On a generally-covariant approach to the averaging problem in cosmology

Juliane Behrend[1]

Otto Nachtmann[2] Thomas Richter[2] Iain A. Brown[2] Georg Robbers[2]

[1]University of Ulm, Germany
[2]University of Heidelberg, Germany

Cosmo08, Madison, USA
August 28, 2008

1. Introduction
2. Averaging Problem
3. Generally Covariant Averaging
4. Conclusions

Introduction

- Universe is homogeneous and isotropic on large scales

$\Rightarrow$ Use exact solution to Einstein equations (FLRW metric) to model the universe

Introduction

- Universe is homogeneous and isotropic on large scales

$\Rightarrow$ Use exact solution to Einstein equations (FLRW metric) to model the universe

- CMB is isotropic with only small anisotropies

$\Rightarrow$ Describe by linear perturbations about the FLRW solution

Introduction

- Universe is homogeneous and isotropic on large scales

$\Rightarrow$ Use exact solution to Einstein equations (FLRW metric) to model the universe

- CMB is isotropic with only small anisotropies

$\Rightarrow$ Describe by linear perturbations about the FLRW solution

- Astronomical observations (galaxy clustering and motions, gravitational lensing, CMB, type Ia supernovae, Lyman α, etc.)

$\Rightarrow$ ΛCDM model with 76% dark energy, 20% dark matter, and 4% baryonic matter

Introduction

- Universe is homogeneous and isotropic on large scales

$\Rightarrow$ Use exact solution to Einstein equations (FLRW metric) to model the universe

- CMB is isotropic with only small anisotropies

$\Rightarrow$ Describe by linear perturbations about the FLRW solution

- Astronomical observations (galaxy clustering and motions, gravitational lensing, CMB, type Ia supernovae, Lyman α, etc.)

$\Rightarrow$ ΛCDM model with 76% dark energy, 20% dark matter, and 4% baryonic matter

- Expansion of universe has started to accelerate when structure formation has grown increasingly non-linear on small scales

$\Rightarrow$ Is dark energy the backreaction of the generation and evolution of inhomogeneities on the evolution of the background Cosmology?

The Averaging Problem

- Standard Cosmology based on

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \left\langle T_{\mu\nu}\right\rangle + \Lambda \left\langle g_{\mu\nu}\right\rangle$$

The Averaging Problem

- Standard Cosmology based on

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \,\langle T_{\mu\nu}\rangle + \Lambda \,\langle g_{\mu\nu}\rangle$$

- Einstein equations are nonlinear

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle \neq G_{\mu\nu}(\langle g_{\mu\nu}\rangle)$$

The Averaging Problem

- Standard Cosmology based on

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \langle T_{\mu\nu}\rangle + \Lambda \langle g_{\mu\nu}\rangle$$

- Einstein equations are nonlinear

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle \neq G_{\mu\nu}(\langle g_{\mu\nu}\rangle)$$

$\Rightarrow$ We are using the wrong metric to describe the universe!

The Averaging Problem

- Standard Cosmology based on

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \langle T_{\mu\nu}\rangle + \Lambda \langle g_{\mu\nu}\rangle$$

- Einstein equations are nonlinear

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle \neq G_{\mu\nu}(\langle g_{\mu\nu}\rangle)$$

$\Rightarrow$ We are using the wrong metric to describe the universe!

- Correct equations

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle = 8\pi G \langle T_{\mu\nu}\rangle + \Lambda \langle g_{\mu\nu}\rangle$$

for some average $\langle A\rangle$ in a domain $\mathcal{D}$

The Averaging Problem

- Standard Cosmology based on

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G\, \langle T_{\mu\nu}\rangle + \Lambda\, \langle g_{\mu\nu}\rangle$$

- Einstein equations are nonlinear

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle \neq G_{\mu\nu}(\langle g_{\mu\nu}\rangle)$$

$\Rightarrow$ We are using the wrong metric to describe the universe!

