Curvature perturbation spectrum from false vacuum inflation

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Based on

- JG and M. Sasaki, `arXiv:0804.4488[astro-ph]`

Outline

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   - Physical picture

2. Two-point Correlation functions
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4. Conclusions

Predictions of slow-roll inflation

Scale invariant spectrum $\mathcal{P}_{\mathcal{R}}$

1. One of the greatest triumphs of inflation
2. Confirmed by recent observations e.g. WMAP5: $n_{\mathcal{R}} \approx 0.96$
3. Naturally generated

Predictions of slow-roll inflation

Scale invariant spectrum $\mathscr{P}_{\mathscr{R}}$

1. One of the greatest triumphs of inflation
2. Confirmed by recent observations e.g. WMAP5: $n_{\mathscr{R}} \approx 0.96$
3. Naturally generated under the **slow-roll approximation**

$$\text{slow-roll} \underset{\text{false}}{\overset{\text{true}}{\rightleftarrows}} \text{inflation}$$

Inflation = nearly scale invariant $\mathscr{P}_{\mathscr{R}}$: **NOT necessarily true**
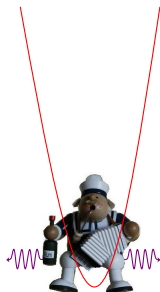
e.g. false vacuum inflation

$\mathscr{R}_c$ during false vacuum inflation

- A drunken sailor cannot move in a deep, narrow hole: $\dot{\phi} = 0$

$$\mathscr{R}_c \sim \frac{H}{\dot{\phi}}\delta\phi$$

No preferred rest frame in pure dS space: Meaningless quantity!

$\mathscr{R}_c$ during false vacuum inflation

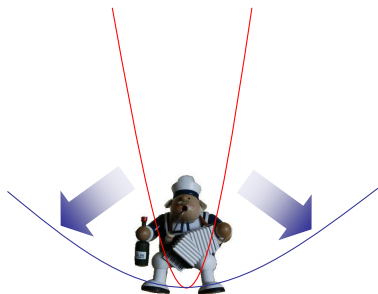

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 No preferred rest frame in pure dS space: Meaningless quantity!

- We **DO** have a preferred frame

$$m_{\text{eff}}^2 = m_\phi^2 + \frac{\mu^2}{a^2} = m_{\text{eff}}(t)$$

Breaking the perfect dS phase

$\mathscr{R}_c$ during false vacuum inflation

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Breaking the perfect dS phase

- He can go home for more rum

How can we proceed?

Situation is OK, but the method is inadequate

Full quantum computation goes as:

1. Calculate the inflaton 2-point correlation function
   $G(x, x') = \langle \phi(x) \phi(x') \rangle$

2. Calculate the energy density 2-point correlation function
   $D(x, x') \sim \langle \delta\rho(x) \delta\rho(x') \rangle / \rho^2 \sim D[G(x, x')]$

3. Calculate $\mathcal{P}_\Phi$ using $\nabla^2 \Phi \sim \delta\rho/\rho$

4. Calculate $\mathcal{P}_\mathcal{R}$ using $\Phi \sim \int \mathcal{R}_c d\eta$

Introduction
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Two-point Correlation functions
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Power spectra
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Conclusions

Inflaton field 2-point correlation function

Given

$$V = V_0 + \frac{1}{2}\left(m_\phi^2 + \frac{\mu^2}{a^2}\right)\phi^2$$

$$G(x, x') = \langle\phi(x)\phi(x')\rangle$$

$$= \left(\frac{H}{2\pi}\right)^2 \int_0^\infty ds \cosh(\nu s) \frac{1 + p\sqrt{2\cosh s - 2(1-u)}}{[2\cosh s - 2(1-u)]^{3/2}} e^{-p\sqrt{2\cosh s - 2(1-u)}}$$

$$p = \sqrt{\mu^2 \eta\eta'} \qquad , u = \frac{r^2 - (\eta - \eta')^2}{2\eta\eta'} \qquad , r^2 = |\mathbf{x} - \mathbf{x}'|^2 , \nu^2 = \frac{9}{4} - \frac{m_\phi^2}{H^2}$$

Introduction
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$$G(x, x') = \langle\phi(x)\phi(x')\rangle$$

