On the Stability of the Einstein Static Universe in f(R)-gravity

Naureen Goheer
University of Cape Town

Outline

Outline

- why are we interested in the Einstein static (ES) model?

Outline

- why are we interested in the Einstein static (ES) model?
    - basic features, historical review

Outline

- why are we interested in the Einstein static (ES) model?
    - basic features, historical review

- why modified gravity, in particular f(R)-gravity?

Outline

- why are we interested in the Einstein static (ES) model?
  - basic features, historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations

Outline

- why are we interested in the Einstein static (ES) model?
    - basic features, historical review
- why modified gravity, in particular f(R)-gravity?
    - derive basic field equations
- dynamical system analysis of the closed FRW state space (including the ES model)

Outline

- why are we interested in the Einstein static (ES) model?
  - basic features, historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical system analysis of the closed FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background

Outline

- why are we interested in the Einstein static (ES) model?
  - basic features, historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical system analysis of the closed FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background
- compare and interpret the results obtained from the two approaches

The Background: Einstein Static (ES)

The Background: Einstein Static (ES)

- originally introduced by Einstein in 1917 to construct a static solution of the GR field equations

The Background: Einstein Static (ES)

- originally introduced by Einstein in 1917 to construct a static solution of the GR field equations

- cosmological constant exactly balances energy content: $\Lambda = 4\pi(\rho_0+3p_0) = 4\pi(1+3w)\rho_0$ for perfect fluid

The Background: Einstein Static (ES)

- originally introduced by Einstein in 1917 to construct a static solution of the GR field equations
- cosmological constant exactly balances energy content: $\Lambda = 4\pi(\rho_0+3p_0) = 4\pi(1+3w)\rho_0$ for perfect fluid
- topology $R\times S^3$, metric: $ds^2 = -dt^2 + a_0^2\left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2\right]$ with fixed finite radius a_0 ($R = {}^3R = 6 / a_0^2$)

The Background: Einstein Static (ES)

- originally introduced by Einstein in 1917 to construct a static solution of the GR field equations
- cosmological constant exactly balances energy content: $\Lambda = 4\pi(\rho_0+3p_0) = 4\pi(1+3w)\rho_0$ for perfect fluid
- topology $R\times S^3$, metric: $ds^2 = -dt^2 + a_0^2\left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2\right]$ with fixed finite radius a_0 ($R = {}^3R = 6 / a_0^2$)
- abandoned when observations showed that the universe is expanding

features of ES model

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)
- ES is unstable against homogeneous linear perturbations (Eddington 1930)

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)
- ES is unstable against homogeneous linear perturbations (Eddington 1930)
  - expansion/contraction

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)
- ES is unstable against homogeneous linear perturbations (Eddington 1930)
  - expansion/contraction
  - allows for transition from decelerated expansion to acceleration in ΛCDM cosmology

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)
- ES is unstable against homogeneous linear perturbations (Eddington 1930)
  - expansion/contraction
  - allows for transition from decelerated expansion to acceleration in ΛCDM cosmology
- ES is neutrally stable against inhomogeneous linear perturbations for $w>1/5$ (Harrison 1967, Gibbons 1987, 1988, Barrow 2003)

features of ES model

- ES maximizes entropy within family of FRW radiation models (Gibbons 1987)
- ES is the unique highest symmetry non-empty FRW model (Ellis 1967)
- ES is unstable against homogeneous linear perturbations (Eddington 1930)
  - expansion/contraction
  - allows for transition from decelerated expansion to acceleration in ΛCDM cosmology
- ES is neutrally stable against inhomogeneous linear perturbations for w>1/5 (Harrison 1967, Gibbons 1987, 1988, Barrow 2003)
  - reason for this "Non-Newtonian" stability: maximum scale (finite "size" of the universe) => fluctuations oscillate rather than grow

possible ES scenarios

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase (“emergent universe”, Ellis & Maartens, 2002)

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase ("emergent universe", Ellis & Maartens, 2002)

  → avoid initial singularity (and maybe quantum regime)

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase ("emergent universe", Ellis & Maartens, 2002)

  → avoid initial singularity (and maybe quantum regime)

  → no horizon problem

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase ("emergent universe", Ellis & Maartens, 2002)

  → avoid initial singularity (and maybe quantum regime)

  → no horizon problem

- transient phase: BB → inflation → decelerating phase → **ES** (inflection point) → accelerating phase (DE/dS regime)

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase ("emergent universe", Ellis & Maartens, 2002)

  → avoid initial singularity (and maybe quantum regime)

  → no horizon problem

- transient phase: BB → inflation → decelerating phase → **ES** (inflection point) → accelerating phase (DE/dS regime)

  → time for structure formation

possible ES scenarios

- initial state: **ES** → inflation → decelerating phase → accelerating phase ("emergent universe", Ellis & Maartens, 2002)

  → avoid initial singularity (and maybe quantum regime)

  → no horizon problem

- transient phase: BB → inflation → decelerating phase → **ES** (inflection point) → accelerating phase (DE/dS regime)

  → time for structure formation

- **find orbits** in the dynamical systems analysis **corresponding to one of the scenarios above**

Outline

- why are we interested in the Einstein static (ES) model?
  - historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical system analysis of FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background
- compare and interpret the results obtained from the two approaches

why modify GR?

why modify GR?

- the “Standard” ΛCDM Model of cosmology fits observational data (CMB, LSS) very well if we assume that the universe is **dominated by Dark Energy** (74%) and **Dark Matter** (22%)

why modify GR?

- the “Standard” ΛCDM Model of cosmology fits observational data (CMB, LSS) very well if we assume that the universe is **dominated by Dark Energy** (74%) and **Dark Matter** (22%)
- shortcomings: dark matter and dark energy **unexplained/ not observed directly**
  - ΛCDM model does not give theoretical explanation for late time acceleration ==> it is more of an empirical fit to data
  - must introduce scalar fields and/or fine-tuned cosmological constant for inflation and DE
  - quantum regime?

why modify GR?