- Correct equations

$$\langle G_{\mu\nu}(g_{\mu\nu})\rangle = 8\pi G\, \langle T_{\mu\nu}\rangle + \Lambda\, \langle g_{\mu\nu}\rangle$$

for some average $\langle A\rangle$ in a domain $\mathcal{D}$

$\Rightarrow$ Modifications can in principle act as a dark enegy

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G\, \langle T_{\mu\nu}\rangle + 8\pi G T^{g}_{\mu\nu} + \Lambda\, \langle g_{\mu\nu}\rangle$$

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A\sqrt{h} d^3\mathbf{x}$$

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A \sqrt{h} d^3\mathbf{x}$$

⇒ Buchert equations [Buchert 00,01]

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A\sqrt{h} d^3\mathbf{x}$$

$\Rightarrow$ Buchert equations [Buchert 00,01]

- Backreaction is a small but non-vanishing effect
  [JB, Brown, Robbers 08, Li and Schwarz 07, Rasanen 08, Paranjape 08]

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A\sqrt{h} d^3\mathbf{x}$$

$\Rightarrow$ Buchert equations [Buchert 00,01]

- Backreaction is a small but non-vanishing effect
  [JB, Brown, Robbers 08, Li and Schwarz 07, Rasanen 08, Paranjape 08]

- Averaging process
  - depends on the choice of slicing
  - depends on choice of coordinate system
  - cannot be used to average vector and tensor quantities

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A \sqrt{h} d^3\mathbf{x}$$

$\Rightarrow$ Buchert equations [Buchert 00,01]

- Backreaction is a small but non-vanishing effect
  [JB, Brown, Robbers 08, Li and Schwarz 07, Rasanen 08, Paranjape 08]

- Averaging process
  - depends on the choice of slicing
  - depends on choice of coordinate system
  - cannot be used to average vector and tensor quantities
- Background free approach

- Decompose Einstein equations into a set of scalar equations and apply averaging process

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A\sqrt{h} d^3\mathbf{x}$$

$\Rightarrow$ Buchert equations [Buchert 00,01]

- Backreaction is a small but non-vanishing effect
  [JB, Brown, Robbers 08, Li and Schwarz 07, Rasanen 08, Paranjape 08]

- Averaging process
  - depends on the choice of slicing
  - depends on choice of coordinate system
  - cannot be used to average vector and tensor quantities
- Background free approach

$\Rightarrow$ Need a generally covariant averaging process

Generally Covariant Averaging Process for the Metric

- Averaging Process must be independent of coordinate system

$\Rightarrow$ Parallel transport tensor quantities along geodesics to the same point before averaging

Generally Covariant Averaging Process for the Metric

- Averaging Process must be independent of coordinate system

$\Rightarrow$ Parallel transport tensor quantities along geodesics to the same point before averaging

- Decompose metric into a right-handed orthochronous Minkowski tetrad

$$g_{\mu\nu}(x) = \eta_{\alpha\beta} E^{\alpha}{}_{\mu}(x) E^{\beta}{}_{\nu}(x)$$

Generally Covariant Averaging Process for the Metric

- Averaging Process must be independent of coordinate system

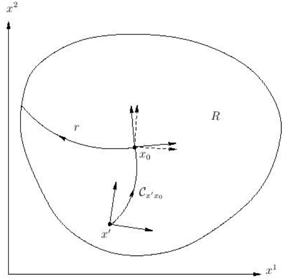
$\Rightarrow$ Parallel transport tensor quantities along geodesics to the same point before averaging

- Decompose metric into a right-handed orthochronous Minkowski tetrad

$$g_{\mu\nu}(x) = \eta_{\alpha\beta} E^{\alpha}{}_{\mu}(x) E^{\beta}{}_{\nu}(x)$$

- Find (up to global Lorentz-transformations) unique tetrad field, the maximally smooth tetrad field, by following Lagrangian

