k- / DBI inflationary Power Spectra and Constraints from WMAP5

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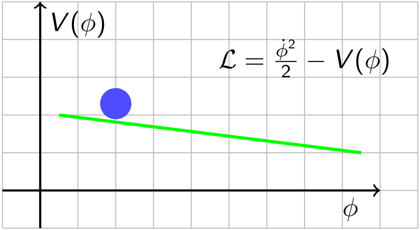
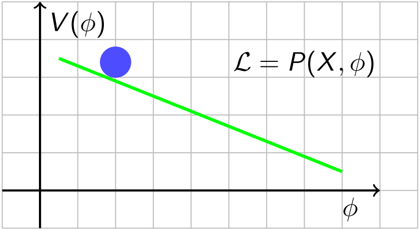
work with J. Martin & C. Ringeval: **arXiv:0807.2414, 0807.3037**

DBI as a special case of k-inflation

Two ways to drive inflation:



flat potential



modified kinetic term

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Garriga and Mukhanov (1999)

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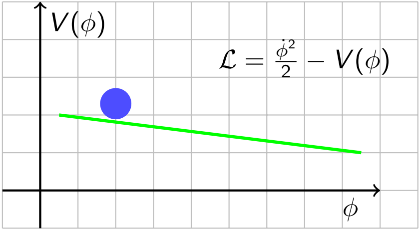

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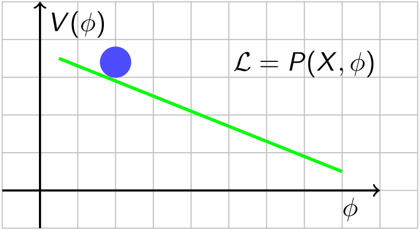

modified kinetic term

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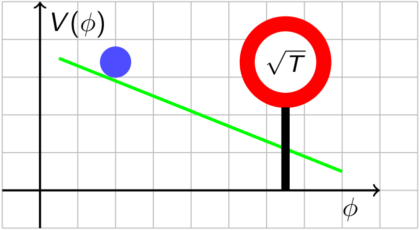
String inflation combines both:

- candidate potentials (Coulomb terms, moduli stabilization etc.)
- DBI kinetic term from brane action (open string mode)

DBI as a special case of k-inflation

Effective 4d DBI inflaton action:

$$S = -\int \mathrm{d}^4x\sqrt{-g}\left[V(\phi) - T(\phi) + T(\phi)\sqrt{1+\frac{1}{T(\phi)}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}\right]$$



$V(\phi)$: potential
$T(\phi)$: warped brane tension

functions of
10d background geometry

"speed limit":

$$\gamma(\dot{\phi},\phi) = \frac{1}{\sqrt{1-\dot{\phi}^2/T(\phi)}}$$

$\gamma \approx 1$: standard regime
$\gamma \gg 1$: "ultrarelativistic" DBI regime

Silverstein and Tong (2003)
Alishahiha, Silverstein and Tong (2004)

Regimes and perturbation equations

Inflation can proceed in two regimes:

$$\epsilon_1 = \frac{2}{\kappa\gamma}\frac{1}{H^2}\left(\frac{dH}{d\phi}\right)^2$$

$\mathrm{d}H/\mathrm{d}\phi \ll 1$

$\gamma \gg 1$

"ultrarelativistic"

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$dH/d\phi \ll 1$

$\gamma \gg 1$

"ultrarelativistic"

Scalar perturbations have a sound speed $c_s \neq 1$:

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0$$

$$z \sim a\,\gamma\sqrt{\epsilon_1}$$

$$c_s = 1/\gamma$$

- sound speed c_s time dependent
  standard solution method fails, initial conditions?
- effective potential z''/z
  contains usual ϵ_i parameters, but also c_s derivatives

New parameter hierarchies

$$\epsilon_{n+1} = \frac{\mathrm{d}\ln|\epsilon_n|}{\mathrm{d}N}, \quad \text{where } \epsilon_0 = \frac{H_{\mathrm{in}}}{H}$$

"Hubble flow parameters"

$$\delta_{n+1} = \frac{\mathrm{d}\ln|\delta_n|}{\mathrm{d}N}, \quad \text{where } \delta_0 = \frac{c_{\mathrm{s,in}}}{c_{\mathrm{s}}}$$

"sound flow parameters"

similar parameter sets defined e.g. in: Shandera and Tye (2006), Bean et al. (2007), Kinney and Tzirakis (2007)

