EFFECTIVE FIELD THEORY $T_{\mu\nu}$

AND THE COSMOLOGICAL CONSTANT PROBLEM

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COSMOLOGICAL CONSTANT PROBLEMS

1. **From QFT : Vacuum Energy contribution to CC**
2. **From QFT : Condensates contribution to CC**
3. **Why ρ_Λ has the tiny value we measure?**
4. **Why ρ_Λ and ρ_{mat} at present time coincide?**
5. **.... Modified gravity, Voids, ...**

THIS TALK : addresses the (QFT) Problems 1 & 2

Mechanism : Zero point Energies & Condensates do not contribute to CC.

How : More careful treatement (Effective Field Theory) of $T_{\mu\nu}$ in the Einstein Equation

VACUUM ENERGY & CC

QUESTION : **What do we usually take as the Vacuum Energy contribution to CC?**

ANSWER :

1. Lorentz Invariance $\Longrightarrow$ $T_{\mu\nu}^{vac} = \rho^{vac} g_{\mu\nu}$ $\quad (\; p^{vac} = -\rho^{vac} \;)$

Now consider $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$

2. Vacuum Energy Density **($\Leftarrow$ Zero Point Energy) :**

$$\rho^{vac} = \frac{1}{V}\sum_{\vec{k}}\frac{1}{2}\sqrt{\vec{k}^2 + m^2} = \frac{1}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}\sqrt{\vec{k}^2 + m^2} \simeq \frac{M_P^4}{16\,\pi^2}$$

3. Continuity Equation **:** $\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$

$\rho^{vac} = Const. \simeq \frac{M_P^4}{16\,\pi^2}$ $\quad \geq 120$ **ord. of magn. problem ...**

.... BUT ....

EINSTEIN EQS. : ENERGY DENSITY & PRESSURE

... Let's start again ...

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 8\,\pi\, G\, T_{\mu\nu}$$

Which $T_{\mu\nu}$ in the Ein. Eq. from our QFT $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$?

Quantum Statistical Average (non diag. terms vanish):

$$T_{00} = \rho = << \hat{T}_{00} >> = \frac{1}{V}\sum_{\vec{k}}\sum_n < n|\varrho|n > \; n_{\vec{k}}\,\omega_{\vec{k}} \; + \; \frac{1}{V}\sum_{\vec{k}}\frac{1}{2}\,\omega_{\vec{k}}$$

$$T_{ii} = p = << \hat{T}_{ii} >> = \frac{1}{V}\sum_{\vec{k}}\sum_n < n|\varrho|n > \; n_{\vec{k}}\,\frac{(k^i)^2}{\omega_{\vec{k}}} \; + \; \frac{1}{V}\sum_{\vec{k}}\frac{(k^i)^2}{2\,\omega_{\vec{k}}}$$

$|n >$ **: generic element Fock space basis;**

ϱ **: density operator ;** $n_{\vec{k}} = < n|a^\dagger_{\vec{k}} a_{\vec{k}}|n >$ **;** $\omega_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$**.**

By performing the sum over n :

$$T_{00} = \rho = \int \frac{d^3\,\vec{k}}{(2\pi)^3}\,\omega_{\vec{k}}\,n_{_{BE}}(\vec{k}^2) + \frac{1}{2}\int \frac{d^3\,\vec{k}}{(2\pi)^3}\,\omega_{\vec{k}} \equiv \rho^{mat} + \rho^{vac}$$

$$T_{ii} = p = \frac{1}{3}\int \frac{d^3\,\vec{k}}{(2\pi)^3}\,\frac{\vec{k}^2}{\omega_{\vec{k}}}\,n_{_{BE}}(\vec{k}^2) + \frac{1}{3}\int \frac{d^3\,\vec{k}}{(2\pi)^3}\,\frac{\vec{k}^2}{2\,\omega_{\vec{k}}} \equiv p^{mat} + p^{vac}$$

$n_{_{BE}}(\vec{k}^2)$ = **Bose-Einstein distribution**

In Blue : Gas of Rel. Part. (Matter/Radiation contribution)

In Red : The corresponding Vacuum contribution

Remark 1 : In the Einstein Eqs., ρ^{mat} & ρ^{vac}, as well as p^{mat} & p^{vac}, enter on an equal footing.