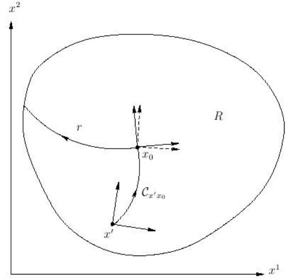
$$\mathcal{L}_{\mathrm{MS}} = \left(D_{\mu} E^{\alpha}{}_{\rho}\right)\left(D_{\nu} E^{\beta}{}_{\lambda}\right) g^{\mu\nu} g^{\rho\lambda} \eta_{\alpha\beta}$$



Parallel transport along geodesics $\mathcal{C}_{x_0x'}$ realized by Wegner-Wilson line operator

$$V(x', x_0; \mathcal{C}_{x_0x'}) = \mathcal{P} \exp\left[-\int_{\mathcal{C}_{x_0x'}} dz^\mu \, \Gamma_\mu(z)\right]$$

where $\Gamma_\mu(x)$ are four matrices with components $(\Gamma_\mu(x))^\lambda_\nu = \Gamma^\lambda_{\mu\nu}(x)$

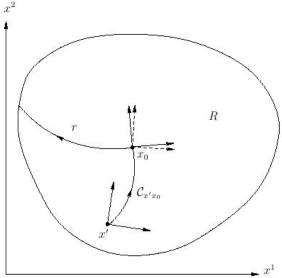


Parallel transport along geodesics $\mathcal{C}_{x_0 x'}$ realized by Wegner-Wilson line operator

$$V(x', x_0; \mathcal{C}_{x_0 x'}) = \mathcal{P} \exp\left[-\int_{\mathcal{C}_{x_0 x'}} dz^\mu \, \Gamma_\mu(z)\right]$$

where $\Gamma_\mu(x)$ are four matrices with components $(\Gamma_\mu(x))^\lambda_\nu = \Gamma^\lambda_{\mu\nu}(x)$

$$\langle E^\alpha{}_\mu(x_0)\rangle = \int_R f(x_0, x'; \mathcal{C}_{x' x_0}) \widehat{V}_\mu{}^\nu(x_0, x'; \mathcal{C}_{x' x_0}) E^\alpha{}_\nu(x') \sqrt{-g(x')} \, d^4x'$$



Parallel transport along geodesics $\mathcal{C}_{x_0 x'}$ realized by Wegner-Wilson line operator

$$V(x', x_0; \mathcal{C}_{x_0 x'}) = \mathcal{P} \exp \left[- \int_{\mathcal{C}_{x_0 x'}} dz^\mu \, \Gamma_\mu(z) \right]$$

where $\Gamma_\mu(x)$ are four matrices with components $(\Gamma_\mu(x))^\lambda_\nu = \Gamma^\lambda_{\mu\nu}(x)$

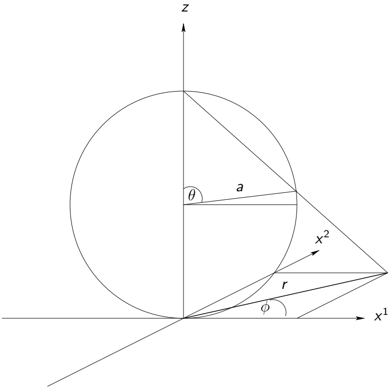
$$\langle E^\alpha{}_\mu(x_0) \rangle$$
$$= \int_R f(x_0, x'; \mathcal{C}_{x' x_0}) \widehat{V}_\mu{}^\nu(x_0, x'; \mathcal{C}_{x' x_0}) E^\alpha{}_\nu(x') \sqrt{-g(x')} \; d^4 x'$$

$\Rightarrow$ Averaged metric:

$$\langle g_{\mu\nu}(x) \rangle = \eta_{\alpha\beta} \langle E^\alpha{}_\mu(x) \rangle \left\langle E^\beta{}_\nu(x) \right\rangle$$

Averaging the Metric of a Two-Sphere

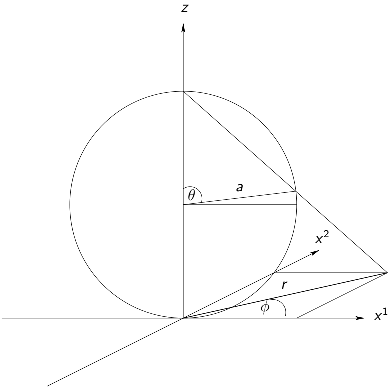
Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$

