Cosmological Perturbations in models of dark-sector interactions.

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Introduction

- DE-DM unknown
- May support interactions?
- Interaction may explain coincidence Wetterich, Amendola, Copeland, Tsujikawa etc
- Can appear to have w<-1
- Interactions exist in perturbation.
- Goal : develop formalism for general setting, few applications

Quintessence

- Lagrangian

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}\phi^{,\mu}\phi_{,mu} - V(\phi) + \boxed{L_{\rm int}}\right)$$

- Equation of motion

$$\Gamma \equiv -\frac{\partial}{\partial\phi}L_{\rm int}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) \quad = \quad \boxed{-\Gamma.}$$

- 'Conservation'

$$\nabla_\nu T^{(c)\nu}_\mu = \boxed{Q^{(c)}_\mu}$$

$$\nabla_\nu T^{(\phi)\nu}_\mu = \boxed{Q^{(\phi)}_\mu}$$

$$Q^{(c)}_\mu = -Q^{(\phi)}_\mu,$$

$$Q = -Q^{(c)}_0 = \Gamma\dot{\phi}$$

$$\dot{\rho_c} + 3H\rho_c \quad = \quad \boxed{Q,}$$

$$\dot{\rho_\phi} + 3H\rho_\phi(1+w_\phi) \quad = \quad \boxed{-Q,}$$

$$w_{\phi,\rm eff} = w_\phi + \frac{Q}{3H\rho_\phi}$$

Phase-space approach

- Dimensionless variables Copeland, Liddle, Wands

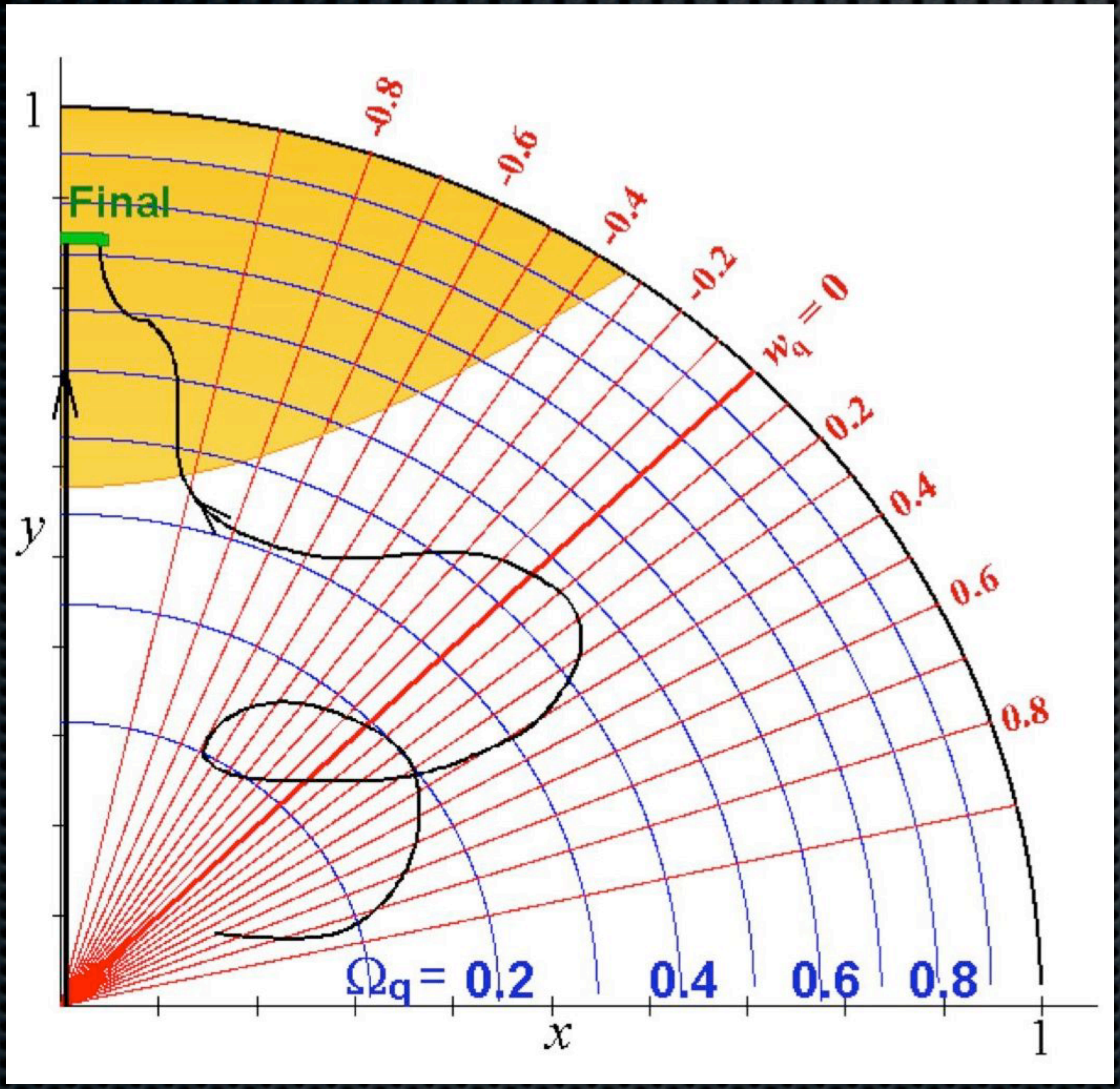
$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \quad y = \frac{\kappa}{H}\sqrt{\frac{V}{3}}, \quad u = \frac{\kappa}{H}\sqrt{\frac{\rho_c}{3}}$$