- the “Standard” ΛCDM Model of cosmology fits observational data (CMB, LSS) very well if we assume that the universe is **dominated by Dark Energy** (74%) and **Dark Matter** (22%)
- shortcomings: dark matter and dark energy **unexplained/ not observed directly**
  - ΛCDM model does not give theoretical explanation for late time acceleration ==> it is more of an empirical fit to data
  - must introduce scalar fields and/or fine-tuned cosmological constant for inflation and DE
  - quantum regime?
- one option to avoid introducing dark components: modify theory of gravity itself on relevant scales
  - interesting to note: unique status of GR was questioned by Weyl (1919) and Eddington (1922) by considering higher order invariants in the GR action

f(R)-gravity

f(R)-gravity

- generalize Einstein-Hilbert action $\mathcal{A}_{EH} = \int d^4x\sqrt{-g}R$

f(R)-gravity

- generalize Einstein-Hilbert action $\mathcal{A}_{EH} = \int d^4x\sqrt{-g}R$
- R → function of Ricci scalar f(R)

f(R)-gravity

- generalize Einstein-Hilbert action $\mathcal{A}_{EH} = \int d^4x\sqrt{-g}R$
- R → function of Ricci scalar f(R)
  - f(R) is good toy model: simple, but has the nice feature of admitting late time accelerating models (alternative to DE)

linearized 1+3 eqs. around FRW

$$\Theta^2 = 3\left[\rho^T + \frac{\Lambda}{f'}\right] - \frac{3}{2}\tilde{R}$$

$$\dot{\rho^m} = -\Theta\rho^m(1+w)$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + \tilde{\nabla}^a A_a - \frac{1}{2}\left(\rho^T + 3p^T\right) + \frac{\Lambda}{f'}$$

$$A^a = \dot{u}^a - \frac{w}{w+1}\frac{\tilde{\nabla}^a \rho^m}{\rho^m}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - E_{ab} + \frac{1}{2}\Pi_{ab} + \tilde{\nabla}_{\langle a}A_{b\rangle}$$

$$\dot{E}_{ab} = -\Theta E_{ab} + curl(H_{ab}) - \frac{1}{2}\left(+p^T\right)\sigma_{ab} - \frac{1}{6}\Theta\Pi_{ab} - \frac{1}{2}\dot{\Pi}_{ab} - \frac{1}{2}\tilde{\nabla}_{\langle a}q_{b\rangle}$$

$$\dot{H}_{ab} = -\Theta H_{ab} - curl(E_{ab}) + \frac{1}{2}curl(\Pi_{ab})$$

$$\dot{\omega}_a = -\frac{2}{3}\Theta\omega_a - \frac{1}{2}curl(A_a)$$

- plus constraint equations

linearized 1+3 eqs. around FRW

$$\Theta^2 = 3\left[\rho^T + \frac{\Lambda}{f'}\right] - \frac{3}{2}\tilde{R}$$

$$\dot{\rho^m} = -\Theta\rho^m(1+w)$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + \tilde{\nabla}^a A_a - \frac{1}{2}\left(\rho^T + 3p^T\right) + \frac{\Lambda}{f'}$$

$$A^a = \dot{u}^a - \frac{w}{w+1}\frac{\tilde{\nabla}^a\rho^m}{\rho^m}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - E_{ab} + \frac{1}{2}\Pi_{ab} + \tilde{\nabla}_{\langle a}A_{b\rangle}$$

$$\dot{E}_{ab} = -\Theta E_{ab} + curl(H_{ab}) - \frac{1}{2}\left(+p^T\right)\sigma_{ab} - \frac{1}{6}\Theta\Pi_{ab} - \frac{1}{2}\dot{\Pi}_{ab} - \frac{1}{2}\tilde{\nabla}_{\langle a}q_{b\rangle}$$

$$\dot{H}_{ab} = -\Theta H_{ab} - curl(E_{ab}) + \frac{1}{2}curl(\Pi_{ab})$$

$$\dot{\omega}_a = -\frac{2}{3}\Theta\omega_a - \frac{1}{2}curl(A_a)$$

- plus constraint equations

- for **FRW background**, A=ω=σ=*0* and ∇^{a}*f=0* for all scalars *f*, and **only the first 3 equations** are **non-zero**

linearized 1+3 eqs. around FRW

$$\Theta^2 = 3\left[\rho^T + \frac{\Lambda}{f'}\right] - \frac{3}{2}\tilde{R}$$

$$\dot{\rho^m} = -\Theta\rho^m(1+w)$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + \tilde{\nabla}^a A_a - \frac{1}{2}\left(\rho^T + 3p^T\right) + \frac{\Lambda}{f'}$$

$$A^a = \dot{u}^a - \frac{w}{w+1}\frac{\tilde{\nabla}^a\rho^m}{\rho^m}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - E_{ab} + \frac{1}{2}\Pi_{ab} + \tilde{\nabla}_{\langle a}A_{b\rangle}$$

$$\dot{E}_{ab} = -\Theta E_{ab} + curl(H_{ab}) - \frac{1}{2}\left(+p^T\right)\sigma_{ab} - \frac{1}{6}\Theta\Pi_{ab} - \frac{1}{2}\dot{\Pi}_{ab} - \frac{1}{2}\tilde{\nabla}_{\langle a}q_{b\rangle}$$

$$\dot{H}_{ab} = -\Theta H_{ab} - curl(E_{ab}) + \frac{1}{2}curl(\Pi_{ab})$$

$$\dot{\omega}_a = -\frac{2}{3}\Theta\omega_a - \frac{1}{2}curl(A_a)$$

- plus constraint equations

- for **FRW background**, A=ω=σ=*0* and $\nabla^a f=0$ for all scalars *f*, and **only the first 3 equations** are **non-zero**
- linearized 1+3 eqs. fully characterize linear perturbations around FRW background