Averaging the Metric of a Two-Sphere

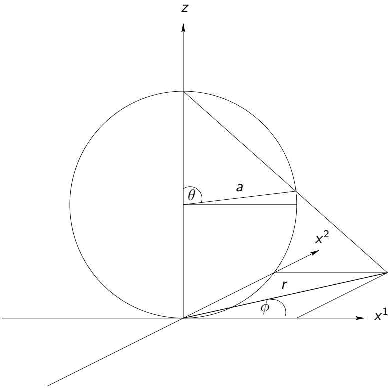
Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$
- Maximally smooth dyad field: $E^a{}_i = (\frac{2a}{L})^2 \delta_{ai}$

Averaging the Metric of a Two-Sphere

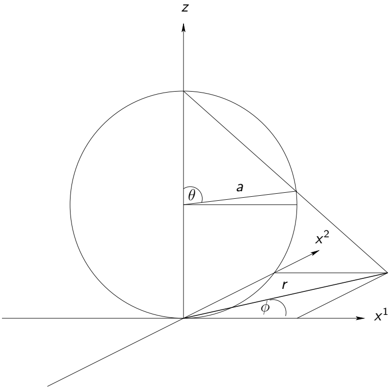
Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$
- Maximally smooth dyad field: $E^a{}_i = (\frac{2a}{L})^2 \delta_{ai}$
- Geodesics through origin: $z^i(\tau) = 2a \tan(\frac{\tau}{2a}) \frac{dz^i}{d\tau}(0)$

Averaging the Metric of a Two-Sphere

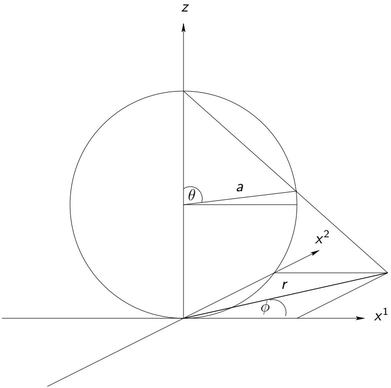
Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$
- Maximally smooth dyad field: $E^a{}_i = (\frac{2a}{L})^2 \delta_{ai}$
- Geodesics through origin: $z^i(\tau) = 2a \tan(\frac{\tau}{2a}) \frac{dz^i}{d\tau}(0)$
- Connector: $\widehat{V}_j{}^i(0, \tau; \mathcal{C}_{\tau 0}) = \cos^{-2}(\frac{\tau}{2a}) \delta_{ij}$

Averaging the Metric of a Two-Sphere

Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$
- Maximally smooth dyad field: $E^a{}_i = (\frac{2a}{L})^2 \delta_{ai}$
- Geodesics through origin: $z^i(\tau) = 2a \tan(\frac{\tau}{2a}) \frac{dz^i}{d\tau}(0)$
- Connector: $\widehat{V}_j{}^i(0, \tau; \mathcal{C}_{\tau 0}) = \cos^{-2}(\frac{\tau}{2a}) \delta_{ij}$

- Averaged metric: $\langle g_{ij}(x) \rangle = g_{ij}(x)$

Averaging the Metric of a Perturbed Two-Sphere

- Perturb spherical coordinates with function $f(x, y, z)$
- Stereographic projection leads to perturbed metric:

$$(g_P)_{ij} = (1 + 2\eta f)(\tfrac{2a}{L})^4 \delta_{ij}$$

Averaging the Metric of a Perturbed Two-Sphere

- Perturb spherical coordinates with function $f(x, y, z)$
- Stereographic projection leads to perturbed metric:

$$(g_P)_{ij} = (1 + 2\eta f)(\tfrac{2a}{L})^4 \delta_{ij}$$

- Maximally Smooth Dyad Field:
  - Reference dyad field $\widetilde{E}^a{}_i$ with $\widetilde{E}^a{}_i \widetilde{E}^b{}_j \ \delta_{ab} = g_{ij}$
  - Maximally smooth dyad $E^a{}_i = U_{ab}(\phi(x))\widetilde{E}^b{}_i$