- Friedmann constraint

$$x^2 + y^2 + u^2 = 1$$

- Phase-space = (part of) circle

Quintessence as trajectories



$$\Omega_q = x^2 + y^2$$

$$w_q = \frac{x^2 - y^2}{x^2 + y^2}$$

$$w_{\rm eff} = x^2 - y^2$$

Quintessence as trajectories

$$\frac{d\ln H}{dN} = -\frac{3}{2}\left(1+x^2-y^2+\frac{1}{3}z^2\right),$$
$$\frac{dx}{dN} = -\gamma + \sqrt{\frac{3}{2}}\lambda y^2 - x\left(3+\frac{d\ln H}{dN}\right),$$
$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}}\lambda xy - y\left(\frac{d\ln H}{dN}\right),$$
$$\frac{du}{dN} = \frac{\gamma x}{u} - u\left(\frac{3}{2}+\frac{d\ln H}{dN}\right),$$

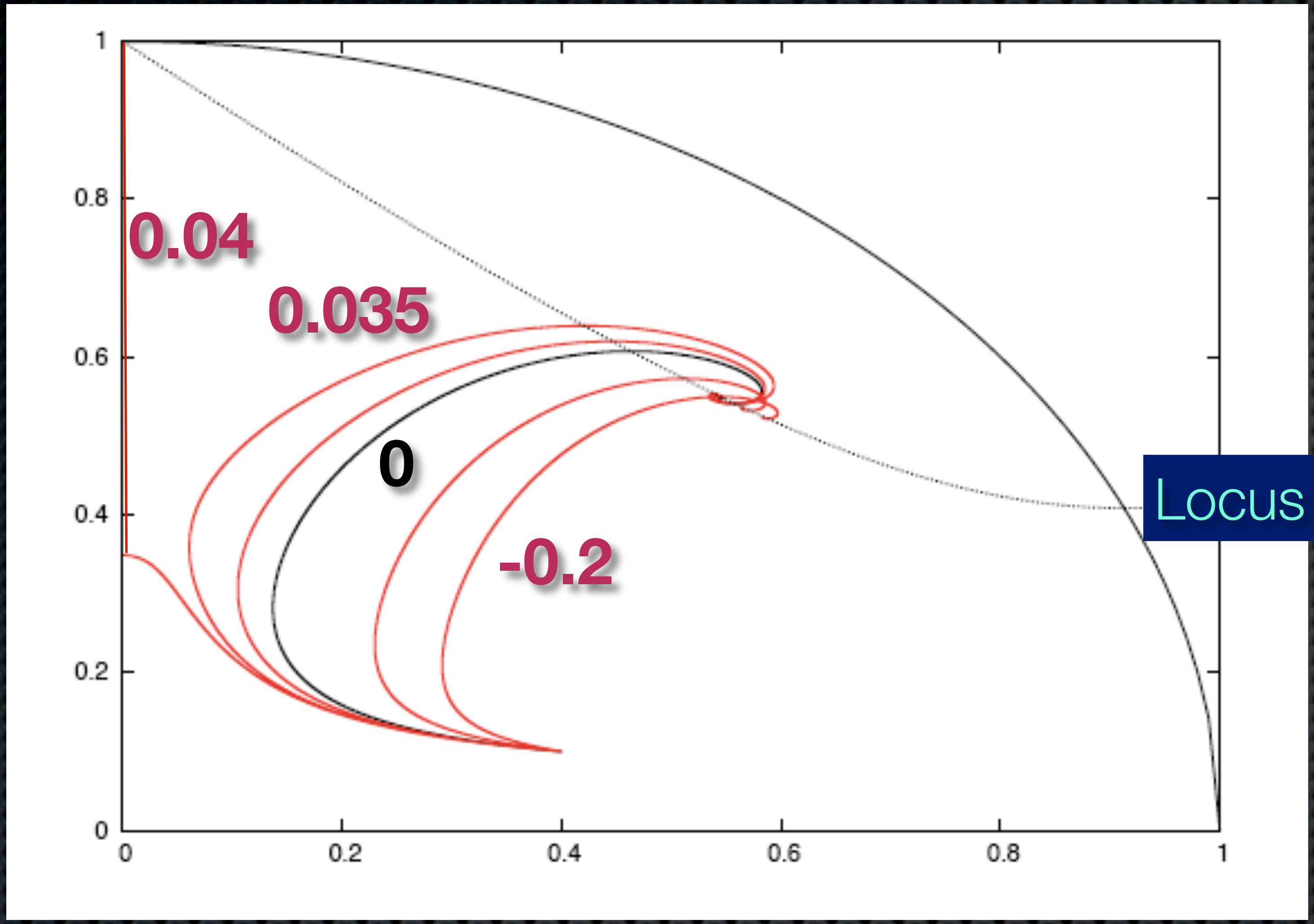
$$\gamma = \frac{\kappa\Gamma}{\sqrt{6}H^2}.$$
$$N = \ln a$$

- 'Roll' variable parametrizes potential:

$$\lambda \equiv -\frac{1}{\kappa V}\frac{dV}{d\phi}$$

- E.g. λ = constant gives exponential potential.
- Attractor dynamics are known for each λ= constant slice...