Remark 2 : UV cutoff needed to compute ρ^{vac} & p^{vac}

On the contrary, thanks to $n_{_{BE}}(\vec{k}^2)$, ρ^{mat} & p^{mat} are finite

Computing ρ^{vac} & p^{vac} with an UV cutoff $\Lambda\,(= M_P)$

for our $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$ **we have :**

$$\rho^{vac} = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

$$p^{vac} = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

Note : For $\Lambda >> m \implies$ **RED dominant :**

$$\rho^{vac} \simeq \frac{\Lambda^4}{16\pi^2} \quad ; \quad p^{vac} \simeq \frac{1}{3}\,\frac{\Lambda^4}{16\pi^2}$$

QUESTION : How do we deal with the Divergences ?

Formal Point of View : The Divergent Terms (which do not respect $T^{vac}_{\mu\nu} \propto g_{\mu\nu}$) are removed via Renormalization

This would "alleviate" the 120 order of magnitude problem ... However, it is said, even if renormalization cures this problem, we still have the condesates (see later) ...

Criticism (De Witt) :

- **For Zero Point Energies : Physical Meaning of Divergences rooted in the harmonic oscillator structure of a QFT.**
- **Lost if we cancel out these terms with a formal procedure such as normal ordering.**
- **Still, popular prescription for the automatic cancellation of these divergences : Dimensional Regularization**

Deeper Physical Point of View $\Longrightarrow$

Effective Field Theory Point of View

Lesson from Wilson RG & Effective Field Theories :

QFT = Effective Theory valid up to Λ ($\Lambda =$ "scale of new physics")

Hyerarchy of Field Theories up to M_P $\leftarrow$ String theory (?)

According to this view, the cutoff Λ is physical $\Longrightarrow$ we do not discard any term in $T_{\mu\nu}$

OK Let's take this point of view

and let's go back to the Effective Field $T_{\mu\nu}$

Effective Field $T_{\mu\nu}$

$$T_{00} = \rho^{vac} = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

$$T_{ii} = p^{vac} = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

We have already noticed that : for $\Lambda\,(= M_P) >> m$ **,**

Red is dominant :

$$\rho^{vac} \simeq \frac{\Lambda^4}{16\pi^2} \quad ; \quad p^{vac} \simeq \frac{1}{3}\,\frac{\Lambda^4}{16\pi^2}$$

.... then .... :

$$p^{vac} \simeq \frac{1}{3}\,\rho^{vac}$$

If matter is relativistic, it is also : $p^{mat} \simeq \frac{1}{3}\,\rho^{mat}$

$$\Rightarrow \quad p = p^{vac} + p^{mat} \simeq \frac{\rho^{vac}+\rho^{mat}}{3} = \frac{\rho}{3}$$

Different from the expectation : $p^{vac} = -\,\rho^{vac} \;\; (w = -1)$

Consequences for the CC problem :

- $\rho = \rho^{vac} + \rho^{mat}$ **follows the known evolution of rel. matter :**

$$\rho(t) \propto a(t)^{-4} \quad \Leftarrow \quad \dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$$

- **The Cont. Eq. holds also for** ρ^{vac} **and** ρ^{mat} **separately**

(When no matter is present, $\rho = \rho^{vac} \quad \Rightarrow$ **Cont. Eq. holds for** ρ^{vac} **alone. If no substantial change in the behaviour of** ρ^{vac} **is induced by matter,** ρ^{mat} **satisfies same Cont. Eq..) ed by**

- **At early times ($T >> m$) Radiation Era :**

$$\rho^{mat}(t) = \frac{\pi^2}{30} T^4 \ \propto a^{-4}$$

- **As long as matter is relativistic : ρ^{mat} & ρ^{vac} same scaling**

$$\Rightarrow \ \ \rho^{vac}(t) = \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\, \rho^{mat}(t) \qquad\qquad (t_P = \textbf{Planck time})$$

- **From $\dot{a}/a = -\dot{T}/T \quad \Rightarrow \quad$ Fried. Eq., $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$, becomes :**

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi G}{3}\left(1 + \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\right)\rho^{mat}(t) = \frac{4\pi^3 G}{45}\left(1 + \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)}\right) T^4$$

By integrating :

$$T = \left(\frac{45}{16\pi^3\, K\, G}\right)^{\frac{1}{4}}\ t^{-\frac{1}{2}}$$

(with $K = 1 + \rho^{vac}(t_P)/\rho^{mat}(t_P)$). In Standard Approach : $\rho^{vac}(t)$ absent $\ \Rightarrow\ $ $K = 1$)