Averaging the Metric of a Perturbed Two-Sphere

- Perturb spherical coordinates with function $f(x, y, z)$
- Stereographic projection leads to perturbed metric:

$$(g_P)_{ij} = (1 + 2\eta f)(\tfrac{2a}{L})^4 \delta_{ij}$$

- Maximally Smooth Dyad Field:
  - Reference dyad field $\widetilde{E}^a{}_i$ with $\widetilde{E}^a{}_i \widetilde{E}^b{}_j \; \delta_{ab} = g_{ij}$
  - Maximally smooth dyad $E^a{}_i = U_{ab}(\phi(x))\widetilde{E}^b{}_i$
  - Solve $\delta S = 0$ with $S = \int_R d^2x \sqrt{g}\; (D_i E^a{}_j)(D_k E^b{}_l) g^{ik} g^{jl} \delta_{ab}$
  - Introduce vector field $u^k = (D_i \widetilde{E}^c{}_j)\widetilde{E}^d{}_l \epsilon_{cd} g^{ik} g^{jl}$

Averaging the Metric of a Perturbed Two-Sphere

- Perturb spherical coordinates with function $f(x, y, z)$
- Stereographic projection leads to perturbed metric:

$$(g_P)_{ij} = (1 + 2\eta f)(\tfrac{2a}{L})^4 \delta_{ij}$$

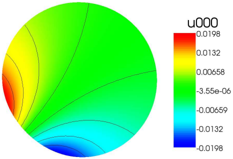
- Maximally Smooth Dyad Field:
  - Reference dyad field $\widetilde{E}^a{}_i$ with $\widetilde{E}^a{}_i \widetilde{E}^b{}_j \; \delta_{ab} = g_{ij}$
  - Maximally smooth dyad $E^a{}_i = U_{ab}(\phi(x))\widetilde{E}^b{}_i$
  - Solve $\delta S = 0$ with $S = \int_R d^2x \sqrt{g}\; (D_i E^a{}_j)(D_k E^b{}_l) g^{ik} g^{jl} \delta_{ab}$
  - Introduce vector field $u^k = (D_i \widetilde{E}^c{}_j)\widetilde{E}^d{}_l \epsilon_{cd} g^{ik} g^{jl}$

$\Rightarrow$ $$g^{ik}(\partial_i \partial_k \phi) = -\tfrac{1}{2} D_k u^k \text{ on } R$$

$\Rightarrow$ $$\frac{\partial \phi}{\partial n} = -\tfrac{1}{2} n_k u^k \text{ on } \partial R$$

The Gaussian Shaped Perturbation

Gascoigne3D



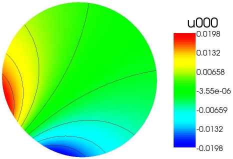
$\left(\frac{\partial^2}{(\partial x^1)^2} + \frac{\partial^2}{(\partial x^2)^2}\right)\phi(x^1, x^2) = 0$ on R

Neumann boundary conditions on ∂R

$\frac{\partial \phi}{\partial n} = \eta \cos^{-2}\left(\frac{r}{2a}\right)\left(h(r,\gamma) + \frac{1}{2a^2}\partial_\gamma \int_0^r f(s',\gamma)ds'\right)$

The Gaussian Shaped Perturbation

Gascoigne3D



$$\left(\frac{\partial^2}{(\partial x^1)^2} + \frac{\partial^2}{(\partial x^2)^2}\right)\phi(x^1, x^2) = 0 \text{ on } R$$

Neumann boundary conditions on ∂R

$$\frac{\partial \phi}{\partial n} = \eta \cos^{-2}\left(\frac{r}{2a}\right)\left(h(r,\gamma) + \frac{1}{2a^2}\partial_\gamma \int_0^r f(s',\gamma)ds'\right)$$