linearized 1+3 eqs. around FRW

$$\Theta^2 = 3\left[\rho^T + \frac{\Lambda}{f'}\right] - \frac{3}{2}\tilde{R}$$

$$\dot{\rho^m} = -\Theta\rho^m(1+w)$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + \tilde{\nabla}^a A_a - \frac{1}{2}\left(\rho^T + 3p^T\right) + \frac{\Lambda}{f'}$$

$$A^a = \dot{u}^a - \frac{w}{w+1}\frac{\tilde{\nabla}^a\rho^m}{\rho^m}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - E_{ab} + \frac{1}{2}\Pi_{ab} + \tilde{\nabla}_{\langle a}A_{b\rangle}$$

$$\dot{E}_{ab} = -\Theta E_{ab} + curl(H_{ab}) - \frac{1}{2}\left(+p^T\right)\sigma_{ab} - \frac{1}{6}\Theta\Pi_{ab} - \frac{1}{2}\dot{\Pi}_{ab} - \frac{1}{2}\tilde{\nabla}_{\langle a}q_{b\rangle}$$

$$\dot{H}_{ab} = -\Theta H_{ab} - curl(E_{ab}) + \frac{1}{2}curl(\Pi_{ab})$$

$$\dot{\omega}_a = -\frac{2}{3}\Theta\omega_a - \frac{1}{2}curl(A_a)$$

- plus constraint equations

- for **FRW background**, A=ω=σ=*0* and ∇^a*f=0* for all scalars *f*, and **only the first 3 equations** are **non-zero**
- linearized 1+3 eqs. fully characterize linear perturbations around FRW background
- $\boldsymbol{\rho^T = \rho^m / f' + \rho^R}$ etc, where $\boldsymbol{\rho^R}$ contains the "curvature corrections"

linearized 1+3 eqs. around FRW

$$\Theta^2 = 3\left[\rho^T + \frac{\Lambda}{f'}\right] - \frac{3}{2}\tilde{R}$$

spatial curvature

$$\dot{\rho^m} = -\Theta\rho^m(1+w)$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + \tilde{\nabla}^a A_a - \frac{1}{2}\left(\rho^T + 3p^T\right) + \frac{\Lambda}{f'}$$

$$A^a = \dot{u}^a - \frac{w}{w+1}\frac{\tilde{\nabla}^a\rho^m}{\rho^m}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - E_{ab} + \frac{1}{2}\Pi_{ab} + \tilde{\nabla}_{\langle a}A_{b\rangle}$$

$$\dot{E}_{ab} = -\Theta E_{ab} + curl(H_{ab}) - \frac{1}{2}\left(+p^T\right)\sigma_{ab} - \frac{1}{6}\Theta\Pi_{ab} - \frac{1}{2}\dot{\Pi}_{ab} - \frac{1}{2}\tilde{\nabla}_{\langle a}q_{b\rangle}$$

$$\dot{H}_{ab} = -\Theta H_{ab} - curl(E_{ab}) + \frac{1}{2}curl(\Pi_{ab})$$

$$\dot{\omega}_a = -\frac{2}{3}\Theta\omega_a - \frac{1}{2}curl(A_a)$$

- plus constraint equations

- for **FRW background**, A=ω=σ=*0* and ∇^{a}*f=0* for all scalars *f*, and **only the first 3 equations** are **non-zero**
- linearized 1+3 eqs. fully characterize linear perturbations around FRW background
- $\boldsymbol{\rho^T = \rho^m / f' + \rho^R}$ etc, where $\boldsymbol{\rho^R}$ contains the "curvature corrections"

ES as a background model in f(R)-gravity

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

- assume ES exist in $f(R)$

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

- assume ES exist in $f(R)$
- use the background field equations

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

- assume ES exist in $f(R)$
- use the background field equations

- $f(R)$: fix w, Λ → fixes $\boldsymbol{f(R) = a+b \cdot R^n}$ with $\boldsymbol{n=3/2 \cdot (1+w)}$, $\boldsymbol{a=2\Lambda}$ and $b=b(n,w)$

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

- assume ES exist in $f(R)$
- use the background field equations

- $f(R)$: fix w, Λ → fixes $\boldsymbol{f(R) = a+b \cdot R^n}$ with $\boldsymbol{n=3/2 \cdot (1+w)}$, $\boldsymbol{a=2\Lambda}$ and $b=b(n,w)$

  → the cosmological constant effectively cancels

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow R={}^3R$)

- assume ES exist in $f(R)$
- use the background field equations

- $f(R)$: fix w, Λ → fixes $f(R) = a+b\cdot R^n$ with $n=3/2\cdot(1+w)$, $a=2\Lambda$ and $b=b(n,w)$

  → the cosmological constant effectively cancels

  → ES in general only exists for specific $f(R)$

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 ($\leftrightarrow$ $R={}^3R$)

- assume ES exist in $f(R)$
- use the background field equations

- $f(R)$: fix w, Λ → fixes $f(R) = a + b \cdot R^n$ with $n = 3/2 \cdot (1+w)$, $a = 2\Lambda$ and $b = b(n,w)$

  → the cosmological constant effectively cancels

  → ES in general only exists for specific $f(R)$

  → ES can exist for any R

ES as a background model in f(R)-gravity

- review GR: fix w, Λ → fix a_0, ρ_0 $(\leftrightarrow R={}^3R)$

- assume ES exist in $f(R)$
- use the background field equations

- $f(R)$: fix w, Λ → fixes $\boldsymbol{f(R) = a+b \cdot R^n}$ with $\boldsymbol{n=3/2 \cdot (1+w)}$, $\boldsymbol{a=2\Lambda}$ and $b=b(n,w)$

  → the cosmological constant effectively cancels

  → ES in general only exists for specific $f(R)$

  → ES can exist for any R

- interesting constraint!

