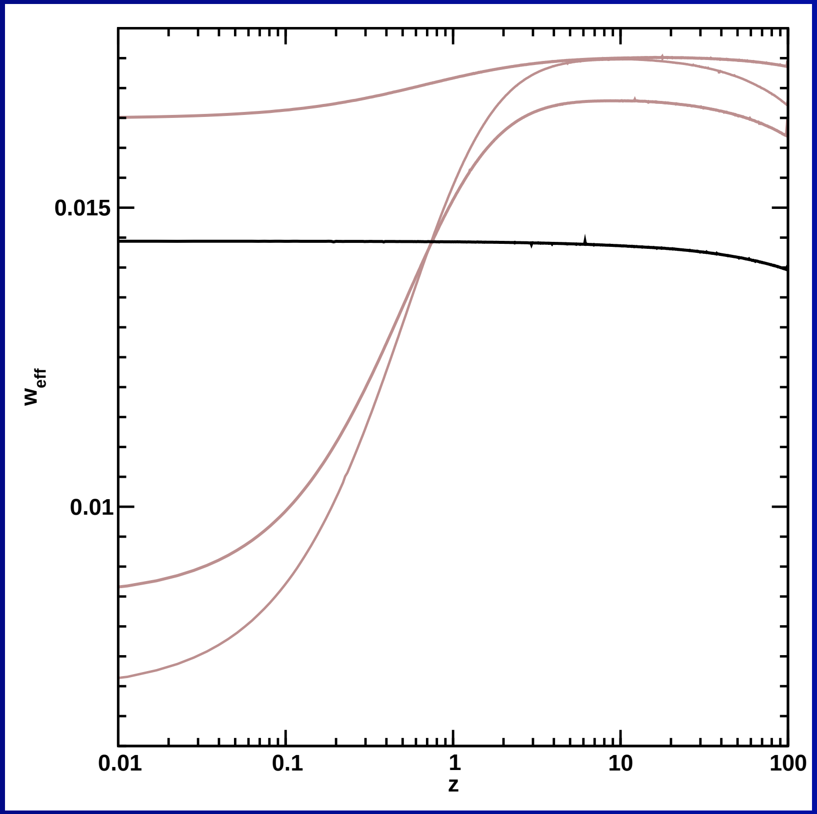




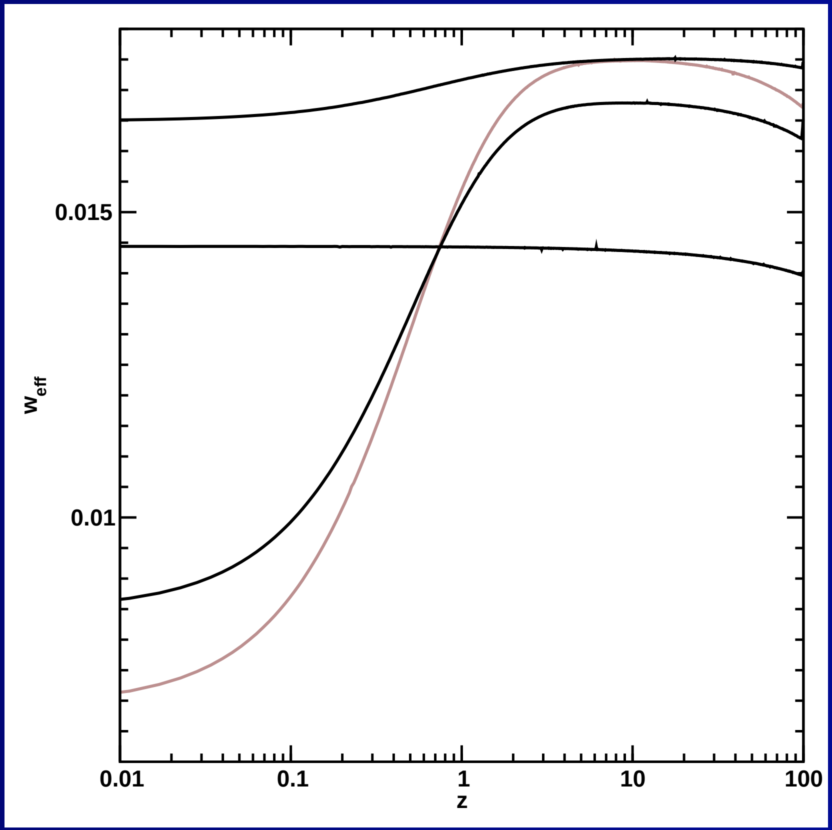
Exponential Potential

- Exponential Potential; $\Omega_\phi = 0.200$, $\Omega_b = 0.041$, $\Omega_m = 0.759$
- Similar to inverse power law





Equations of State



- $w_{\mathrm{eff}} > 0$ – as before acts against acceleration
- But: this includes quintessence perturbations!
- These differences far too small to observe, but smaller-scale study looks vital

Summary

Summary

- Have expressed Buchert equations in multifluid form easily incorporated into general Boltzmann codes for wide variety of models
- Differing *linear* models barely change impact on Friedmann equation; on Raychaudhuri equation it's similar and remains $\sim 10^{-5}$
- Impact at recombination is close to observable anisotropies $\Rightarrow$ possible chance of detection?
  - ΛCDM: $w_{\rm eff} \approx 0.007$
  - Early dark energy: $w_{\rm eff} \approx 0.009$
  - Exponential: $w_{\rm eff} \approx 0.014$
  - Inverse power law: $w_{\rm eff} \approx 0.016$
- Equation of state from quintessence perturbations > -1: is there a problem with clustering quintessence models? Small scale study is needed (c.f. Wetterich '02).
- CMB observables?
- Non-Linear models
- Modified averaging procedure (Behrend/Nachtmann?)