COSMO '08 August, 2008

Is the growth equation useful on large scales?

0806.2689 ; Submitted to PRD

Collaborator: James Dent

Sourish Dutta
Vanderbilt University

Outline

- The assumptions that underlie the growth equation
- When these assumptions break down
- An “improved” growth equation
- Testing beyond-Einstein physics

The growth equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0$$

- Simple equation governing growth of perturbations
- Scale invariant[1]
- Can be used to distinguish GR from MG[1,2]

1. Acquaviva,Das,Hajian and Spergel, (2008)
2. Linder (2005), Linder and Cahn (2007), Polarski and Gannouji (2007)

Deriving the growth equation

Growth of perturbations of a generic cosmic fluid[1]:

$$\ddot{\delta} + \dot{\delta}(2 - 3w)H + \frac{k^2}{a} w\delta + \frac{k^2}{a^2}(1+w)\Phi = 3(1+w)[\ddot{\Phi} + \dot{\Phi}(2-3w)H] + 3\dot{w}\dot{\Phi} - 3H\dot{\Theta} + [-\frac{k^2}{a^2} + \frac{3H^2}{2}(1+9w)]\Theta$$

$$\Theta \equiv (\delta p/\delta\rho - w)\delta$$

1. Dutta, Dent, Weiler (2008)

Deriving the growth equation

- Assume constant w

$$c_s^2 = w, \quad \Theta = 0, \quad \dot{\Theta} = 0$$

- Assume matter domination

$$w \simeq 0, \quad \dot{\Phi} \simeq 0, \quad \ddot{\Phi} \simeq 0$$

- Assume the Poisson equation is true

$$-4\pi G\delta\rho = \frac{k^2}{a^2}\Phi$$

Testing the growth equation

- Test against a direct numerical solution of ΛCDM:

Background:

$$
\begin{aligned}
2\dot{H} + 3H^2 &= 8\pi G \rho_\Lambda \\
\dot{\rho} &= -3H\rho
\end{aligned}
$$

Perturbations:

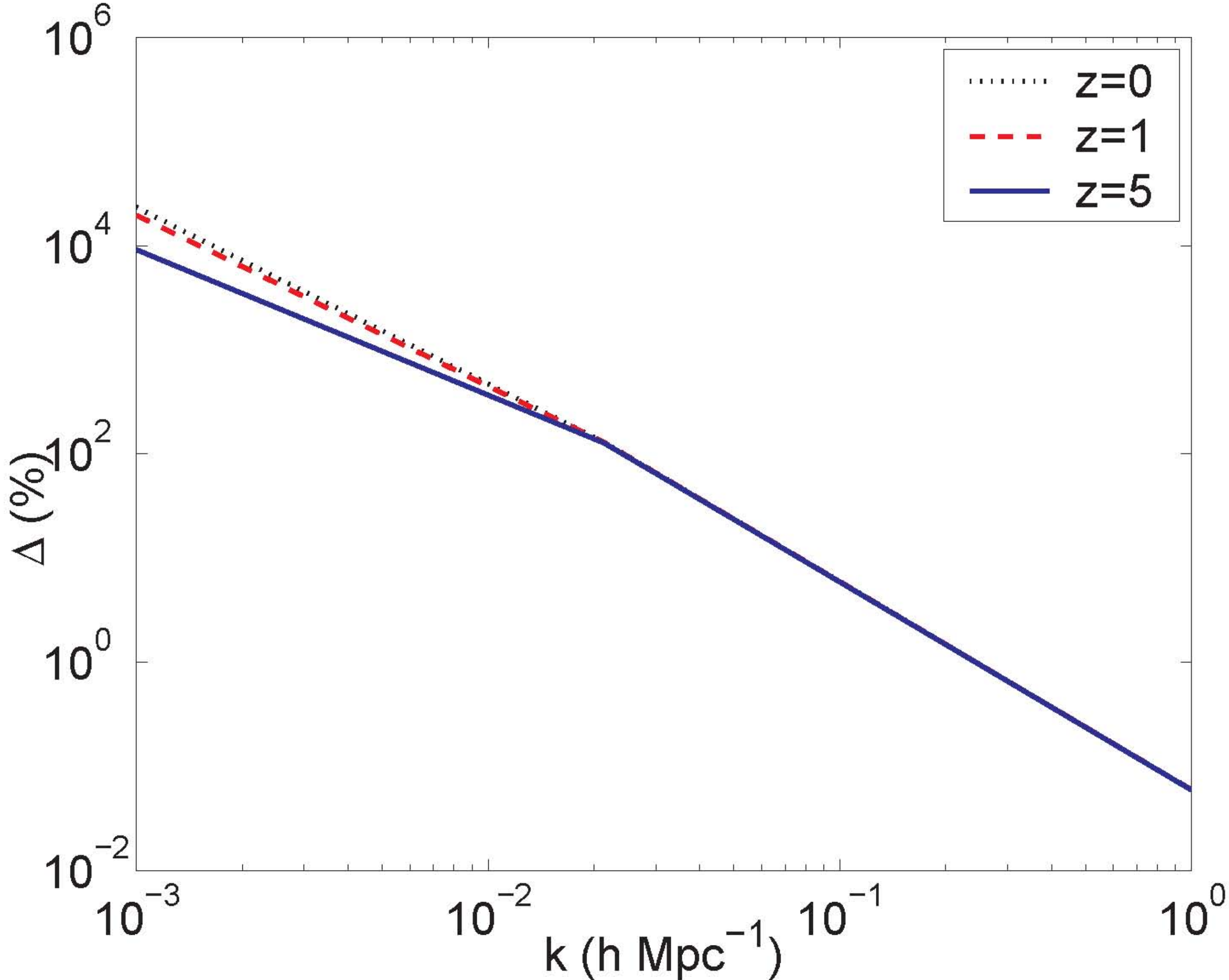
$$
\begin{aligned}
\ddot{\Phi} &= -4H\dot{\Phi} - 8\pi G \rho_\Lambda \Phi \\
\dot{\delta} &= 3\dot{\Phi} + \frac{k^2}{a^2} v_f \\
\dot{v}_f &= -\Phi
\end{aligned}
$$