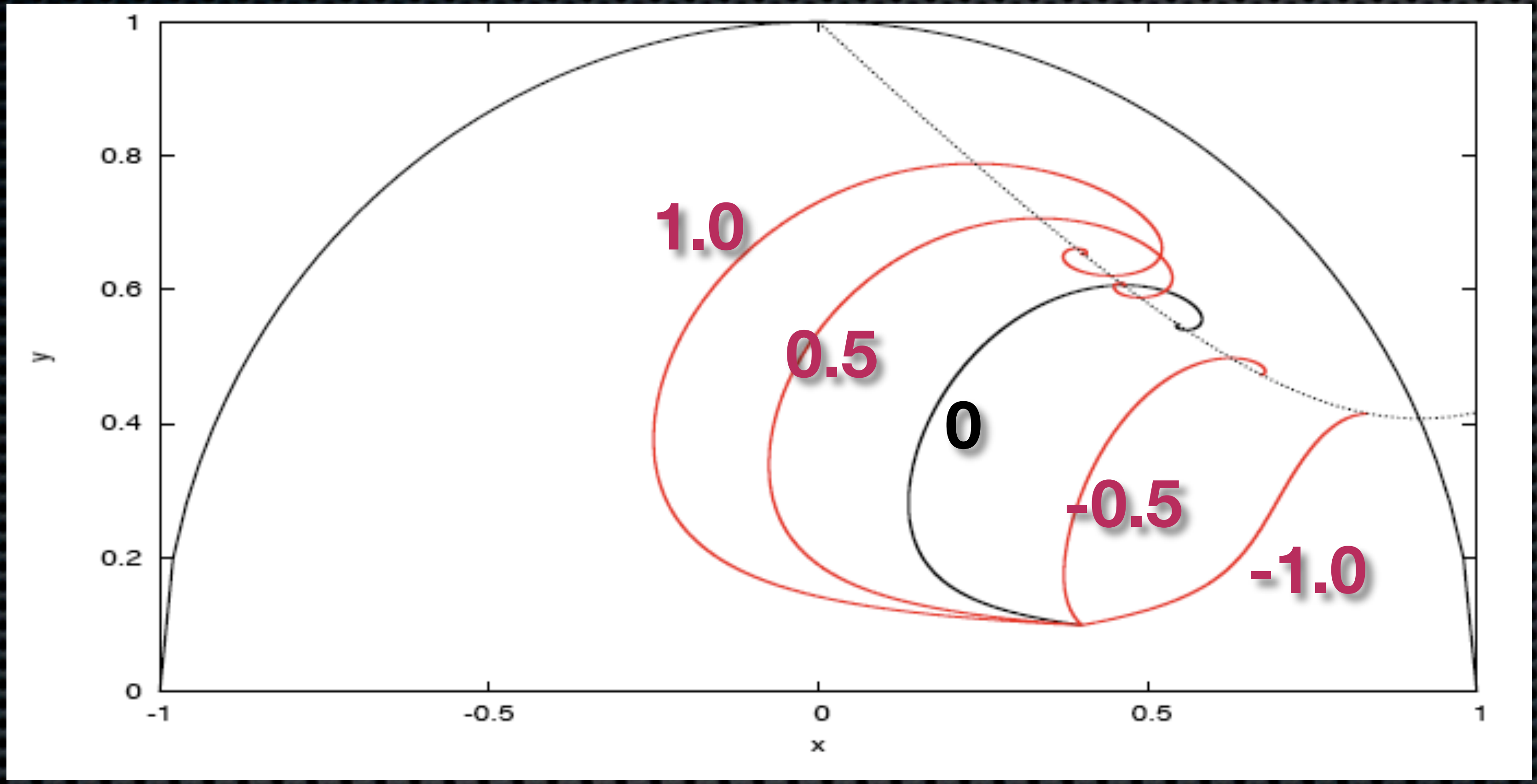
Example 1: $Q = \beta H \rho_c$ Billyard+Coley, Zimdahl etc.



$$\gamma = \frac{\beta u^2}{2x}$$

- DM→DE conversion speeds up scaling process - we can't live in a scaling solution. [Amendola]
- DE →DM conversion works against field motion

Example 2: $Q = \sqrt{2/3} b\kappa\dot{\phi}\rho_c$ Wetterich, Amendola



$\gamma = bu^2$

- DM→DE conversion speeds up scaling process
- DE →DM conversion pushes field 'uphill'

Perturbations

$$\delta_c = \frac{\delta\rho_c}{\bar{\rho}_c}, \quad \phi = \bar{\phi} + \delta\phi,$$