Outline

- why are we interested in the Einstein static (ES) model?
  - historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical systems analysis of the closed FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background
- compare and interpret the results obtained from the two approaches

Dynamical Systems

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations
- equilibrium points characterized by vanishing of all derivatives

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations
- equilibrium points characterized by vanishing of all derivatives
  - if the system is in this state once it will remain there forever

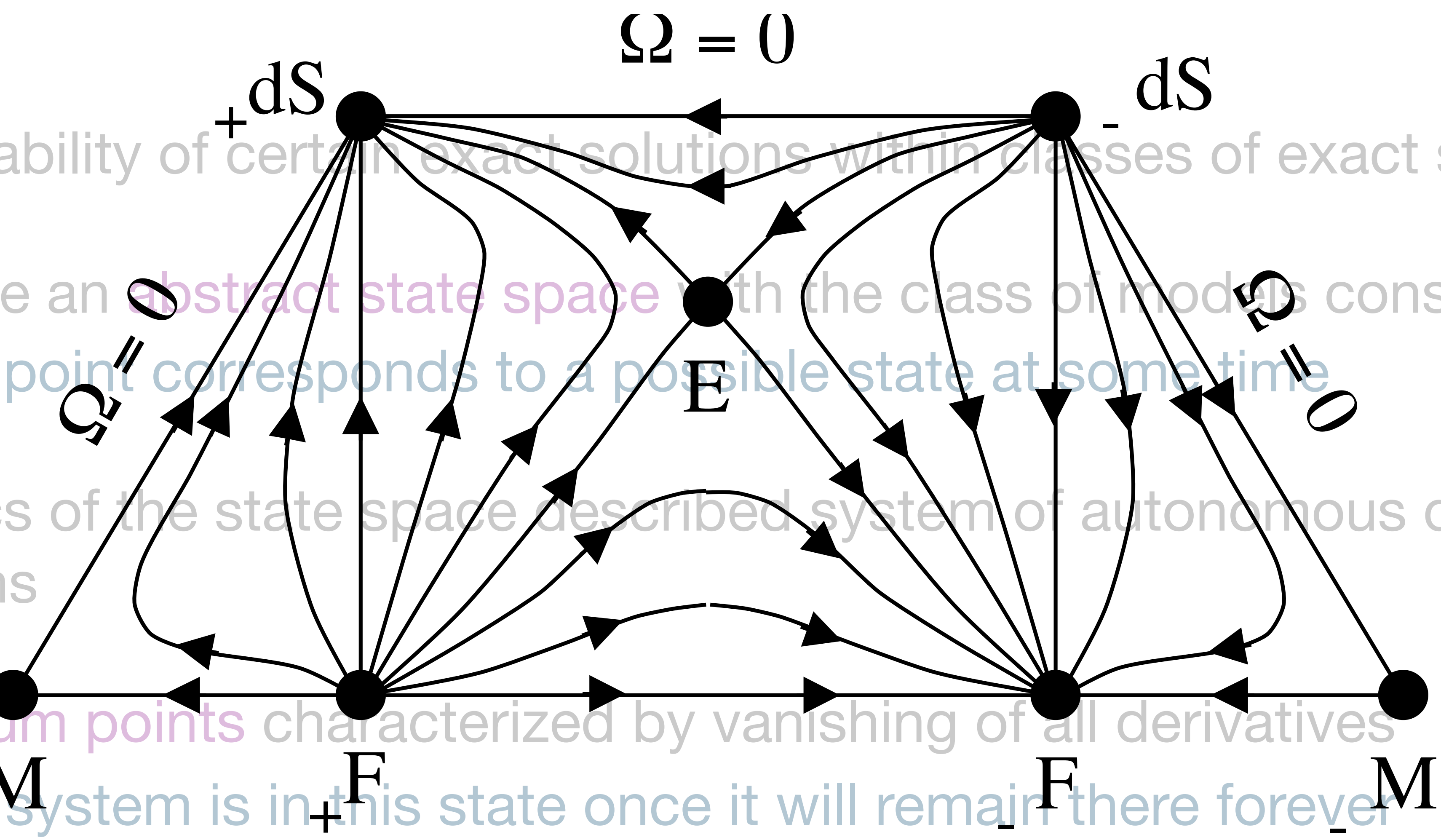
Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations
- equilibrium points characterized by vanishing of all derivatives
  - if the system is in this state once it will remain there forever
  - correspond to solutions with special symmetries

Dynamical Systems

- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations
- equilibrium points characterized by vanishing of all derivatives
  - if the system is in this state once it will remain there forever
  - correspond to solutions with special symmetries
  - can be classified as sources (repellers), sinks (attractors) and saddles according to the sign if their eigenvalues (i.e. linearize the system around each equilibrium point)

Dynamical Systems



- study stability of certain exact solutions within classes of exact solutions
- associate an abstract state space with the class of models considered
  - each point corresponds to a possible state at some time
- dynamics of the state space described system of autonomous differential equations
- equilibrium points characterized by vanishing of all derivatives
  - if the system is in this state once it will remain there forever
  - correspond to solutions with special symmetries
  - can be classified as sources (repellers), sinks (attractors) and saddles according to the sign if their eigenvalues (i.e. linearize the system around each equilibrium point)

Choice of variables for closed FRW models in f(R)

Choice of variables for closed FRW models in f(R)

- basic concept:

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the “time” along each DS orbit

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the "time" along each DS orbit
  - must find a normalization that accomplishes this (see Goliath & Ellis, 1999)

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the "time" along each DS orbit
  - must find a normalization that accomplishes this (see Goliath & Ellis, 1999)

- choose

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the “time” along each DS orbit
  - must find a normalization that accomplishes this (see Goliath & Ellis, 1999)

- choose
  - dynamical systems variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\ , \quad y = \frac{3R}{2nD^2}(n-1)\ , \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\ , \quad K = \frac{3\tilde{R}}{2D^2}\ , \qquad Q = \frac{\Theta}{D}$$

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the "time" along each DS orbit
  - must find a normalization that accomplishes this (see Goliath & Ellis, 1999)