Testing the growth equation

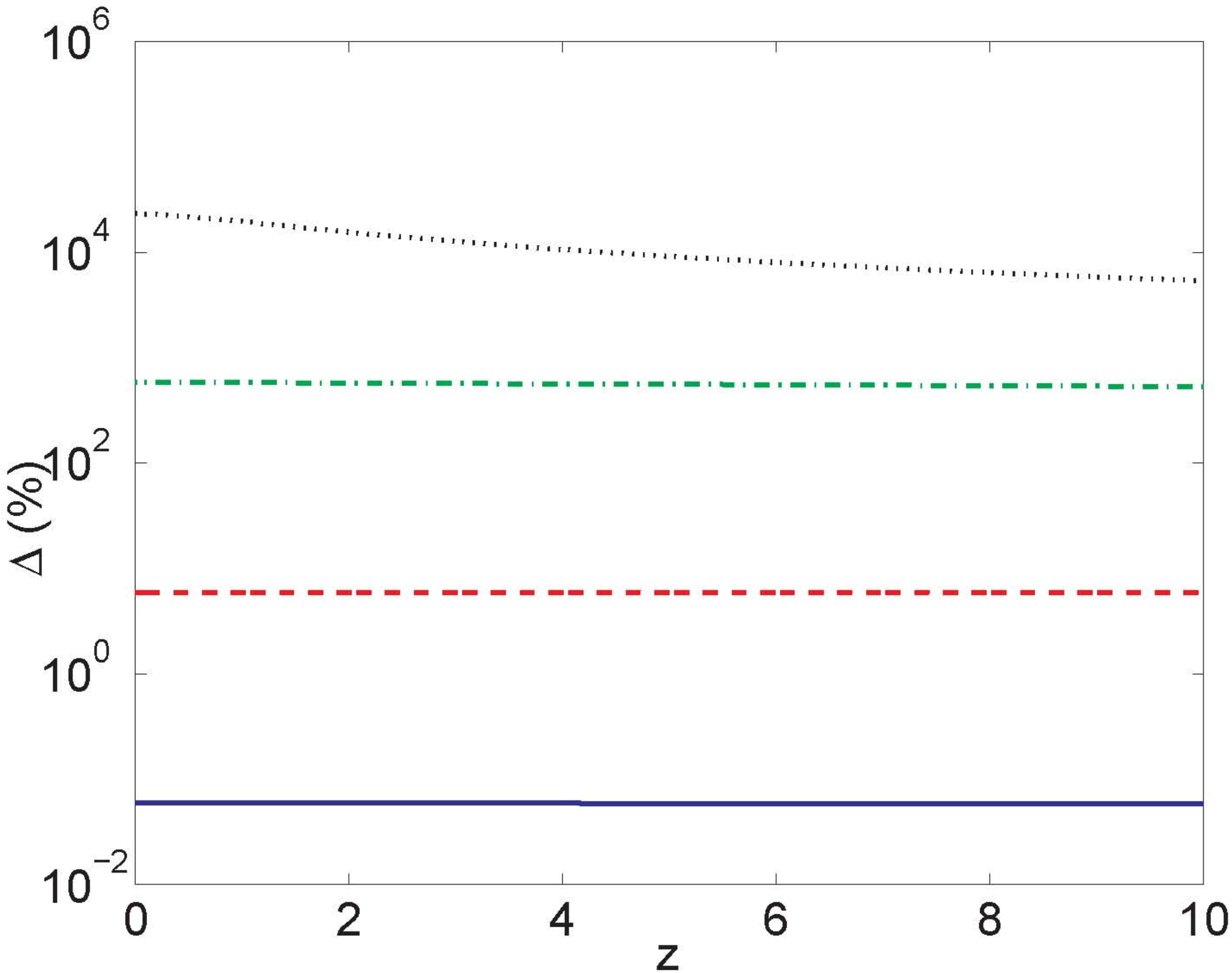
- δ : growth from numerical integration
- δ_g : growth from growth equation
- Δ : comparison variable

$$\Delta \equiv \frac{(\delta_g - \delta)}{\delta}$$

Testing the growth equation



Testing the growth equation



k=.001 h/Mpc

k=.01 h/Mpc

k=.1 h/Mpc

k=1 h/Mpc

What goes wrong on large scales?