- Hwang+Noh approch, in comoving gauge

$$\begin{aligned}
\ddot{\delta}_c + 2H\dot{\delta}_c - \frac{\kappa^2}{2}\rho_c\delta_c &= \kappa^2\left(2\dot{\phi}\delta\dot{\phi} - V'\delta\phi\right) - \frac{1}{a^2}\frac{d}{dt}\left\{\frac{a^2}{\rho_c}\left[\Gamma\dot{\phi}\delta_c - \dot{\phi}\delta\Gamma - \Gamma(\delta\dot{\phi} + 3H\delta\phi)\right]\right\} \\
&+ \left(3\dot{H} + 2\kappa^2\dot{\phi}^2 - \frac{k^2}{a^2}\right)\frac{\Gamma}{\rho_c}\delta\phi. \\
\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V''\right)\delta\phi &= \dot{\phi}\dot{\delta}_c + \frac{\Gamma\dot{\phi}^2}{\rho_c}\delta_c - (1 + \frac{\dot{\phi}^2}{\rho_c})\delta\Gamma - \frac{\Gamma\dot{\phi}}{\rho_c}\delta\dot{\phi} - \dot{\phi}\frac{d}{dt}\left(\frac{\Gamma\delta\phi}{\rho_c}\right) \\
&+ 2(\ddot{\phi} + 3H\dot{\phi})\frac{\Gamma\delta\phi}{\rho_c},
\end{aligned}$$

- Possible to do perturbations in phase-space (useful for perturbations at attractors)

Perturbations

- In phase-space variables

$$\frac{d^2\delta_c}{dN^2} + A\frac{d\delta_c}{dN} + B\delta_c = C\frac{d\delta\gamma}{dN} + D\delta\gamma + E\frac{d\left(\kappa\delta\phi\right)}{dN} + F(\kappa\delta\phi),$$

$$\frac{d^2\left(\kappa\delta\phi\right)}{dN^2} + \hat{A}\frac{d\left(\kappa\delta\phi\right)}{dN} + \hat{B}(\kappa\delta\phi) = \hat{C}\frac{d\delta_c}{dN} + \hat{D}\delta_c + \hat{E}\delta\gamma,$$

- Just a set of coupled ODEs, perturbed interactions = extra source terms.

Perturbations

- In phase-space variables

$$\frac{d^2\delta_c}{dN^2} + A\frac{d\delta_c}{dN} + B\delta_c = C\frac{d\delta\gamma}{dN} + D\delta\gamma + E\frac{d(\kappa\delta\phi)}{dN} + F(\kappa\delta\phi),$$

$$\frac{d^2(\kappa\delta\phi)}{dN^2} + \hat{A}\frac{d(\kappa\delta\phi)}{dN} + \hat{B}(\kappa\delta\phi) = \hat{C}\frac{d\delta_c}{dN} + \hat{D}\delta_c + \hat{E}\delta\gamma,$$

$$A = \frac{d\ln H}{dN} + 2 \qquad C = \frac{2x}{u^2}$$

$$\hat{A} = \frac{d\ln H}{dN} + \frac{4\gamma x}{u^2} + 3 \qquad \hat{C} = \sqrt{6}x$$

Perturbations

- In phase-space variables

$$\frac{d^2\delta_c}{dN^2} + A\frac{d\delta_c}{dN} + B\delta_c = C\frac{d\delta\gamma}{dN} + D\delta\gamma + E\frac{d(\kappa\delta\phi)}{dN} + F(\kappa\delta\phi),$$

$$\frac{d^2(\kappa\delta\phi)}{dN^2} + \hat{A}\frac{d(\kappa\delta\phi)}{dN} + \hat{B}(\kappa\delta\phi) = \hat{C}\frac{d\delta_c}{dN} + \hat{D}\delta_c + \hat{E}\delta\gamma,$$