Expansion near $s \approx 0$

$$= \left(\frac{H}{2\pi}\right)^2 \int_0^\infty ds\cosh(\nu s)\frac{1 + p\sqrt{2\cosh s - 2(1-u)}}{[2\cosh s - 2(1-u)]^{3/2}}e^{-p\sqrt{2\cosh s - 2(1-u)}}$$

$$p = \sqrt{\mu^2\eta\eta'} \gg 1 \,,\, u = \frac{r^2 - (\eta - \eta')^2}{2\eta\eta'} \gg 1 \,,\, r^2 = |\mathbf{x} - \mathbf{x}'|^2 \,,\, \nu^2 = \frac{9}{4} - \frac{m_\phi^2}{H^2}$$

Early times
Super-horizon separation

$$G(x, x') \approx \left(\frac{H}{2\pi}\right)^2 \sqrt{\frac{\pi}{2}}\frac{\sqrt{\mu}\eta\eta'}{\left[r^2 - \left(\eta - \eta'\right)^2\right]^{3/4}}e^{-\mu\left[r^2 - (\eta - \eta')^2\right]^{1/2}}$$

Energy density 2-point correlation function (1/2)

Density perturbation on the comoving hypersurface

$$\nabla^2(\rho\Delta) = \nabla^2\left(-T^0{}_0\right) + 3H\partial^i\left(-T^0{}_i\right)$$

2-point correlation function of $\rho\Delta$:

$$D(x, x') = \left\langle \nabla_x^2\left[\rho\Delta(x)\right] \nabla_{x'}^2\left[\rho\Delta(x')\right]\right\rangle$$
$$= f_i^{\rho\mu\nu}(t) f_{j'}^{\sigma'\alpha'\beta'}(t)\partial^i\partial^{j'}\left\{\left[\partial_\rho\partial_{\alpha'}\partial_{\beta'}G(x,x')\right]\left[\partial_{\sigma'}\partial_\mu\partial_\nu G(x,x')\right]\right.$$
$$\left. + \left[\partial_\rho\partial_{\sigma'}G(x,x')\right]\left[\partial_\mu\partial_\nu\partial_{\alpha'}\partial_{\beta'}G(x,x')\right]\right\}$$

with the time dependent coefficients

$$f_i^{00j} = f_i^{0j0} = \frac{1}{2}\delta_i{}^j$$
$$f_i^{j00} = -\delta_i{}^j$$
$$f_i^{jkl} = a^{-2}\left[\delta_i{}^j\delta^{kl} + \frac{1}{2}\left(\delta_i{}^k\delta^{jl} + \delta_i{}^l\delta^{jk}\right)\right]$$

Energy density 2-point correlation function (2/2)

Useful properties of $G(x, x')$:

1. Function of $r = |\mathbf{x} - \mathbf{x}'|$: $G = G(r)$
2. Anti-symmetric w.r.t. spatial derivative: $\partial_{x'} = -\partial_x$
3. Symmetric w.r.t. time: $G(r; t, t') = G(r; t', t)$

$\cdots$ After some calculations, we find that to leading order

$$D(r, \eta) \approx \left(\frac{H}{2\pi} \right)^4 16\pi (H\eta)^4 \frac{(\mu\eta)^4}{(\mu r)^3} \mu^8 e^{-2\mu r}$$

2-point correlation function of Φ in configuration space

Φ: gauge invariant curvature perturbation in the Newtonian gauge

$$\text{Poisson equation: } \frac{\nabla^2}{a^2}\Phi = -\frac{\rho\Delta}{2m_{\text{Pl}}^2}$$

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Φ: gauge invariant curvature perturbation in the Newtonian gauge

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Super-horizon separation: $\nabla^2 \to (2\mu)^2$

$$D(x,x') = 4m_{\text{Pl}}^4 (H\eta)^4 \left\langle \nabla_x^2 \left[\nabla_x^2 \Phi(x)\right] \nabla_{x'}^2 \left[\nabla_{x'}^2 \Phi(x')\right]\right\rangle$$
$$\approx 4m_{\text{Pl}}^4 (H\eta)^4 (2\mu)^8 \langle \Phi(x)\Phi(x')\rangle$$

Thus the 2-point correlation function of Φ in *configuration space*

$$\xi_\Phi(x) \equiv \langle \Phi(\mathbf{x})\Phi(\mathbf{x}+\mathbf{r})\rangle$$
$$= \frac{\pi}{64}\left(\frac{H}{2\pi m_{\text{Pl}}}\right)^4 \frac{(\mu\eta)^4}{(\mu r)^3} e^{-2\mu r}$$