Now : Consider the theory defined at t_P with $\Lambda = M_P$

From : $\rho^{mat}(t_P) = \frac{\pi^2}{30} T_P^4$; $\rho^{vac}(t_P) = \frac{M_P^4}{16\pi^2}$ **(and $G = M_P^{-2} = t_P^2$) :**

$$\frac{\rho^{mat}(t_P)}{\rho^{vac}(t_P)} = \frac{3\pi}{2} - 1 \simeq 3.71$$

As long as matter is rel. : from $\rho^{vac}(t) = \frac{\rho^{vac}(t_P)}{\rho^{mat}(t_P)} \rho^{mat}(t)$

$$\rho^{mat}(t)/\rho^{vac}(t) \simeq 3.71$$

For a massless field : $\rho^{mat}(t)/\rho^{vac}(t) = 3.71$ **up to now.**

Lesson : if today Relat. Mat. contribution to ρ negligible

$\Rightarrow$

The Zero Point Energy contribution is negligible too !

As T decreases : matter $\rightarrow$ non rel. regime ($T << m$)

$$\Rightarrow \quad \rho^{mat} \propto a^{-3}(t) \ , \quad \textbf{while} \quad \rho^{vac} \propto a^{-4}(t)$$

Now : define $t = t_{eq}$: $\quad \rho_{rel}(t_{eq}) = \rho_{nrel}(t_{eq})$

- **Integrating Cont. Eq. for ρ^{vac} during Rad. Era ($t_P \rightarrow t_{eq}$) :**

$$\rho^{vac}(t_{eq}) = \rho^{vac}(t_P) \left(\frac{t_P}{t_{eq}}\right)^2 \qquad (\Leftarrow \ a(t) \propto t^{1/2} \ )$$

- **During Matter Era : $\rho^{vac} \propto a^{-4}$ still, but $a(t) \propto t^{2/3}$**

$\Rightarrow$ Integrating Cont. Eq. for ρ^{vac} from t_{eq} down to t_0 :

$$\rho^{vac}(t_0) = \rho^{vac}(t_{eq}) \left(\frac{t_{eq}}{t_0}\right)^{\frac{8}{3}}$$

$\Rightarrow$ at present time $\rho^{vac}(t_0)$ is :

$$\rho^{vac}(t_0) = \rho^{vac}(t_P)\left(\frac{t_P}{t_0}\right)^2 \cdot \left(\frac{t_{eq}}{t_0}\right)^{\frac{2}{3}} = \rho^{vac}(t_P)\left(\frac{t_P}{t_0}\right)^2 \cdot \frac{a_{eq}}{a_0}$$

Inserting : $\rho^{vac}(t_P) = \frac{M_P^4}{16\pi^2}$ **,** $t_P \simeq 5 \cdot 10^{-44}\, s$ **,** $t_0 \simeq 2/(3H_0)$ **(** $H_0^{-1} \simeq 13.7\, Gy$**) and** $\frac{a_{eq}}{a_0} \simeq 1/3048 \;\Rightarrow$

$$\rho^{vac}(t_0) \simeq \left(1.93 \times 10^{-4}\,\mathrm{eV}\right)^4$$

Compare $\rho^{vac}(t_0)$ with ρ_γ measured at present time :

$$\rho_\gamma \sim \left(2.11 \times 10^{-4}\,\mathrm{eV}\right)^4$$

Central Result : Contribution of ρ^{vac} to ρ : Negligible !

LESSONS FROM EFFECTIVE FIELD THEORY :

- **Cosmic evolution provides the mechanism to dilute (~ 0) Zero Point Energy ρ^{vac} contribution to ρ ;**
- **ρ^{vac} does not contribute to the CC ;**
- **Although at t_P $\quad T^{vac}_{\mu\nu} = < \hat{T}_{\mu\nu} >$ not Lorentz invariant, the cosmic evolution gives Lorentz invariance at t_0 ;**

Now note that :

- **At t_P physics is described by one (a small number of) quantum field(s).**
- **Lower Energy Theories : born during cosmic evolution.**
- **Low Energy Fields : convenient way to parametrize physics at lower scales (NO NEW DOF).** $\Longrightarrow$

$\Rightarrow$ **When computing Vacuum Energy : do not include Zero Point Energies of Lower Energy Effective Theories. Otherwise : multiple counting of dof. Zero Point Energies of the original High Energy Theory give the whole contribution to Vacuum Energy.**

ANOTHER IMPORTANT LESSON

Some Low Energy Theories have Condensates. They enter $T_{\mu\nu}$ as : $T_{\mu\nu} = \rho^{cond} g_{\mu\nu}$ $\Rightarrow$ should give large contributions to CC.

..... BUT .....

Effective Field Theory scenario : no such terms. Taking into account these terms would again result in a multiple counting of dof.