$$F = 3y^2\lambda + \frac{\sqrt{6}}{u^2}\left[5\gamma + 4\gamma\frac{d\ln H}{dN} + \frac{d\gamma}{dN} + 4\gamma x^2 - \gamma\eta y^2 - \frac{2\gamma}{3}\left(\frac{k}{aH}\right)^2\right]$$

$$+ 2\gamma\lambda y^2 - \frac{2}{3}\sqrt{6}\gamma^2 - 2\sqrt{6}\gamma x - \frac{2}{3}\sqrt{6}\gamma x\frac{d\ln H}{dN} - \frac{\sqrt{6}x}{3}\frac{d\gamma}{dN}\frac{2\gamma}{u^4} + \frac{4\sqrt{6}}{3}\frac{\gamma^3 x^2}{u^6}$$

Perturbations

- In phase-space variables

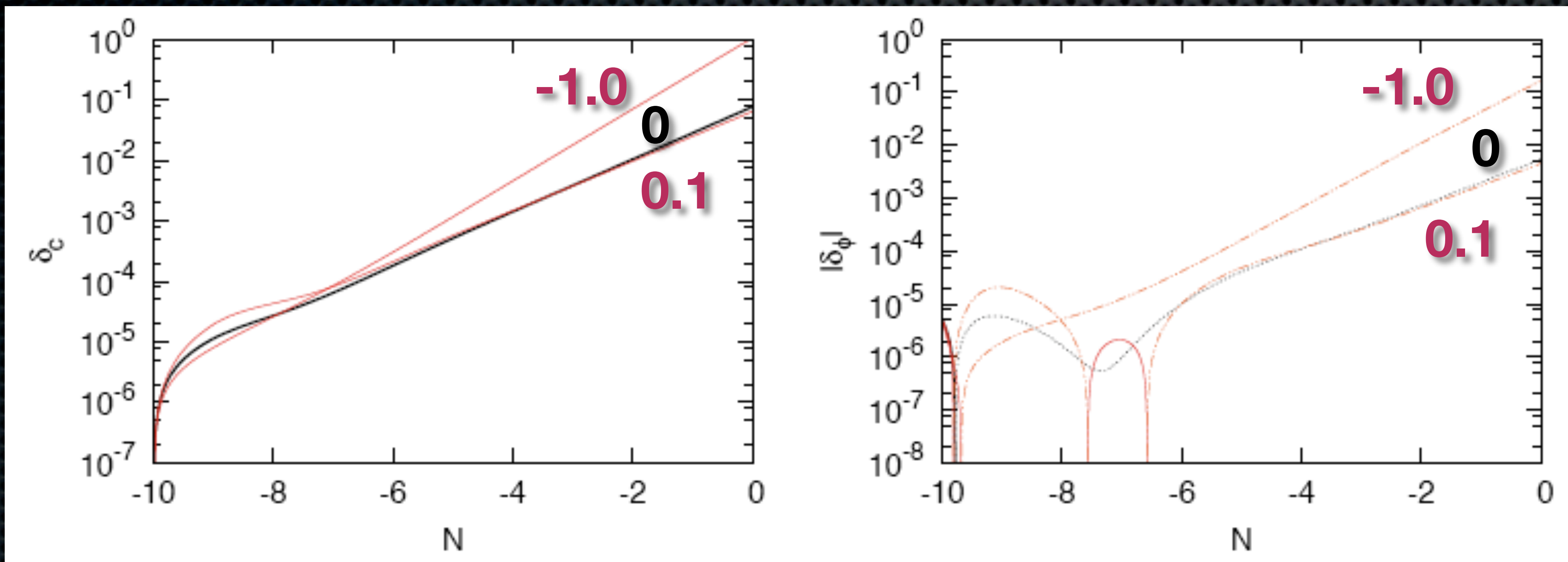
$$\frac{d^2\delta_c}{dN^2} + A\frac{d\delta_c}{dN} + B\delta_c = C\frac{d\delta\gamma}{dN} + D\delta\gamma + E\frac{d(\kappa\delta\phi)}{dN} + F(\kappa\delta\phi),$$

$$\frac{d^2(\kappa\delta\phi)}{dN^2} + \hat{A}\frac{d(\kappa\delta\phi)}{dN} + \hat{B}(\kappa\delta\phi) = \hat{C}\frac{d\delta_c}{dN} + \hat{D}\delta_c + \hat{E}\delta\gamma,$$