- choose
  - dynamical systems variables

$$x=\frac{3\dot{R}}{2RD}(n-1)\;,\quad y=\frac{3R}{2nD^2}(n-1)\;,\quad z=\frac{3\rho^m}{nR^{n-1}D^2}\;,\quad K=\frac{3\tilde{R}}{2D^2}\;,\qquad Q=\frac{\Theta}{D}$$

  - time variable $' \equiv \frac{d}{d\tau} \equiv \frac{1}{D}\frac{d}{dt}\,.$

Choice of variables for closed FRW models in f(R)

- basic concept:
  - define dimensionless compact variables labeling each point in the state space, and a dimensionless well-defined time-variable measuring the "time" along each DS orbit
  - must find a normalization that accomplishes this (see Goliath & Ellis, 1999)

- choose
  - dynamical systems variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\,, \quad y = \frac{3R}{2nD^2}(n-1)\,, \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\,, \quad K = \frac{3\tilde{R}}{2D^2}\,, \qquad Q = \frac{\Theta}{D}$$

  - time variable $\;{}' \equiv \frac{d}{d\tau} \equiv \frac{1}{D}\frac{d}{dt}\,.$
  - together with the normalization $\;D \equiv \sqrt{\left(\Theta + \frac{3(n-1)}{2}\frac{\dot{R}}{R}\right)^2 + \frac{3}{2}\tilde{R}}$

Compactness of variables

$$x=\frac{3\dot{R}}{2RD}(n-1)\ ,\quad y=\frac{3R}{2nD^2}(n-1)\ ,\quad z=\frac{3\rho^m}{nR^{n-1}D^2}\ ,\quad K=\frac{3\tilde{R}}{2D^2}\ ,\qquad Q=\frac{\Theta}{D}$$

Compactness of variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\,, \quad y = \frac{3R}{2nD^2}(n-1)\,, \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\,, \quad K = \frac{3\tilde{R}}{2D^2}\,, \qquad Q = \frac{\Theta}{D}$$

- look at the class of FRW models with positive spatial curvature and R>0

Compactness of variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\,, \quad y = \frac{3R}{2nD^2}(n-1)\,, \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\,, \quad K = \frac{3\tilde{R}}{2D^2}\,, \qquad Q = \frac{\Theta}{D}$$

- look at the class of FRW models with positive spatial curvature and R>0
- re-write Friedman equation in terms of the new variables:

$$x^2 + y + z = 1$$

Compactness of variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\,, \quad y = \frac{3R}{2nD^2}(n-1)\,, \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\,, \quad K = \frac{3\tilde{R}}{2D^2}\,, \qquad Q = \frac{\Theta}{D}$$

- look at the class of FRW models with positive spatial curvature and R>0

- re-write Friedman equation in terms of the new variables:

$$x^2 + y + z = 1$$

- from the definition of normalization *D* we get:

$$(Q + x)^2 + K = 1$$

Compactness of variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\,,\quad y = \frac{3R}{2nD^2}(n-1)\,,\quad z = \frac{3\rho^m}{nR^{n-1}D^2}\,,\quad K = \frac{3\tilde{R}}{2D^2}\,,\qquad Q = \frac{\Theta}{D}$$

- look at the class of FRW models with positive spatial curvature and R>0
- re-write Friedman equation in terms of the new variables:

$$x^2 + y + z = 1$$

- from the definition of normalization *D* we get:

$$(Q + x)^2 + K = 1$$

- *K, y, z ≥ 0* by definition ⇒ all variables are compact:

$$x \in [-1, 1]\,, y \in [0, 1]\; z \in [0, 1]\,, Q \in [-2, 2]\,, K \in [0, 1]\,.$$

Compactness of variables

$$x = \frac{3\dot{R}}{2RD}(n-1)\ , \quad y = \frac{3R}{2nD^2}(n-1)\ , \quad z = \frac{3\rho^m}{nR^{n-1}D^2}\ , \quad K = \frac{3\tilde{R}}{2D^2}\ , \qquad Q = \frac{\Theta}{D}$$

- look at the class of FRW models with positive spatial curvature and R>0

- re-write Friedman equation in terms of the new variables:

$$x^2 + y + z = 1$$

- from the definition of normalization *D* we get:

$$(Q + x)^2 + K = 1$$

- *K, y, z ≥ 0* by definition ⇒ all variables are compact:

$$x \in [-1, 1]\ , y \in [0, 1]\ z \in [0, 1]\ , Q \in [-2, 2]\ , K \in [0, 1]\ .$$

- five variables together with two constraints ⇒ three-dimensional system

The dynamical system

The dynamical system

- the system is fully described by the equations

$$
\begin{aligned}
Q' &= \left[(3-n)x^2 - n(y-1) - 1\right]\frac{Q^2}{3} + \left[(3-n)x^2 - n(y-1) + 1\right]\frac{Qx}{3} + \frac{1}{3}\left[x^2 - 1 + \frac{ny}{n-1}\right]\,, \\
y' &= \frac{2yx^2}{3}(3-n)(x+Q) + \frac{2xy}{3}\left[\frac{(n^2-2n+2)}{n-1} - ny\right] + \frac{2}{3}Qny(1-y)\,, \\
x' &= \frac{x^3}{3}(3-n)(Q+x) + \frac{x^2}{3}\left[n(2-y)-5\right] + \frac{Qx}{3}\left[n(1-y)-3\right] + \frac{1}{3}\left[\frac{n(n-2)}{n-1} - n + 2\right].
\end{aligned}
$$

The dynamical system

- the system is fully described by the equations

$$
\begin{aligned}
Q' &= \left[(3-n)x^2 - n(y-1) - 1\right]\frac{Q^2}{3} + \left[(3-n)x^2 - n(y-1) + 1\right]\frac{Qx}{3} + \frac{1}{3}\left[x^2 - 1 + \frac{ny}{n-1}\right] , \\
y' &= \frac{2yx^2}{3}(3-n)(x+Q) + \frac{2xy}{3}\left[\frac{(n^2-2n+2)}{n-1} - ny\right] + \frac{2}{3}Qny(1-y) , \\
x' &= \frac{x^3}{3}(3-n)(Q+x) + \frac{x^2}{3}\left[n(2-y) - 5\right] + \frac{Qx}{3}\left[n(1-y) - 3\right] + \frac{1}{3}\left[\frac{n(n-2)}{n-1} - n + 2\right] .
\end{aligned}
$$