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"Hubble flow parameters" **"sound flow parameters"**

similar parameter sets defined e.g. in: Shandera and Tye (2006), Bean et al. (2007), Kinney and Tzirakis (2007)

Recall: $z \sim a\sqrt{\epsilon_1}/c_{\rm s}$, $c_{\rm s} = 1/\gamma$ in DBI

At $\mathcal{O}(\epsilon_i, \delta_i)$, the effective potential becomes:

$$\frac{z''}{z} \simeq \frac{1}{\eta^2}\left(2 + 3\epsilon_1 + \frac{3}{2}\epsilon_2 + 3\delta_1\right)$$

Initial conditions $v_k'' + \left(k^2/\gamma^2 - z''/z\right) v_k = 0$

Parametric oscillator with frequency: $\boldsymbol{\omega^2(k,\eta) = \frac{k^2}{\gamma^2} - \frac{z''}{z}}$

WKB initial state well-defined if

$$\left|\frac{Q}{\omega^2}\right| \ll 1 \qquad \text{with } Q(k,\eta) = \frac{3}{4}\frac{\omega'^2}{\omega^2} - \frac{\omega''}{2\omega}$$

$\mathrm{standard:\ adiabaticity}$

$\omega \sim k$

$\mathrm{DBI:\ non-trivial}$

$\omega \sim c_\mathrm{s} k$

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- wavelength must be smaller than the **sound horizon** $k^2/\gamma^2 \gg z''/z$
- **and:** evolution of the sound speed must be **adiabatic** !

$$\frac{Q}{\omega^2} \simeq \frac{a^2 H^2 \gamma^2}{2k^2}\left(\delta_1 - \epsilon_1\delta_1 + \delta_1\delta_2 + \frac{1}{2}\delta_1^2\right)$$

Calculating the scalar power spectrum

Expand the spectrum around a pivot scale k_P:

$$\mathcal{P}_\zeta(k) = \mathcal{A}_\zeta(k_\mathrm{P}) \sum_{n=0}^{+\infty} \frac{a_n}{n!} \ln^n \left(\frac{k}{k_\mathrm{P}} \right)$$

- physical spectrum independent of k_P: recursion for $a_{n+1} = f(a_n)$
- **goal:** obtain $a_n = a_n(\epsilon_i, \delta_i)$ from uniform approximation
- if a_0 is known at $\mathcal{O}(p)$ in slow-roll, a_n is known at $\mathcal{O}(p+n)$

Schwarz and Terrero-Escalante (2004)

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coeff	quantity	order
$a_0(k_{\mathrm{P}})$	amplitude	ϵ_i, δ_i
$a_1(k_{\mathrm{P}})$	index $n_{\mathrm{S}} - 1$	$\epsilon_i^2, \delta_i^2, \epsilon_i\delta_i$
$a_2(k_{\mathrm{P}})$	running α_{S}	$\epsilon_i^k\delta_i^l$, $k+l=3$

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Note:
amplitude at $\mathcal{O}(0)$
suffices for n_S at $\mathcal{O}(1)$

Solving the mode equation

Using $z''/z = (\nu^2 - 1/4)/\eta^2$, write the mode equation as

$$\mathbf{v}_\mathbf{k}'' + \left[\mathbf{c}_\mathbf{s}^2\mathbf{k}^2 - \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right)\right]\mathbf{v}_\mathbf{k} = 0$$

where $\nu^2(\eta) \simeq 9/4 + 3\epsilon_1 + 3/2\,\epsilon_2 + 3\delta_1$

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Three distinct situations:

$c_\mathrm{s} \equiv 1$ standard case Bessel function solutions ($\delta_1 = 0!$)

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$c_\mathrm{s} = c_\mathrm{s}(\eta)$	general case	**solve with uniform approximation**

Uniform approximation

General mode equation solved by

Habib et al. (2002, 2004)

$$v_k(\eta) \sim g^{-1/4} B_k \exp\left[\int_{\eta_*}^{\eta} \mathrm{d}\tau \sqrt{g(\tau)}\right]$$

constant, from initial conditions

turning point

$$g(\eta) = \frac{\nu^2}{\eta^2} - c_\mathrm{s}^2 k^2$$

"turning point": where $k\eta_* = -\frac{\nu_*}{c_{\mathrm{s}\,*}}$

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Expand ν and $1/c_\mathrm{s}$ around η_*:

$$\nu^2(\eta) \simeq 9/4 + 3\epsilon_{1*} + 3/2\,\epsilon_{2*} + 3\delta_{1*}$$

$$1/c_\mathrm{s} \simeq 1/c_{\mathrm{s}*}\left[1 - \delta_{1*} \ln(\eta/\eta_*)\right]$$