- Valid for any potential and interaction.
- Can calculate DE-density contrast (clustering)

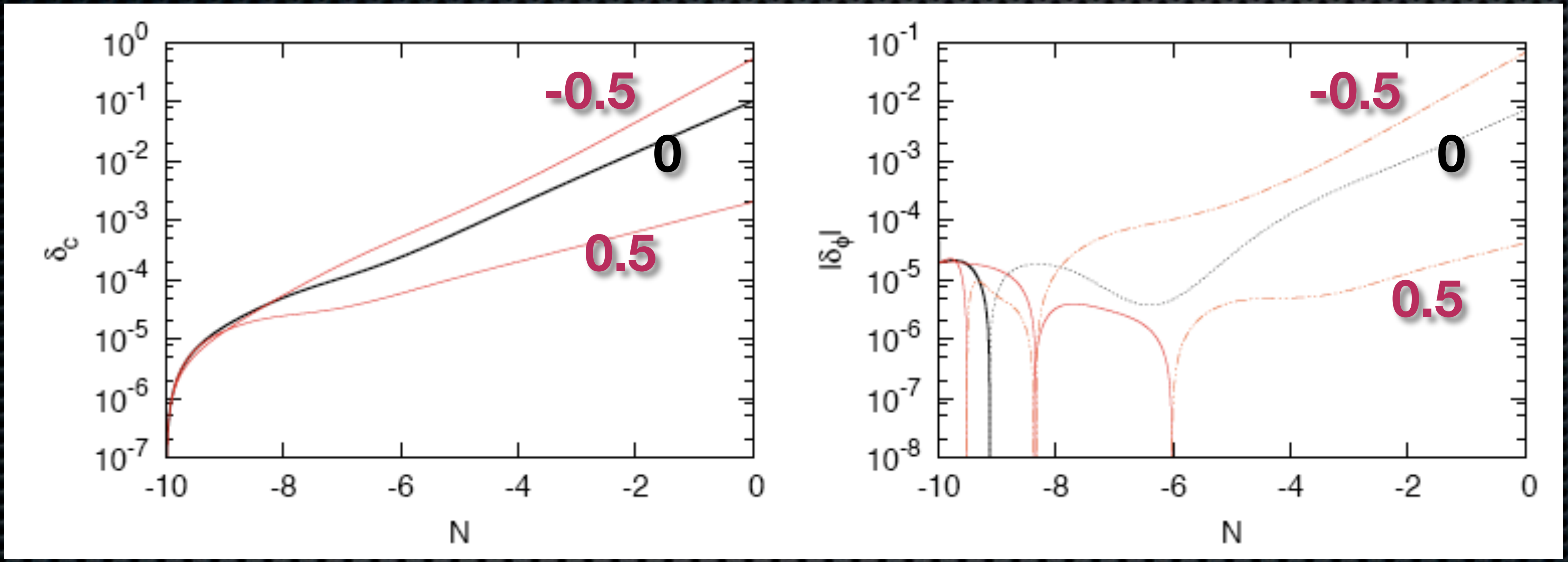
$$\delta_\phi = \frac{1}{3(x^2+y^2)}\sqrt{6}x\frac{d(\kappa\delta\phi)}{dN} + \left(\boxed{2\sqrt{6}\gamma\frac{x^2}{u^2}} - 3\lambda y^2\right)\kappa\delta\phi.$$

Example 1: $Q = \beta H \rho_c$ Billyard+Coley, Zimdahl etc.