  - find equilibrium points defined by *Q'=y'=x'=0*

The dynamical system

- the system is fully described by the equations

$$
\begin{aligned}
Q' &= \left[(3-n)x^2 - n(y-1) - 1\right]\frac{Q^2}{3} + \left[(3-n)x^2 - n(y-1) + 1\right]\frac{Qx}{3} + \frac{1}{3}\left[x^2 - 1 + \frac{ny}{n-1}\right] , \\
y' &= \frac{2yx^2}{3}(3-n)(x+Q) + \frac{2xy}{3}\left[\frac{(n^2-2n+2)}{n-1} - ny\right] + \frac{2}{3}Qny(1-y) , \\
x' &= \frac{x^3}{3}(3-n)(Q+x) + \frac{x^2}{3}\left[n(2-y)-5\right] + \frac{Qx}{3}\left[n(1-y)-3\right] + \frac{1}{3}\left[\frac{n(n-2)}{n-1} - n + 2\right] .
\end{aligned}
$$

  - find equilibrium points defined by $Q'=y'=x'=0$
  - to each equil. point, find the eigenvalues ⇒ local stability

Equilibrium points

Point	(Q, x, y)	constraints	Solution/Description
$\mathcal{N}_\epsilon$	$(0,\ \epsilon,\ 0)$	$n \in [1, 3]$	Vacuum Minkowski
$\mathcal{L}_\epsilon$	$(2\epsilon,\ -\epsilon,\ 0)$	$n \in [1, 3]$	Vacuum Minkowski
$\mathcal{B}_\epsilon$	$\left(\frac{\epsilon}{3-n}\ \ \epsilon\frac{n-2}{n-3},\ 0\right)$	$n \in [1, 2.5]$	Vacuum Minkowski
$\mathcal{A}_\epsilon$	$\left(\epsilon\frac{2n-1}{3(n-1)}, \epsilon\frac{n-2}{3(n-1)}, \frac{8n^2-14n+5}{9(n-1)^2}\right)$	$n \in [1.25, 3]$	Vacuum, Flat, Acceleration$\neq 0$ Decelerating for $P_+ < n < 2$ $a(t) = a_0\left(a_1 + k(n)t\right)^{-3k(n)}$
Line $\mathcal{LC}$	$\left(Q,\ -Q(n-1),\ \frac{j(n)Q+n-1}{n}\right)$	$\lvert Q\rvert \leq \frac{1}{2-n}$ for $n \in [1, P_+]$ $\lvert Q\rvert \leq \frac{1}{\sqrt{3}(n-1)}$ for $n \in [P_+, 3]$	Non-Accelerating curved $a(t) = a_2 t + a_3$, $\rho^m(t) > 0$

- recover all the points from standard R^n-gravity
- the line $\mathcal{LC}$ including the ES model is an artifact of the n-w correspondence - in R^n-gravity we only get a point, and no ES model

Stability properties

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

point	type	range of n				
		$(1, 5/4)$	$(5/4, P_+)$	$(P_+, 3/2)$	$(3/2, 5/2)$	$(5/2, 3)$
$\mathcal{A}_+$	expanding	–	saddle	**sink**		
$\mathcal{A}_-$	collapsing	–	saddle	**source**		
$\mathcal{B}_\pm$	static	saddle				–
$\mathcal{L}_+$	static	saddle	**source**			saddle
$\mathcal{L}_-$	static	saddle	**sink**			saddle
$\mathcal{N}_+$	static	**source**				
$\mathcal{N}_-$	static	**sink**				
$\mathcal{LC}^{\mathrm{exp}}$	expanding	**sink**		**sink** (for $Q < Q_b$) saddle (for $Q > Q_b$)	saddle	
$\mathcal{ES}$	static	**center**			saddle	
$\mathcal{LC}^{\mathrm{coll}}$	collapsing	**source**		**source** (for $\lvert Q\rvert < Q_b$) saddle (for $\lvert Q\rvert > Q_b$)	saddle	

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

Stability properties

point	type	range of n				
		$(1, 5/4)$	$(5/4, P_+)$	$(P_+, 3/2)$	$(3/2, 5/2)$	$(5/2, 3)$
$\mathcal{A}_+$	expanding	–	saddle	**sink**		
$\mathcal{A}_-$	collapsing	–	saddle	**source**		
$\mathcal{B}_\pm$	static	saddle				–
$\mathcal{L}_+$	static	saddle	**source**			saddle
$\mathcal{L}_-$	static	saddle	**sink**			saddle
$\mathcal{N}_+$	static	**source**				
$\mathcal{N}_-$	static	**sink**				
$\mathcal{LC}^{\mathrm{exp}}$	expanding	**sink**		**sink** (for $Q < Q_b$) saddle (for $Q > Q_b$)	saddle	
$\mathcal{ES}$	static	**center**			saddle	
$\mathcal{LC}^{\mathrm{coll}}$	collapsing	**source**		**source** (for $\lvert Q\rvert < Q_b$) saddle (for $\lvert Q\rvert > Q_b$)	saddle	

- for any equation of state, no expanding past attractor

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

Stability properties

point	type	range of n				
		$(1, 5/4)$	$(5/4, P_+)$	$(P_+, 3/2)$	$(3/2, 5/2)$	$(5/2, 3)$
$\mathcal{A}_+$	expanding	–	saddle	**sink**		
$\mathcal{A}_-$	collapsing	–	saddle	**source**		
$\mathcal{B}_\pm$	static	saddle				–
$\mathcal{L}_+$	static	saddle	**source**			saddle
$\mathcal{L}_-$	static	saddle	**sink**			saddle
$\mathcal{N}_+$	static	**source**				
$\mathcal{N}_-$	static	**sink**				
$\mathcal{LC}^{\text{exp}}$	expanding	**sink**		**sink** (for $Q < Q_b$) saddle (for $Q > Q_b$)	saddle	
$\mathcal{ES}$	static	**center**			saddle	
$\mathcal{LC}^{\text{coll}}$	collapsing	**source**		**source** (for $\lvert Q\rvert < Q_b$) saddle (for $\lvert Q\rvert > Q_b$)	saddle	