The momentum constraint equation:

$$\frac{k^2}{a^2}\Phi + 3H^2\Phi + 3H\dot{\Phi} = -4\pi G\delta\rho$$

Can be ignored under matter domination assumption

Become comparable when k~aH

An improved growth equation

$$\ddot{\delta} + 2H\dot{\delta} - \frac{4\pi G\rho}{1+\xi}\delta = 0$$

$$(\xi = 3a^2H^2/k^2)$$

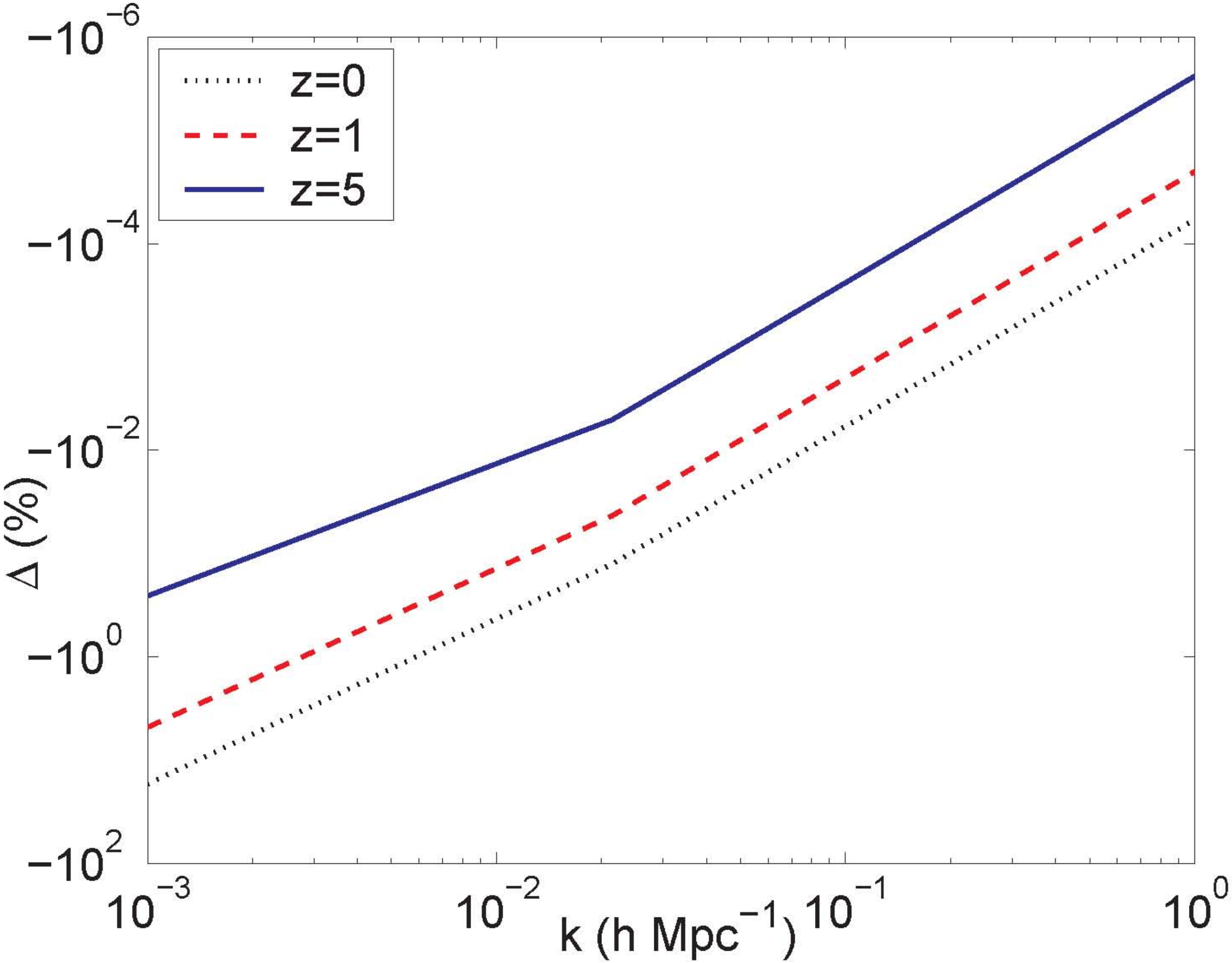
- Simple equation governing growth of perturbations
- Scale dependent
- Accurate on large scales