Integral exactly solvable !
$\mathcal{P}_\zeta$ determined at NL order

But: $\eta_* = \eta_*(k)$, therefore another expansion is necessary

Uniform approximation

After expansion around $-k_\mathrm{P}\eta_\mathrm{P} = 1/c_\mathrm{s}(k_\mathrm{P})$, the spectrum at $\mathcal{O}(\epsilon_i, \delta_i)$ is:

$$\mathcal{P}_\zeta = \underbrace{\frac{H^2\gamma}{\pi m_\mathrm{Pl}^2\epsilon_1}\left(18e^{-3}\right)}_{\mathcal{A}_\zeta(k_\mathrm{P})} \times \left[1 - 2(D+1)\epsilon_1 - D\epsilon_2 + (D+2)\delta_1\right]$$

intrinsic error $\sim 10\%$ of uniform app.

a_0 at $\mathcal{O}(\epsilon_i, \delta_i)$

where $D \equiv 1/3 - \ln 3$

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a_0 at $\mathcal{O}(1)$, hence a_1 at $\mathcal{O}(2)$ $\Rightarrow n_\mathrm{S} - 1$ at NNL:

$$\begin{aligned} n_\mathrm{S} - 1 &= -2\epsilon_1 - \epsilon_2 + \delta_1 \\ &\quad -2\epsilon_1^2 + 3\epsilon_1\delta_1 - (2D+3)\epsilon_1\epsilon_2 - D\epsilon_2\epsilon_3 + \epsilon_2\delta_1 - \delta_1^2 + (D+2)\delta_1\delta_2 \end{aligned}$$

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Can go on to calculate α_{S} at third order...

A note on tensors

Tensor perturbations are not affected by $c_\mathrm{s} \neq 1$ since they do not couple to the matter sector.

But:
If we want the same k_P for scalars and tensors, there is a subtle effect:

$$\mathcal{P}_h = \frac{16H^2}{\pi m_\mathrm{Pl}^2} \left(18e^{-3}\right) \left[1 - 2\left(D + 1 + \ln\frac{1}{c_\mathrm{s}}\right)\epsilon_1 - 2\epsilon_1 \ln\frac{k}{k_\mathrm{P}}\right]$$

Spectral index unchanged: $\mathbf{n_T = -2\epsilon_1}$

But the tensor to scalar ratio is now: $\mathbf{r = 16c_\mathrm{s}\epsilon_1 = \frac{16\epsilon_1}{\gamma}}$

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Standard case:
two parameters (ϵ_1, ϵ_2)

k-inflation case:
four parameters $(c_\mathrm{s}, \epsilon_1, \epsilon_2, \delta_1)$

WMAP5 and k- / DBI inflation – w/o non-gaussianities

Reminder: standard case

$$n_S - 1 = -2\epsilon_1 - \epsilon_2$$
$$r = 16\epsilon_1$$

WMAP3 limits:

Martin and Ringeval (2006)

$\epsilon_1 < 0.022$, $|\epsilon_2| < 0.07$

[for WMAP5, see Alabidi and Lidsey (2008)]

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k-inflation:

$$\begin{aligned} n_{\mathrm{S}} - 1 &= -2(\epsilon_1 - \delta_1) \\ &\quad -(\epsilon_2 + \delta_1) \\ r &= 16 c_{\mathrm{s}} \epsilon_1 \end{aligned}$$

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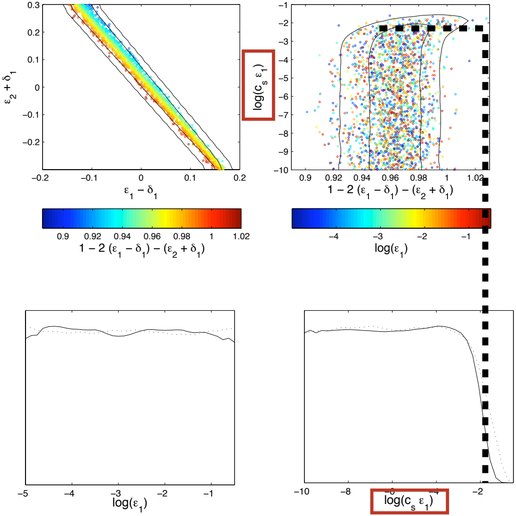
k-inflation:

$$n_S - 1 = -2(\epsilon_1 - \delta_1) - (\epsilon_2 + \delta_1)$$
$$r = 16 c_S \epsilon_1$$