Power spectrum of Φ

By inverse Fourier transform

$$\mathscr{P}_\Phi = \frac{k^3}{2\pi^2} \int d^3 r \, \xi_\Phi(r) e^{-i\mathbf{k}\cdot\mathbf{r}}$$

We have to integrate

$$\int_0^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr): \text{ blows up to infinity at } r = 0$$

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We are interested in the correlations of 2 points with $r \gg |\eta| \cdots$

- The singularity at $r = 0$ should NOT matter
- Cutoff scale $1/\mu$

$$\int_0^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr) \rightarrow \int_{1/\mu}^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr) \approx -\text{Ei}(-2)$$

$$\mathscr{P}_\Phi(k;\eta) \approx \frac{-\text{Ei}(-2)}{32} \left(\frac{H}{2\pi m_{\text{Pl}}} \right)^4 (\mu\eta)^4 \left(\frac{k}{\mu} \right)^3$$

$\mathscr{R}_c$ and Φ

On super-horizon scales the general solution for Φ

$$\Phi = \frac{3}{2} C_1 \frac{\mathscr{H}}{a^2} \int_{\eta_i}^{\eta} (1+w) a^2(\eta') \, d\eta'$$

Given Φ, $\mathscr{R}_c$ is expressed as

$$\mathscr{R}_c = \frac{2\Phi' + (5+3w)\mathscr{H}\Phi}{3(1+w)\mathscr{H}} = C_1$$

$\mathscr{R}_c$ and Φ

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We need the information of a and $1 + w$

① a: perfect dS expansion $a = -1/(H\eta)$

② $1 + w$: we need to evaluate $\langle \rho + p \rangle = $???

$\langle \rho + p \rangle$ during dS stage

$$\langle \rho + p \rangle = \left\langle \dot{\phi}^2 + \frac{(\nabla \phi)^2}{3a^2} \right\rangle$$

We take into account$\cdots$

① Fourier mode expansion

$$\phi(x) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left[a_{\mathbf{k}} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^\dagger \phi_k^*(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

② Cutoff at a large physical momentum, $k/a \leq (k/a)_c = H\Lambda$

③ Evaluate in the limit $\eta \to -\infty$

'Regularized / renormalized' $\langle \rho + p \rangle$:

$$\langle \rho + p \rangle_{\text{ren}} = A \frac{H^4}{16\pi^2} \frac{\left| m_\phi^2 \right|}{H^2} (\mu\eta)^2 \; ; \qquad A = \mathcal{O}(1) > 0$$

Power spectrum of $\mathscr{R}_c$

$$1 + w = \frac{A}{48\pi^2} \frac{\left|m_\phi^2\right|}{m_{\text{Pl}}^2} (\mu\eta)^2$$

We can explicitly evaluate the integral

$$\Phi = -C_1 \kappa (\mu\eta)^2 \; ; \qquad \kappa = \frac{A}{32\pi^2} \frac{\left|m_\phi\right|^2}{m_{\text{Pl}}^2}$$

On large scales,

$$\mathscr{P}_{\mathscr{R}} \quad = \frac{\mathscr{P}_\Phi(k\,;\eta)}{\kappa^2(\mu\eta)^4}$$

$$\approx \frac{-2\text{Ei}(-2)}{A^2} \left(\frac{H^2}{m_\phi^2}\right)^2 \left(\frac{k}{\mu}\right)^3 ,$$

Power spectrum of $\mathcal{R}_c$

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On large scales,

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{\mathcal{P}_\Phi(k;\eta)}{\kappa^2 (\mu\eta)^4}$$

$$\approx \frac{-2\text{Ei}(-2)}{A^2} \left(\frac{H^2}{m_\phi^2}\right)^2 \left(\frac{k}{\mu}\right)^3 ,$$

$$n_{\mathcal{R}} = 4$$

Conclusions

1. NOT all inflation models incorporate slowly rolling inflaton field
   - e.g. false vacuum inflation
   - Difficulty: comoving hypersurfaces are not well defined
2. We have calculated the power spectrum $\mathscr{P}_{\mathscr{R}}$ and the spectral index $n_{\mathscr{R}}$
   - Purely quantum field theory approach: regularizing mass μ
   - Highly scale dependent

$$\mathscr{P}_{\mathscr{R}} \sim \mathscr{O}(0.1) \left(\frac{H^2}{m_{\phi}^2} \right)^2 \left(\frac{k}{\mu} \right)^3$$

$$n_{\mathscr{R}} = 4$$