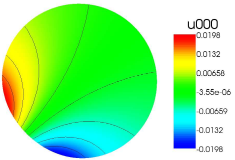
R is the area inside ∂R given by

$$\alpha^1(\gamma) = 2a\tan\left(\frac{r}{2a}\right)\cos\gamma + \eta\frac{v(r,\gamma)}{\cos^2\left(\frac{r}{2a}\right)}\sin\gamma - \eta\frac{\cos\gamma}{\cos^2\left(\frac{r}{2a}\right)}\int_0^r f(s',\gamma)ds'$$

$$\alpha^2(\gamma) = 2a\tan\left(\frac{r}{2a}\right)\sin\gamma - \eta\frac{v(r,\gamma)}{\cos^2\left(\frac{r}{2a}\right)}\cos\gamma - \eta\frac{\cos\gamma}{\cos^2\left(\frac{r}{2a}\right)}\int_0^r f(s,\gamma)ds'$$

The Gaussian Shaped Perturbation

Gascoigne3D



$$\left(\frac{\partial^2}{(\partial x^1)^2}+\frac{\partial^2}{(\partial x^2)^2}\right)\phi(x^1,x^2)=0 \text{ on } R$$

Neumann boundary conditions on ∂R

$$\frac{\partial\phi}{\partial n}=\eta\cos^{-2}\left(\frac{r}{2a}\right)\left(h(r,\gamma)+\frac{1}{2a^2}\partial_\gamma\int_0^r f(s',\gamma)ds'\right)$$

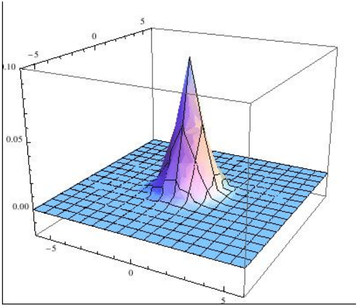
R is the area inside ∂R given by

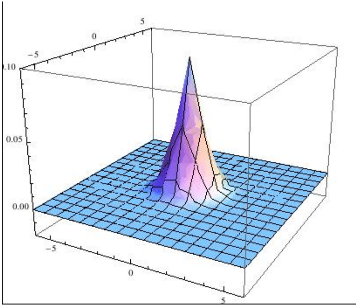
$$\alpha^1(\gamma)=2a\tan\left(\frac{r}{2a}\right)\cos\gamma+\eta\frac{v(r,\gamma)}{\cos^2\left(\frac{r}{2a}\right)}\sin\gamma -\eta\frac{\cos\gamma}{\cos^2\left(\frac{r}{2a}\right)}\int_0^r f(s',\gamma)ds'$$

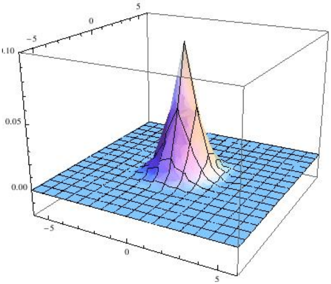
$$\alpha^2(\gamma)=2a\tan\left(\frac{r}{2a}\right)\sin\gamma-\eta\frac{v(r,\gamma)}{\cos^2\left(\frac{r}{2a}\right)}\cos\gamma -\eta\frac{\cos\gamma}{\cos^2\left(\frac{r}{2a}\right)}\int_0^r f(s,\gamma)ds'$$

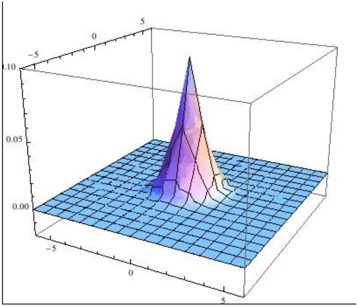
where $h(\tau)=\frac{\partial f}{\partial x^2}\Big|_{(x^1,x^2)=(z^1(\tau),z^2(\tau))}\frac{dz^1}{d\tau}(0)-\frac{\partial f}{\partial x^1}\Big|_{(x^1,x^2)=(z^1(\tau),z^2(\tau))}\frac{dz^2}{d\tau}(0)$