Testing the improved growth equation

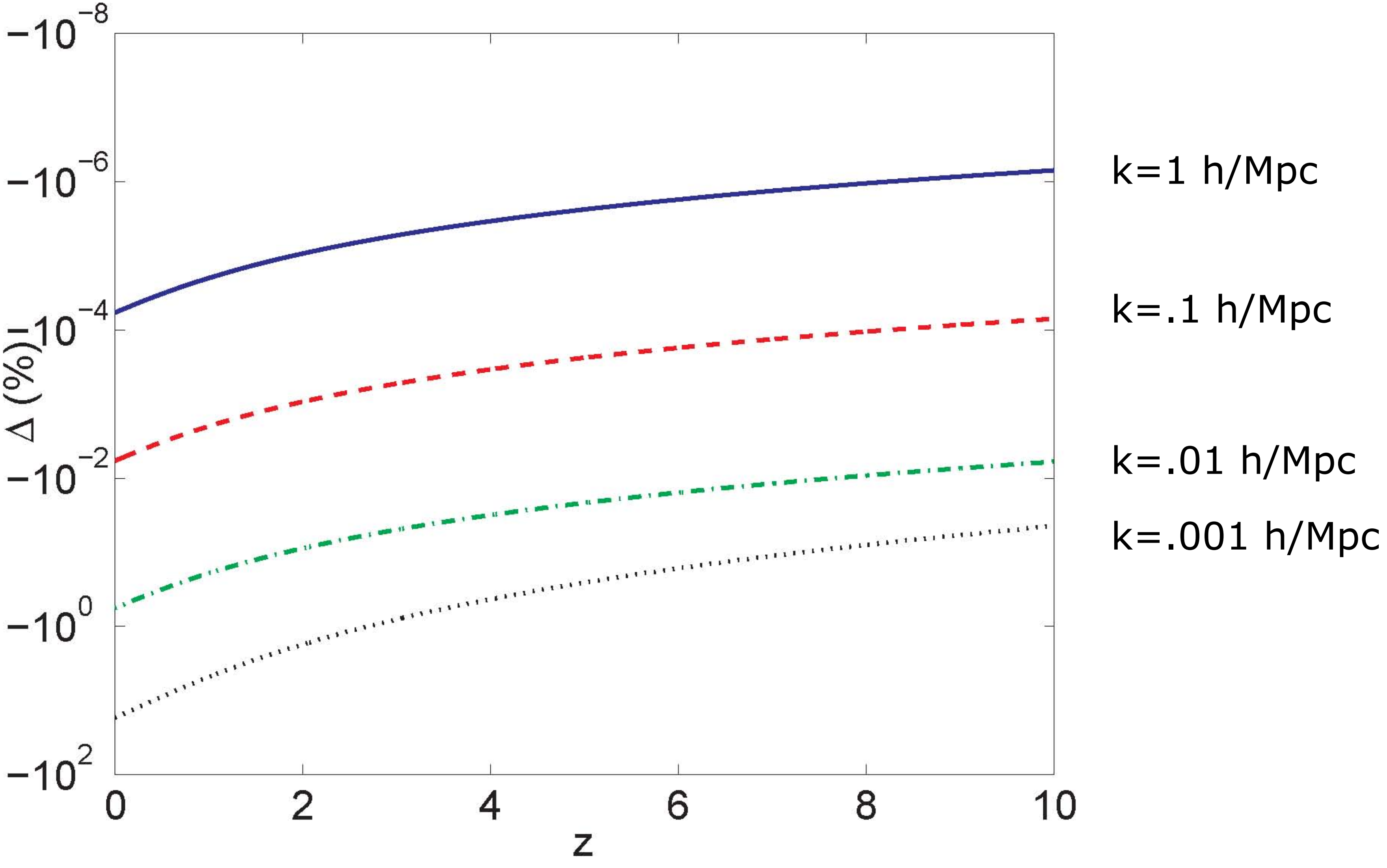
- δ : growth from numerical integration
- δ_{gi} : growth from improved growth equation
- Δ : comparison variable

$$\Delta \equiv \frac{(\delta_{gi} - \delta)}{\delta}$$

Testing the improved growth equation



Testing the improved growth equation



The major correction ξ

$$\xi = 3a^2H^2/k^2$$

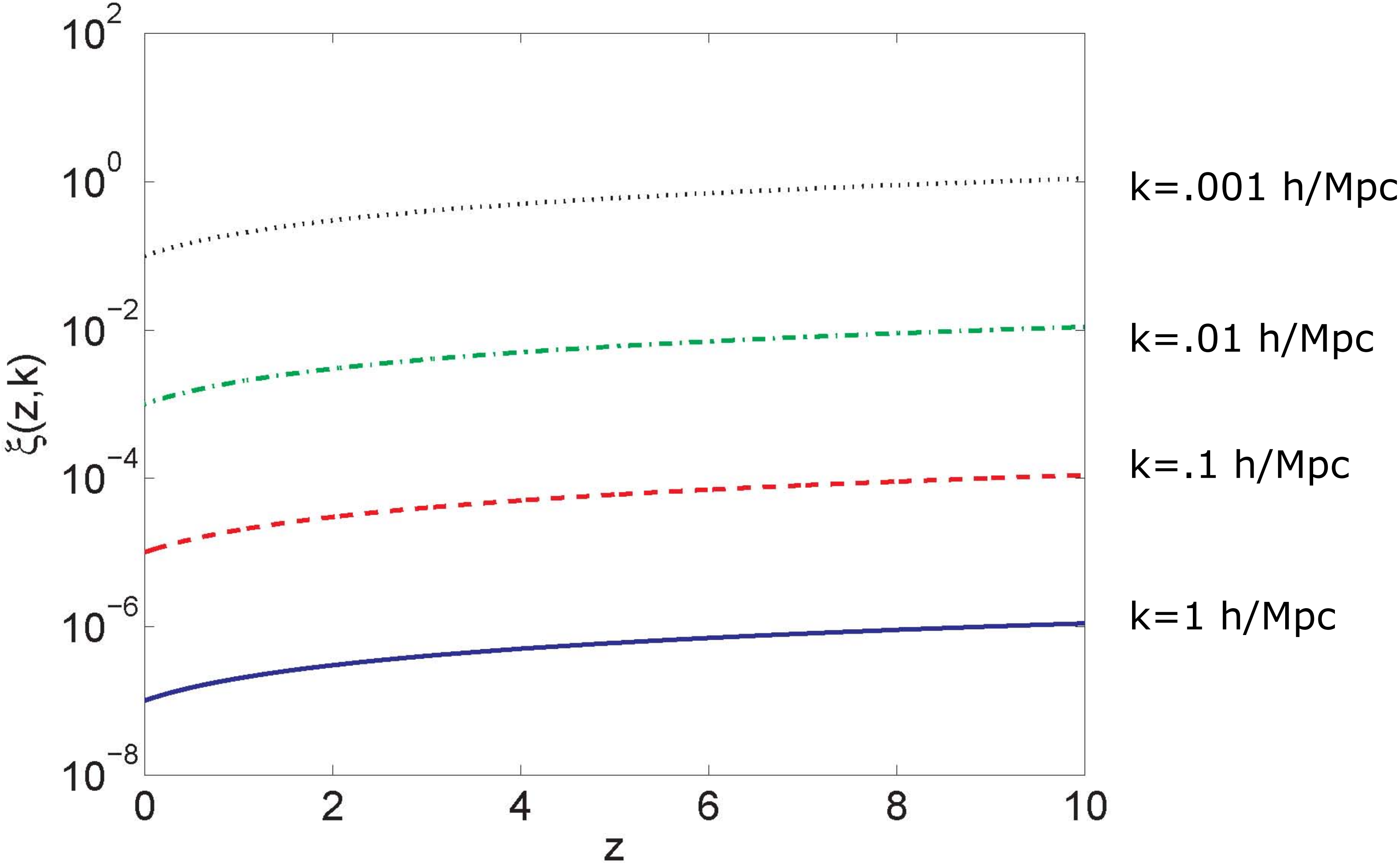
ξ is the ratio of the (discarded) $3H^2\Phi$ term to the (retained) $(k^2/a^2)\Phi$ term in Poisson eqn.:

Evolution of ξ (under matter domination):

$$\xi = (1+z)10^{2n-7}$$

(writing $k=10^{-n}$ h/Mpc)

The major correction ξ



How does the error grow?

Growth Equation:

$$\ddot{\delta}_g + 2H\dot{\delta}_g - 4\pi G\rho\delta_g = 0$$

Improved Growth Equation:

$$\ddot{\delta}_{gi} + 2H\dot{\delta}_{gi} - \frac{4\pi G\rho}{1+\xi}\delta_{gi} = 0$$

Define a comparison (error) variable:

$$\tilde{\Delta} \equiv \frac{(\delta_g - \delta_{gi})}{\delta_{gi}}.$$

How does the error grow?

Error satisfies the differential equation:

$$\ddot{\tilde{\Delta}} + 2\left(\frac{2+\xi}{1+\xi}\right) H\dot{\tilde{\Delta}} - 4\pi G\rho \frac{\xi}{1+\xi}\left(1+\tilde{\Delta}\right) = 0$$

Subject to the initial conditions:

$$\tilde{\Delta}(t_i) = \dot{\tilde{\Delta}}(t_i) = 0$$

Plot of solution almost exactly replicates the Δ of the growth equation compared to the true growth.

Probing beyond-Einstein physics

- Linder and Cahn[1] define a "growth index" γ:

$$f(a) \equiv d\ln\delta / d\ln a = \Omega_m(a)^\gamma$$

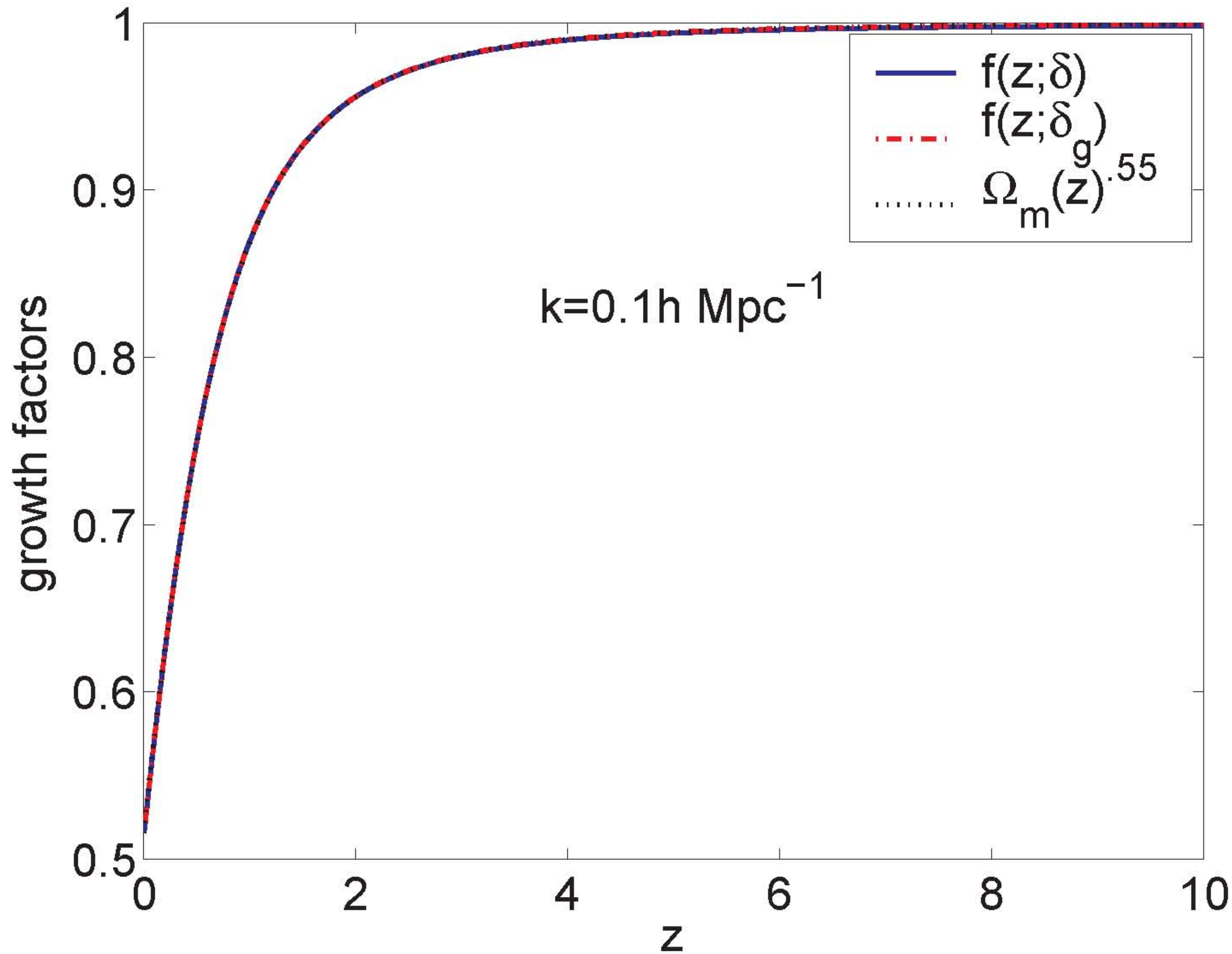
- Provide fitting formulas for γ over 0<a<1:

$$\begin{aligned} \gamma &= 0.55 + .02\left[1 + w(z=1)\right], \quad w < -1 \\ \gamma &= 0.55 + .05\left[1 + w(z=1)\right], \quad w > -1 \end{aligned}$$

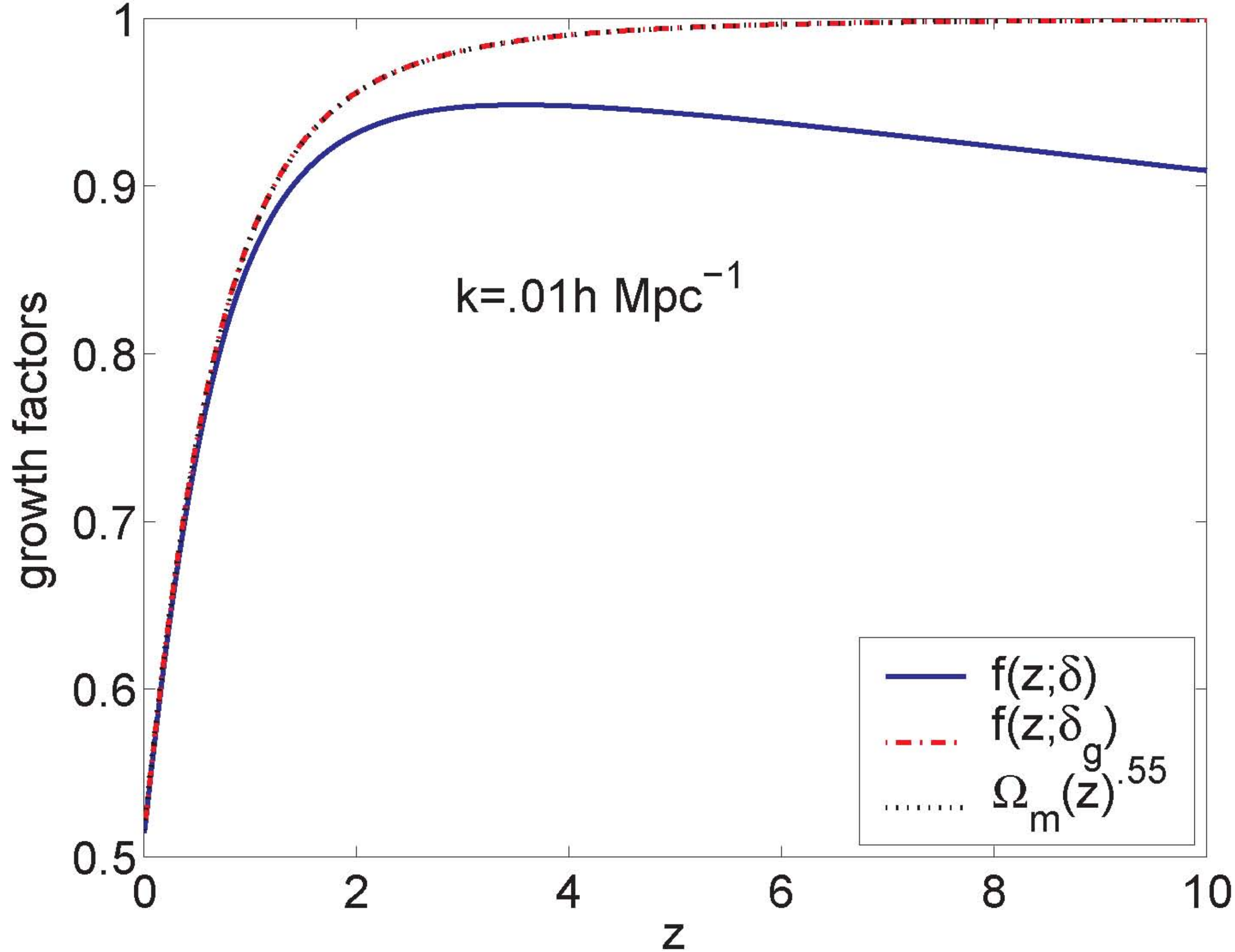
- We now test this for
  - The growth according to the growth equation δ_g
  - The true growth δ

