Non-Perturbative Decay of Flat Directions

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MSSM Flat Directions

MSSM Potential: $V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 \left(\phi^\dagger T^a \phi\right)^2$

$F_i \equiv \frac{\partial W_{\rm MSSM}}{\partial \phi_i}$,

$W_{\rm MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d$.

	$B-L$		$B-L$
$H_u H_d$	0	$QQQ\bar{u}$	1
LH_u	-1	$QQ\bar{u}\bar{u}\bar{e}$	1
		$LL\bar{d}\bar{d}\bar{d}$	-3
$\bar{u}\bar{d}\bar{d}$	-1	$\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$	1
$QL\bar{d}$	-1		
$LL\bar{e}$	-1	$QLQL\bar{d}\bar{d}$	-2
		$QQLL\bar{d}\bar{d}$	-2
$QQ\bar{u}\bar{d}$	0	$\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{d}$	-2
$QQQL$	0		
$QL\bar{u}\bar{e}$	0	$QQQQ\bar{d}LL$	-1
$\bar{u}\bar{u}\bar{d}\bar{e}$	0	$QLQLQL\bar{e}$	-1
		$QL\bar{u}QQ\bar{d}\bar{d}$	-1
		$\bar{u}\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{e}$	-1

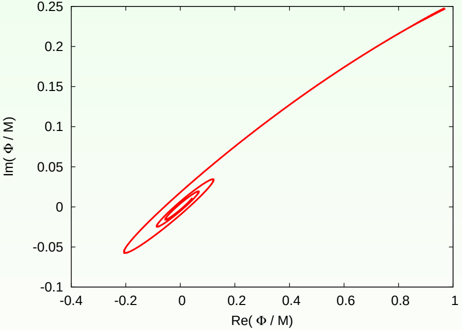
Dine, Randall, Thomas '95
Gherghetta, Kolda, Martin '95

Potential of the flat directions:

$$V = m_\phi^2 |\phi|^2 + \frac{|\lambda|^2 \, |\phi|^{2\,d-2}}{M^{2\,d-6}} + \left(A \, \frac{\lambda \, \phi^d}{M^{d-3}} + \mathrm{h.c} \right)$$

Evolution of Flat Directions

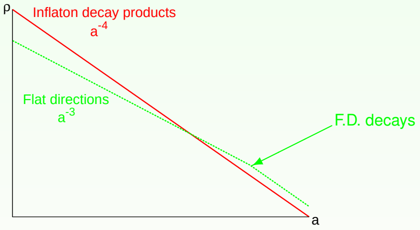
$$V = m_\phi^2|\phi|^2 + \frac{|\lambda|^2\,|\phi|^{2\,d-2}}{M^{2\,d-6}} + \left(A\,\frac{\lambda\,\phi^d}{M^{d-3}} + \mathrm{h.c}\right)$$



- During inflation, $m_\phi \ll H \Rightarrow$ a large vev generated.
- Growth of F.D. is stopped by the non-renormalizable terms $\Rightarrow$ $\langle\phi\rangle \lesssim \left(\frac{m_\phi\,M^{d-3}}{|\lambda|}\right)^{1/(d-2)}$
- After inflation, F.D. nearly frozen, as long as $H > m_\phi$.
- From $H \sim m_\phi$ on, spiral motion towards origin

Effect on Thermal History

- Due to the large VEV of the F.D. , the fields that they couple to acquire a large mass. (Affleck-Dine '85) $\Rightarrow$ very small perturbative decay, $\Gamma \sim m_\phi^3/\phi^2$.
- If they are long lived:
  1. F.D. can delay thermalization (slow down scatterings)
  2. F.D. can dominate the energy density of the universe.



Allahverdi, Mazumdar '06

Non Perturbative Decay

- Complex scalar field χ coupled to flat directions: $\Delta V = g^2 \, |\phi|^2 |\chi|^2$
- Eq. of motion of fluctuations: $\delta\chi'' + \left[p^2 + m_\chi^2 + g^2 \, |\phi(t)|^2\right] \delta\chi = 0$.
- Time dependent frequency $\Rightarrow$ particle production when $\omega' > \omega^2$.

ω' / ω^2

$$\left.\frac{\omega'}{\omega^2}\right|_{\rm max} \simeq \frac{m_\phi}{g \, \epsilon \, \phi_0} \sim \frac{10^{-14}}{\epsilon}$$

ϵ : (Semi-minor)/(Semi-major) $\quad \epsilon = \sqrt{1 - (eccentricity)^2}$

$\Rightarrow$ No non-perturbative decay.