- for any equation of state, no expanding past attractor
  - no BB scenario, only possible bounce or expansion after asymptotic initial Minkowski phase

Stability properties

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

point	type	range of n				
		$(1, 5/4)$	$(5/4, P_+)$	$(P_+, 3/2)$	$(3/2, 5/2)$	$(5/2, 3)$
$\mathcal{A}_+$	expanding	–	saddle	**sink**		
$\mathcal{A}_-$	collapsing	–	saddle	**source**		
$\mathcal{B}_\pm$	static	saddle				–
$\mathcal{L}_+$	static	saddle	**source**			saddle
$\mathcal{L}_-$	static	saddle	**sink**			saddle
$\mathcal{N}_+$	static	**source**				
$\mathcal{N}_-$	static	**sink**				
$\mathcal{LC}^{\text{exp}}$	expanding	**sink**		**sink** (for $Q < Q_b$) saddle (for $Q > Q_b$)	saddle	
$\mathcal{ES}$	static	**center**			saddle	
$\mathcal{LC}^{\text{coll}}$	collapsing	**source**		**source** (for $\lvert Q\rvert < Q_b$) saddle (for $\lvert Q\rvert > Q_b$)	saddle	

- for any equation of state, no expanding past attractor
  - no BB scenario, only possible bounce or expansion after asymptotic initial Minkowski phase
- ES point is unstable saddle for w>0, but neutrally stable center for -1/3<w<0

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

Stability properties

<table>
<tr><th rowspan="2">point</th><th rowspan="2">type</th><th colspan="5">range of n</th></tr>
<tr><th>$(1, 5/4)$</th><th>$(5/4, P_+)$</th><th>$(P_+, 3/2)$</th><th>$(3/2, 5/2)$</th><th>$(5/2, 3)$</th></tr>
<tr><td>$\mathcal{A}_+$</td><td>expanding</td><td>–</td><td>saddle</td><td colspan="3">sink</td></tr>
<tr><td>$\mathcal{A}_-$</td><td>collapsing</td><td>–</td><td>saddle</td><td colspan="3">source</td></tr>
<tr><td>$\mathcal{B}_\pm$</td><td>static</td><td colspan="4">saddle</td><td>–</td></tr>
<tr><td>$\mathcal{L}_+$</td><td>static</td><td>saddle</td><td colspan="3">source</td><td>saddle</td></tr>
<tr><td>$\mathcal{L}_-$</td><td>static</td><td>saddle</td><td colspan="3">sink</td><td>saddle</td></tr>
<tr><td>$\mathcal{N}_+$</td><td>static</td><td colspan="5">source</td></tr>
<tr><td>$\mathcal{N}_-$</td><td>static</td><td colspan="5">sink</td></tr>
<tr><td>$\mathcal{LC}^{\text{exp}}$</td><td>expanding</td><td colspan="2">sink</td><td>sink (for $Q < Q_b$)
saddle (for $Q > Q_b$)</td><td colspan="2">saddle</td></tr>
<tr><td>$\mathcal{ES}$</td><td>static</td><td colspan="3">center</td><td colspan="2">saddle</td></tr>
<tr><td>$\mathcal{LC}^{\text{coll}}$</td><td>collapsing</td><td colspan="2">source</td><td>source (for $|Q| < Q_b$)
saddle (for $|Q| > Q_b$)</td><td colspan="2">saddle</td></tr>
</table>

- for any equation of state, no expanding past attractor
  - no BB scenario, only possible bounce or expansion after asymptotic initial Minkowski phase
- ES point is unstable saddle for w>0, but neutrally stable center for -1/3<w<0
- numerically found orbits linking collapsing decelerating model to expanding accelerating model via Einstein static point (bouncing solutions)

dust: w=0 (n=3/2)

radiation: w=1/3 (n=2)

Stability properties

point	type	range of n				
		$(1, 5/4)$	$(5/4, P_+)$	$(P_+, 3/2)$	$(3/2, 5/2)$	$(5/2, 3)$
$\mathcal{A}_+$	expanding	–	saddle	**sink**		
$\mathcal{A}_-$	collapsing	–	saddle	**source**		
$\mathcal{B}_\pm$	static	saddle				–
$\mathcal{L}_+$	static	saddle	**source**			saddle
$\mathcal{L}_-$	static	saddle	**sink**			saddle
$\mathcal{N}_+$	static	**source**				
$\mathcal{N}_-$	static	**sink**				
$\mathcal{LC}^{\text{exp}}$	expanding	**sink**		**sink** (for $Q < Q_b$) saddle (for $Q > Q_b$)	saddle	
$\mathcal{ES}$	static	**center**			saddle	
$\mathcal{LC}^{\text{coll}}$	collapsing	**source**		**source** (for $\lvert Q\rvert < Q_b$) saddle (for $\lvert Q\rvert > Q_b$)	saddle	

- for any equation of state, no expanding past attractor
  - no BB scenario, only possible bounce or expansion after asymptotic initial Minkowski phase
- ES point is unstable saddle for w>0, but neutrally stable center for -1/3<w<0
- numerically found orbits linking collapsing decelerating model to expanding accelerating model via Einstein static point (bouncing solutions)
  - recover the GR result but without the need of cosmological constant!