1. Linder (2005), Linder and Cahn (2007)

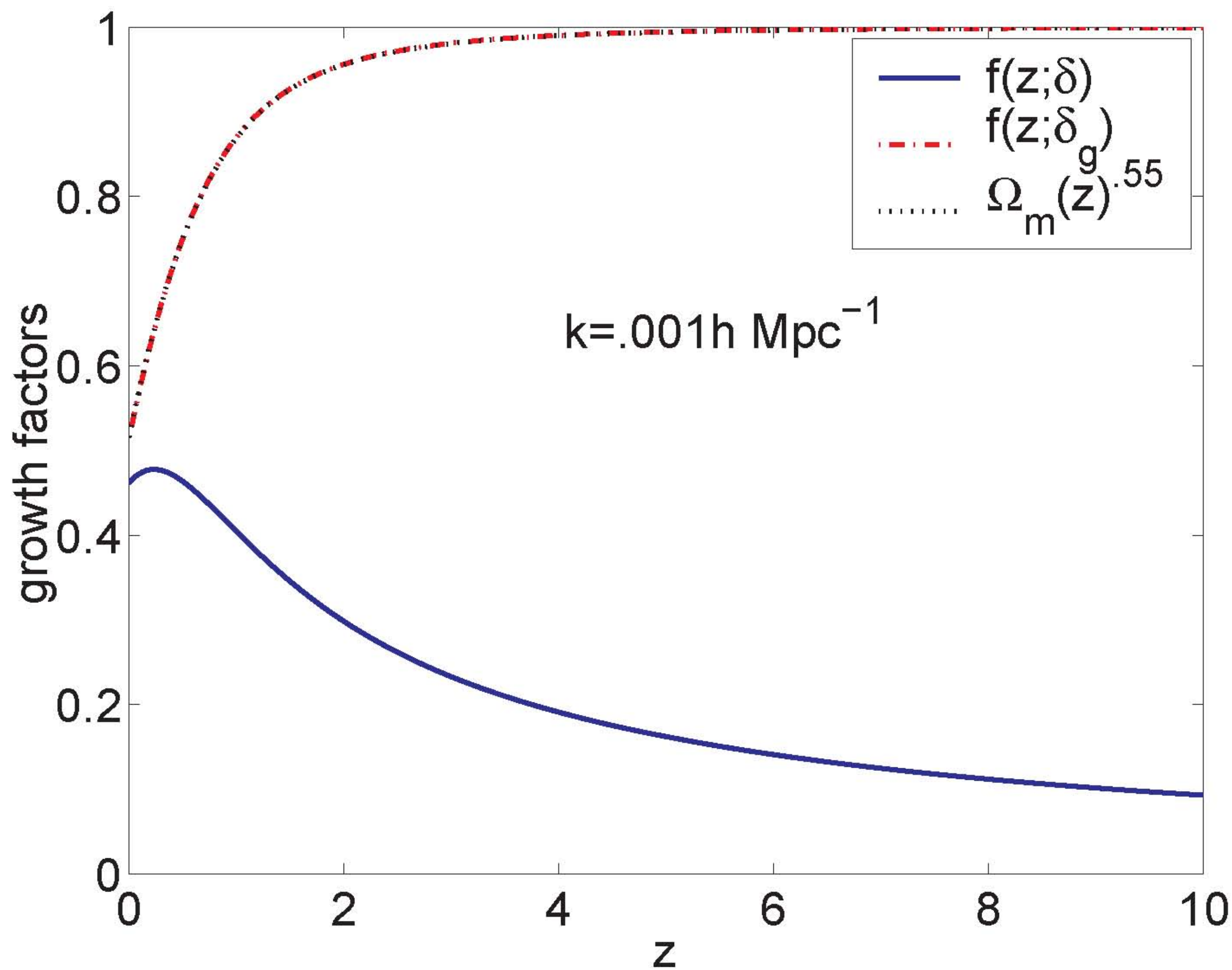
The growth parameter γ



The growth parameter γ



The growth parameter γ



The growth parameter γ

Polarski and Gannouji[1] suggest using the relationship:

$$\gamma_0' = \left(\ln\Omega_{m,0}^{-1}\right)^{-1}\left[-\Omega_{m,0}^{\gamma_0} - 3(\gamma_0 - \frac{1}{2})w_{\mathrm{eff},0} + \frac{3}{2}\Omega_{m,0}^{1-\gamma_0} - \frac{1}{2}\right]$$

$$(w_{\mathrm{eff},0} \equiv w_{DE,0}\Omega_{DE,0})$$

This yields the constraint $|\gamma'_0| < .02$ for ΛCDM

1. Polarski and Gannouji (2007)

The growth parameter γ

Re-deriving this using the modified growth equation:

$$\gamma_0' = \left(\ln\Omega_{m,0}^{-1}\right)^{-1}\left[-\Omega_{m,0}^{\gamma_0} - 3(\gamma_0 - \frac{1}{2})w_{\mathrm{eff},0} + \frac{3}{2}\frac{\Omega_{m,0}^{1-\gamma_0}}{1+\xi} - \frac{1}{2}\right]$$

$$(w_{\mathrm{eff},0} \equiv w_{DE,0}\Omega_{DE,0})$$

Plugging numbers, one can check that $|\gamma_0|<.02$ constraint no longer valid for ΛCDM with k<.01 h/Mpc