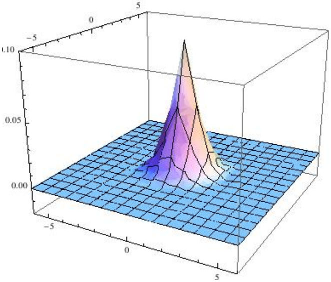
and $v(\tau,\gamma)$ fulfills the differential equation $\frac{d^2v(\tau,\gamma)}{d\tau^2}+\frac{v(\tau,\gamma)}{a^2}=\frac{h(\tau,\gamma)}{\cos^2\left(\frac{\tau}{2a}\right)}$


5
0
−5
0.10
0.05
0.00
−5
0
5


5
0
−5
.10
0.05
0.00
−5
0
5


5
0
−5
.10
0.05
0.00
−5
0
5





- Averaging effect in investigated example is too small

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

⇒ Use different Lagrangian to define initial tetrad field:

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

$\Rightarrow$ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab}\left(t^i t^j + g^{ij} R s^2\right)$

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

$\Rightarrow$ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab}\left(t^i t^j + g^{ij} R s^2\right)$
- $\mathcal{L} = \left(L_n E^a{}_i\right)\left(L_n E^b{}_j\right)\delta_{ab} n^i n^j$

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

$\Rightarrow$ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab}\left(t^i t^j + g^{ij} R s^2\right)$
- $\mathcal{L} = \left(L_n E^a{}_i\right)\left(L_n E^b{}_j\right)\delta_{ab} n^i n^j$
- $\mathcal{L} = \left(L_\xi E^a{}_i\right)\left(L_\xi E^b{}_j\right)\delta_{ab} \xi^i \xi^j$

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right) \left(L_t E^b{}_j\right) \delta_{ab} t^i t^j$$

$\Rightarrow$ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right) \left(L_t E^b{}_j\right) \delta_{ab} \left(t^i t^j + g^{ij} R s^2\right)$
- $\mathcal{L} = \left(L_n E^a{}_i\right) \left(L_n E^b{}_j\right) \delta_{ab} n^i n^j$
- $\mathcal{L} = \left(L_\xi E^a{}_i\right) \left(L_\xi E^b{}_j\right) \delta_{ab} \xi^i \xi^j$
- ...

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

$$\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab} t^i t^j$$

$\Rightarrow$ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab}\left(t^i t^j + g^{ij} R s^2\right)$
- $\mathcal{L} = \left(L_n E^a{}_i\right)\left(L_n E^b{}_j\right)\delta_{ab} n^i n^j$
- $\mathcal{L} = \left(L_\xi E^a{}_i\right)\left(L_\xi E^b{}_j\right)\delta_{ab} \xi^i \xi^j$
- ...

$\Rightarrow$ They all fail to define a dyad field that can be used to average the perturbed plane in the desired way

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR
- Apply it to different perturbation functions to study their interaction with each other and with the background sphere

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR
- Apply it to different perturbation functions to study their interaction with each other and with the background sphere
- Apply it to three-sphere and three-plane corresponding to hypersurfaces of closed and flat FLRW models

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR
- Apply it to different perturbation functions to study their interaction with each other and with the background sphere
- Apply it to three-sphere and three-plane corresponding to hypersurfaces of closed and flat FLRW models
- Apply it to four-dimensional example which involves choice of boundary conditions on the congruence of light-like geodesics

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR
- Apply it to different perturbation functions to study their interaction with each other and with the background sphere
- Apply it to three-sphere and three-plane corresponding to hypersurfaces of closed and flat FLRW models
- Apply it to four-dimensional example which involves choice of boundary conditions on the congruence of light-like geodesics

$\Rightarrow$ Apply averaging process to Cosmology and combine the two lines of research