Outline

- why are we interested in the Einstein static (ES) model?
  - historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical systems analysis of the closed FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background
- compare and interpret the results obtained from the two approaches

linear perturbations around ES

(see Phys. Rev. D78:044011, 2008)

- define **perturbation quantities** that **vanish for this background**
  ⇒ gauge-invariant

- **harmonic decomposition**: use the trace-free symmetric tensor eigenfunctions of the spatial Laplace-Beltrami operator defined by

- **decompose** into scalar, vector and tensor parts

- in each case, **expand all first order quantities** as

- note: for **spatially closed** models, the **spectrum of eigenvalues** is discrete $\boldsymbol{k^2 = n\,(n + 2)}$, where the co-moving wave number n is $n=1,2,3...$ ($n=1$ is a gauge mode)

linear perturbations around ES

(see Phys. Rev. D78:044011, 2008)

- define **perturbation quantities** that **vanish for this background** ⇒ gauge-invariant

- **harmonic decomposition**: use the trace-free symmetric tensor eigenfunctions of the spatial Laplace-Beltrami operator defined by

$$\tilde{\nabla}^2 Q = -\frac{k^2}{a_0^2} Q \ , \ \dot{Q} = 0$$

- **decompose** into scalar, vector and tensor parts

- in each case, **expand all first order quantities** as $X(t, \mathrm{x}) = \sum X^k(t) Q^k(\mathrm{x})$

- note: for **spatially closed** models, the **spectrum of eigenvalues** is discrete ***k^2 = n (n + 2)***, where the co-moving wave number *n* is *n=1,2,3...* (*n=1* is a gauge mode)

linear perturbations around ES

- ES **neutrally stable** against **vector, tensor** perturbations for all *w, k*
- ES **neutrally stable** against **scalar** perturbations for all $k^2 \geq 8$ if $w > 0.21$
- the homogeneous mode (n=0)
  - was not considered previously, since it corresponds to a change in the background (reflecting the fact that the model is unstable against homog, perturbations and will expand/collapse)
  - perturbations oscillate for w<0
  - one growing and one decaying mode for w>0
  - perturbation constant in time for dust (w=0) => must include higher order terms
  - exactly matches the results from the dynamical systems analysis

Outline

- why are we interested in the Einstein static (ES) model?
  - historical review
- why modified gravity, in particular f(R)-gravity?
  - derive basic field equations
- dynamical systems analysis of the closed FRW state space (including the ES model)
- briefly summarize linear covariant perturbations around the ES background
- compare and interpret the results obtained from the two approaches

Stability of Einstein Static

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
  - Einstein static point is unstable saddle for w>0

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
  - Einstein static point is unstable saddle for w>0
  - ES is a neutrally stable center for -1/3<w<0

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
  - Einstein static point is unstable saddle for w>0
  - ES is a neutrally stable center for -1/3<w<0
  - must consider higher order perturbations for dust (w=0)

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with $n=0$):
  - Einstein static point is unstable saddle for $w>0$
  - ES is a neutrally stable center for $-1/3<w<0$
  - must consider higher order perturbations for dust ($w=0$)
  - contrast to GR, where ES is unstable for all $-1/3<w<1$

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
    - Einstein static point is unstable saddle for w>0
    - ES is a neutrally stable center for -1/3<w<0
    - must consider higher order perturbations for dust (w=0)
    - contrast to GR, where ES is unstable for all -1/3<w<1
- **inhomogeneous** perturbations (linear perturbation theory (n>1)

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
  - Einstein static point is unstable saddle for w>0
  - ES is a neutrally stable center for -1/3<w<0
  - must consider higher order perturbations for dust (w=0)
  - contrast to GR, where ES is unstable for all -1/3<w<1
- **inhomogeneous** perturbations (linear perturbation theory (n>1)
  - ES static stable against inhomogeneous perturbations if *w > 0.21..*

Stability of Einstein Static

- **homogeneous** perturbations (dynamical systems and linear perturbations with n=0):
  - Einstein static point is unstable saddle for w>0
  - ES is a neutrally stable center for -1/3<w<0
  - must consider higher order perturbations for dust (w=0)
  - contrast to GR, where ES is unstable for all -1/3<w<1

- **inhomogeneous** perturbations (linear perturbation theory (n>1)
  - ES static stable against inhomogeneous perturbations if $w > 0.21..$
  - similar to GR, where same result hold for $w > 1/5 = 0.2$

Summary

Summary

- while **in GR** a_0 and R_0 are fixed given w, Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)

Summary

- while **in GR** a_0 and R_0 are fixed given w, Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)
- for w=1/3, the ES model is unstable against **homogeneous** perturbations, but stable against **inhomogeneous** perturbations

Summary

- while **in GR** a_0 and R_0 are fixed given *w,* Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)
- for w=1/3, the ES model is unstable against **homogeneous** perturbations, but stable against **inhomogeneous** perturbations
- in the closed FRW state space, we find

Summary

- while **in GR** a_0 and R_0 are fixed given w, Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)

- for $w=1/3$, the ES model is unstable against **homogeneous** perturbations, but stable against **inhomogeneous** perturbations

- in the closed FRW state space, we find
  - an accelerating future attractor without the need for a cosmological constant

Summary

- while **in GR** a_0 and R_0 are fixed given *w,* Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)
- for w=1/3, the ES model is unstable against **homogeneous** perturbations, but stable against **inhomogeneous** perturbations
- in the closed FRW state space, we find
  - an accelerating future attractor without the need for a cosmological constant
  - no expanding past attractor (=> NO Big Bang)

Summary

- while **in GR** a_0 and R_0 are fixed given w, Λ , suprisingly **in f(R)** ES only exists in general for the specific form of $f(R) = a+b \cdot R^n$, with $n=3/2 \cdot (1+w)$ (but a_0 and R_0 not fixed)

- for w=1/3, the ES model is unstable against **homogeneous** perturbations, but stable against **inhomogeneous** perturbations

- in the closed FRW state space, we find
  - an accelerating future attractor without the need for a cosmological constant
  - no expanding past attractor (=> NO Big Bang)
  - orbits connecting the collapsing decelerating point to the expanding accelerating point via ES (=> bouncing solutions?)