ϵ_1 not limited by WMAP5

$\log(c_S\epsilon_1) < -2.3$



WMAP5 and DBI inflation – w/ non-gaussianities

For DBI: $c_s = 1/\gamma$ and

$$f_{\rm NL}^{\rm eq} = \frac{35}{108}\left(1-\gamma^2\right) + \mathcal{O}(\epsilon_i, \delta_i)$$

Chen et al. (2006)

Use WMAP5 bound

$$-151 \leq f_{\rm NL}^{\rm eq} \leq 256$$

as a flat prior for γ^2.

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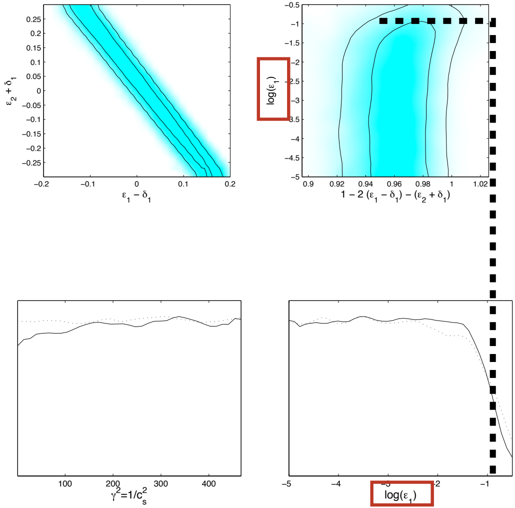
Disentangles $c_s \epsilon_1 \Rightarrow \epsilon_1$!

$$\log \epsilon_1 \leq -1.1$$

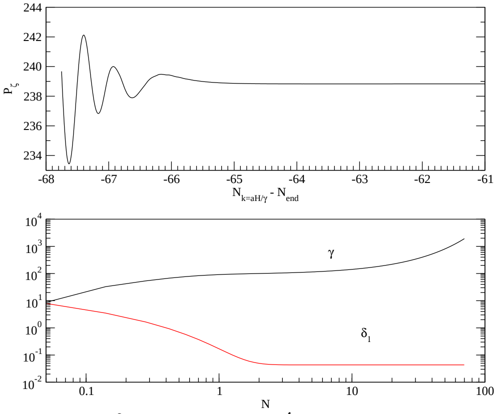


Conclusions

- The uniform approximation is a generic method to obtain scalar and tensor perturbation spectra at next-to-leading order.
- This gives $n_{\mathrm{S}} - 1$ at $\mathcal{O}(2)$ and α_{S} at $\mathcal{O}(3)$, in terms of the new (ϵ_i, δ_i) hierarchy.
- Choosing k inside the sound horizon initially is not enough to ensure adiabaticity. Evolution of c_{s} enters.
- Instead of 2 parameters (ϵ_1, ϵ_2), there are now 4 $(c_{\mathrm{s}}, \epsilon_1, \epsilon_2, \delta_1)$. WMAP5 does not constrain these separately, only combinations.
- Non-gaussianities help break this degeneracy. Future measurements?

Not in this talk, but in the paper:
method applied to DBI power law, KKLMMT, chaotic Klebanov-Strassler ($+$ constant term) inflation

Initial conditions – example



$V(\phi) = \frac{m^2}{2}\phi^2, \ T(\phi) = \frac{\phi^4}{\lambda}$

"chaotic Klebanov-Strassler inflation"

$\delta_1 \sim \mathcal{O}(10)$ initially

WKB condition

$$\frac{Q}{\omega^2} \simeq \frac{a^2 H^2 \gamma^2}{2k^2}\Big(\delta_1 - \epsilon_1\delta_1 + \delta_1\delta_2 + \frac{1}{2}\delta_1^2\Big)$$

not satisfied at early times

Spectral features on large scales possible!

Backup 2

$$n_{\mathrm{S}} - 1 \equiv \left(\frac{\mathrm{d}\ln \mathcal{P}_\zeta}{\mathrm{d}\ln k}\right)_{k=k_{\mathrm{P}}} = \frac{a_1}{a_0}$$

$$\alpha_{\mathrm{S}} = \left(\frac{\mathrm{d}^2\ln \mathcal{P}_\zeta}{\mathrm{d}(\ln k)^2}\right)_{k=k_{\mathrm{P}}} = \frac{a_2}{a_0} - \frac{a_1^2}{a_0^2}$$