The null-test parameter ε

Acquaviva et. al.[1] suggest a null test parameter defined as:

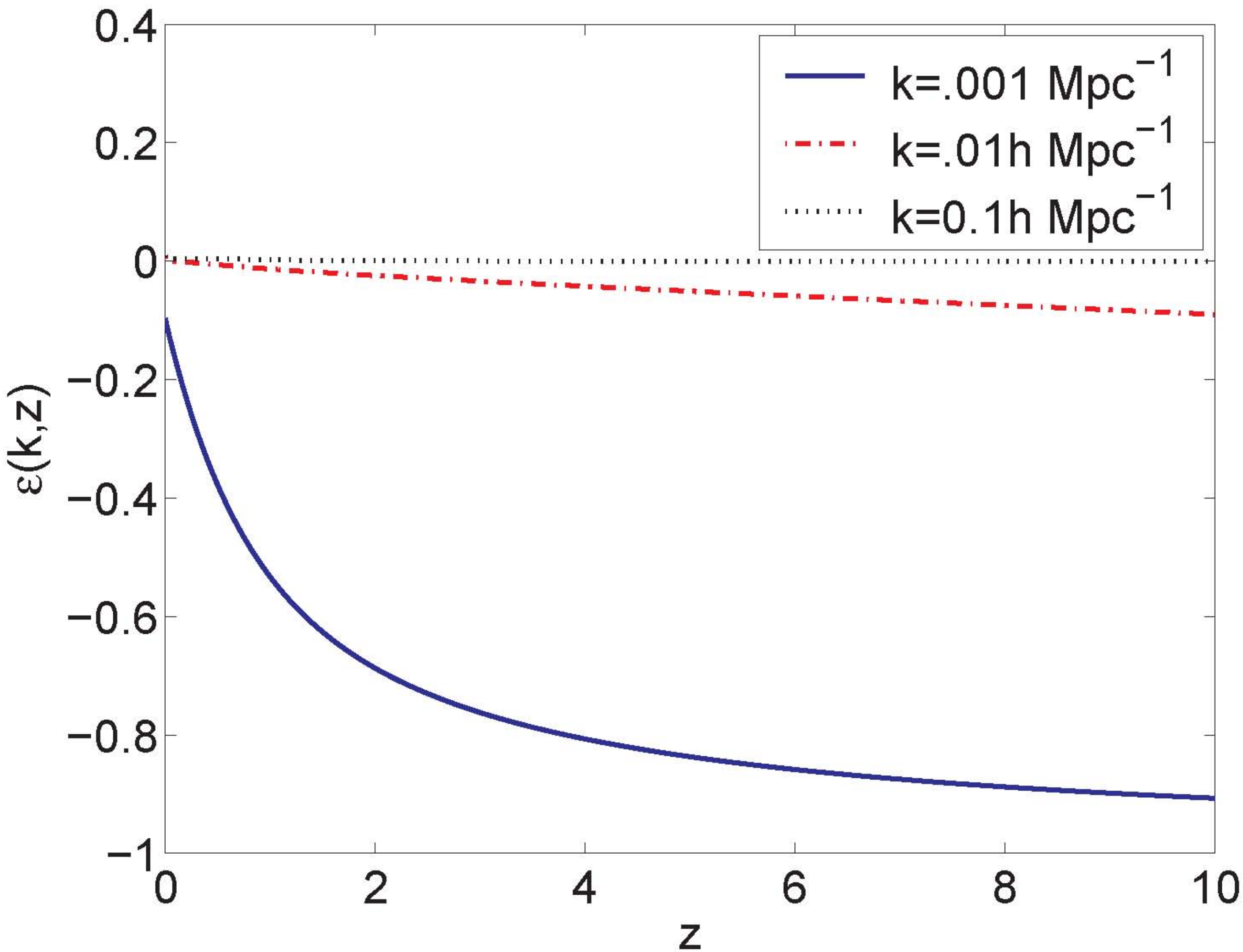
$$\epsilon\left(k,a\right)\equiv\Omega_m\left(a\right)^{-\gamma(z)}f\left(a\right)-1$$

Should be identically 0 if Linder et. al. fits hold true.

Gives non-zero results on large scales and large redshifts because of the departure of the true growth from growth equation.

1. Acquaviva, Das, Hajian, Spergel (2008)

The null-test parameter ε



Conclusions

- Showed that growth equation cannot be trusted at all on scales k<.1 h/Mpc.
- Suggested an improved growth equation which has a higher accuracy on large scales.
- Showed that tests of MG based on growth equation not reliable on large scales and large redshifts.
- Relevant for future survey (e.g. BOSS, ADEPT) which operate at large scale and high redshift.