Let us elucidate these two points with an example

Inspired to the analysis of top condensate models by Bardeen, Hill and Lindner

- **NJL high energy theory , defined at the scale Λ :**

$$Z = \int D\bar{\psi}\, D\psi \exp\left[i \int d^4 x \left(\bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \frac{g^2}{2m_0^2} \bar{\psi}\psi\bar{\psi}\psi \right) \right]$$

- **Hubbard-Stratonovic $\Rightarrow$ auxiliary scalar field ϕ :**

$$Z = \frac{1}{\mathcal{N}} \int D\bar{\psi}\, D\psi\, D\phi \exp\left[i \int d^4 x \left(\bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - \frac{m_0^2}{2}\phi^2 + g\bar{\psi}\psi\phi \right) \right]$$

- **Normaliz. factor $\mathcal{N}$: ensures the equality of the two Eqs.**

- **Integrating high frequency modes from Λ to μ:**
  $Z = \frac{\mathcal{Q}}{\mathcal{N}} \int D\bar{\psi}_l \, D\psi_l \, D\,\phi_l \exp\Big[i \int \, d^4\, x \Big(\bar{\psi}_l (i\gamma^\mu \partial_\mu - M - \delta M)\psi_l +$
  $g\bar{\psi}_l\psi_l\phi_l + \frac{1}{2} Z_\phi \partial^\mu \phi_l \partial_\mu \phi_l - \frac{m_0^2 + \delta m_0^2}{2}\phi_l^2 - \frac{\lambda}{24}\phi_l^4 \Big) \Big]$
  ϕ_l and ψ_l : fields with Fourier components from 0 to μ.
  New (dynamical) dof in the Effective Lagrangian.

- **Normally, the factor $\mathcal{Q}/\mathcal{N}$ is not (cannot be !) considered : no knowledge of the High Energy Theory ! No effect for evaluating Green's fcts. (scattering processes). However :**
  If we compute the Vacuum Energy from this Effective Lagrangian, we end up with a result which differs from the one obtained from the "Fundamental" = "High Energy" NJL theory unless the factor $\mathcal{Q}/\mathcal{N}$ is taken into account.
  Origin of the mismatch : erroneous counting of dof.

- **Condensates : The same argument applies when additional contributions to the vacuum energy come from the appearance of condensates such as, for instance, a vacuum expectation value for ϕ_l.**

Equivalent formulation of the
suppression of Λ^4 and Condensates.

Consider the appearance of a Condensate below some temperature T_{SB} through a Symmetry Breaking mechanism. The cutoff of this Low Energy theory coincides with T_{SB} , the temperature at which the transition takes place.

Cutoff and Condensate contributions to ρ^{vac} and p^{vac} come in the combination as Λ^4 and m^4 in Eqs. :

$$\rho^{vac} = \frac{1}{16\pi^2}\left[\Lambda(\Lambda^2+m^2)^{\frac{3}{2}} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} - \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

$$p^{vac} = \frac{1}{16\pi^2}\left[\frac{\Lambda^3(\Lambda^2+m^2)^{\frac{1}{2}}}{3} - \frac{\Lambda m^2(\Lambda^2+m^2)^{\frac{1}{2}}}{2} + \frac{m^4}{4}\ln\left(\frac{(\Lambda+(\Lambda^2+m^2)^{\frac{1}{2}})^2}{m^2}\right)\right]$$

Add v^4 , the condensate contribution (v = value of the condensate). As the $\Lambda^4 = T_{SB}^4$ term always dominates , for ρ^{vac} we obtain the same scaling as before , regardless of the Lorentz invariant nature of the condensate contribution. Finally, being T_{SB} the cutoff, again we find that these contributions at present time are suppressed.

EFFECTIVE FIELD THEORY SCENARIO : CONCLUSIONS

- **At t_P physics described by an Effective Field Theory with UV cutoff M_P $\Rightarrow$ Vacuum Energy Density ρ^{vac} undergoes a cosmic scaling : ρ^{vac} is negligible at present time t_0**
- **Reason for this behaviour : For an Effective Field Theory $T_{\mu\nu} = << \hat{T}_{\mu\nu} >>$ is such that $p^{vac} \sim \rho^{vac}/3$**
- **ρ^{vac} does not contribute to CC ($w^{vac} \sim 1/3$ while $w^{CC} \sim -1$)**
- **Condensates do not contribute to CC (otherwise : multiple counting of DOF)**
- **Lorentz Invariance : recovered at present time. Worth to be further explored.**