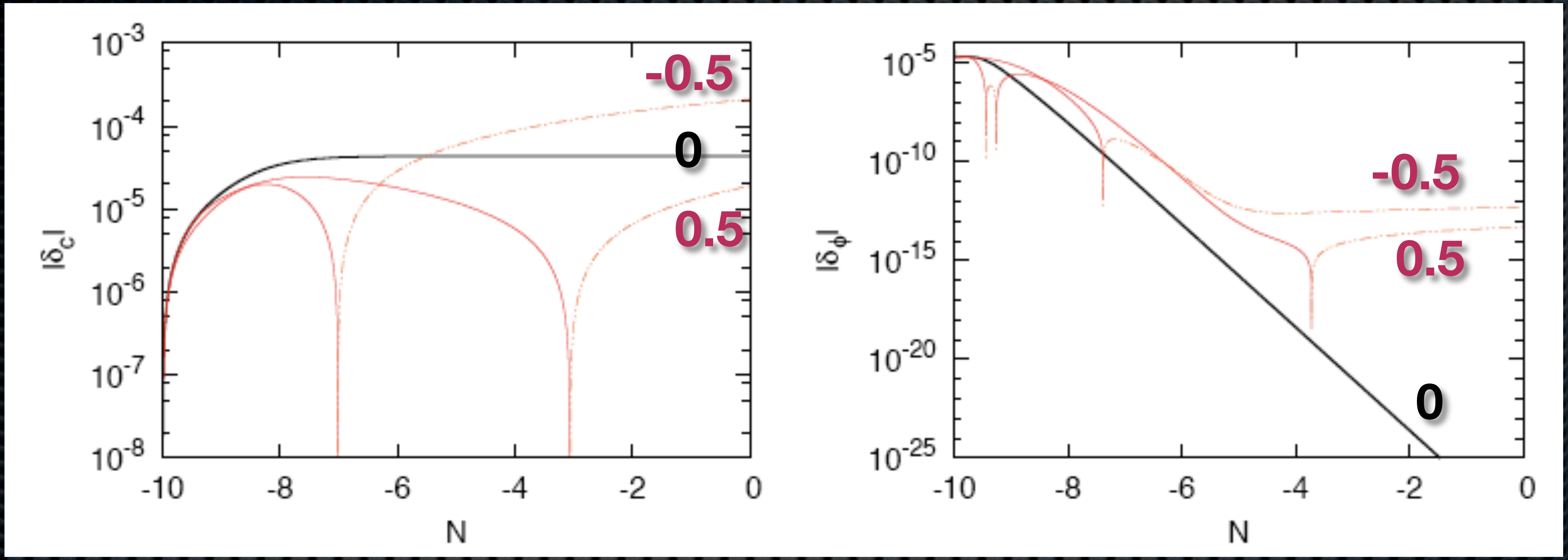
- Growth rate increases with β<0.
- Bigger β make perturbations blow up
- Need large β to be distinguishable from uncoupled case

Example 2: $Q = \sqrt{2/3} b\kappa\dot{\phi}\rho_c$ Wetterich, Amendola



- More sensitive to changes in interaction

Example 2: $Q = \sqrt{2/3}b\kappa\dot{\phi}\rho_c$ Wetterich, Amendola



- Almost flat / 'skater' model [Sahnen+Liddle+Parkinson]

$$\delta_c = C_1 + C_2 N + C_3 e^{-2N}, \quad \kappa\delta\phi = \frac{C_2}{\sqrt{6}b} + C_4 e^{-3N},$$

- Very hard to distinguish without including DE perturbations [SC+Efstathiou '07]

Conclusions

- DM-DE interactions means that their perturbations are also coupled with extra source terms.
- Developed a formalism for applications to any potential/interaction.
- Perturbations and background are evolved together in phase-space. Easy to see behaviour near attractors and identify any instability.
- Ongoing : numerical work on CMB/LSS and prospects for detection.