Allahverdi, Shaw, Campbell '99
Postma, Mazumdar '03

Realistic Case: $H_u H_d$ Flat Direction

- MSSM, with $H_u H_d$ F.D.:
$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} h_u \\ \phi + \xi_u \end{pmatrix} , \; H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi + \xi_d \\ h_d \end{pmatrix} \quad , \quad VEV \to \phi = |\phi| \, \mathrm{e}^{i\,\sigma}$$
- The F and D terms, quadratic in fluctuations:
$$\begin{aligned} V = & \frac{\lambda_u^2}{2} |\phi|^2 \left(|Q_u|^2 + |u|^2 \right) + \frac{\lambda_d^2}{2} |\phi|^2 \left(|Q_d|^2 + |d|^2 \right) + \frac{\lambda_e^2}{2} |\phi|^2 \left(|L_d|^2 + |e|^2 \right) \\ & + \frac{g^2 + g'^2}{16} |\phi|^2 \left(\mathrm{Re}[\xi_u - \xi_d] \, , \, \mathrm{Im}[\xi_u - \xi_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Re}[\xi_u - \xi_d] \\ \mathrm{Im}[\xi_u - \xi_d] \end{pmatrix} \\ & + \frac{g^2}{8} |\phi|^2 \left(\mathrm{Re}[h_u + h_d] \, , \, \mathrm{Im}[h_u + h_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Re}[h_u + h_d] \\ \mathrm{Im}[h_u + h_d] \end{pmatrix} \\ & + \frac{g^2}{8} |\phi|^2 \left(\mathrm{Im}[-h_u + h_d] \, , \, \mathrm{Re}[h_u - h_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Im}[-h_u + h_d] \\ \mathrm{Re}[h_u - h_d] \end{pmatrix} \\ & + \mathcal{O}(\mathrm{TeV}) \text{ mass terms} \end{aligned}$$
- $\mathcal{M}^2 = \begin{pmatrix} \cos^2\sigma & \cos\sigma \, \sin\sigma \\ \cos\sigma \, \sin\sigma & \sin^2\sigma \end{pmatrix} \Rightarrow$ Eigenvalues $(|\phi|^2, 0)$, but eigenvectors have t dependence through $\sigma(t)$.

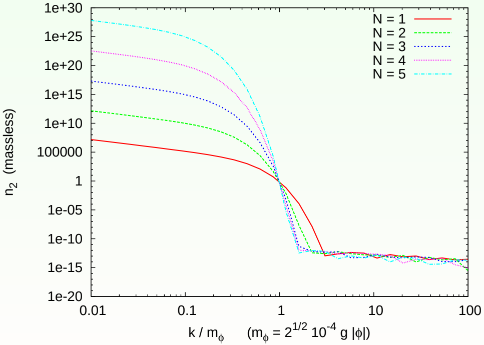
- Using a mass matrix of this type:

$$\mathcal{M}^2 = 2\,g^2|\phi|^2 \begin{pmatrix} \cos^2\sigma & \cos\sigma\,\sin\sigma \\ \cos\sigma\,\sin\sigma & \sin^2\sigma \end{pmatrix} + \begin{pmatrix} m_\chi^2 & 0 \\ 0 & m_\chi^2 \end{pmatrix}$$

- Eigenfrequencies: $\omega_1 = \sqrt{k^2 + m_\chi^2 + 2\,g^2\,|\phi|^2} \quad , \quad \omega_2 = \sqrt{k^2 + m_\chi^2}$
- Use extended Bogoliubov formalism to calculate production

  Nilles, Peloso, Sorbo '01

- Exponential particle production due to rotation of eigenstates, if $k < \sqrt{m_\phi^2 - m_\chi^2}$.



Olive, Peloso '06

Toy Model $\rightarrow$ Gauged Computation

- In realistic models, vev of F.D. break gauge symmetries.
- Goldstone boson can be eliminated in unitary gauge.
- 1 F.D. $\rightarrow$ no degrees of freedom for mixing.
- By counting the degrees of freedom, Olive-Peloso 2006 concluded: “Non-perturbative decay is a real effect only for two or more flat directions.”
- Requires $=$ vevs $\rightarrow$ unrealistic. (Allahverdi, Mazumdar '07)
- Time dependent mixing found in different examples: Toy model (Basbøll, Maybury, Riva, West '07), MSSM F.D. (Basbøll '08)

Gauged model with 2 FD : Complete Calculation

AEG, Olive, Peloso, Sexton '08

- We include gauge field, complete computation in complete model.

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi_i|^2 - V \,, \\
V &= m^2 \left(|\Phi_1|^2 + |\Phi_2|^2 \right) + \tilde{m}^2 \left(|\Phi_3|^2 + |\Phi_4|^2 \right) + \lambda \left(\Phi_1^2 \Phi_2^2 + \mathrm{h.c.} \right) \\
&\quad + \tilde{\lambda} \left(\Phi_3^2 \Phi_4^2 + \mathrm{h.c.} \right) + \frac{g^2}{8} \left(q\,|\Phi_1|^2 - q\,|\Phi_2|^2 + q'\,|\Phi_3|^2 - q'\,|\Phi_4|^2 \right)^2
\end{aligned}
$$

with $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{F}{R}\, \mathrm{e}^{i\,\Sigma}\,, \quad \langle \Phi_3 \rangle = \langle \Phi_4 \rangle = \frac{G}{R}\, \mathrm{e}^{i\,\tilde{\Sigma}}$

- Degrees of freedom in unitary gauge:

$$
\begin{array}{ccccccccc}
\{\Phi_i & , & A_\mu\} & \rightarrow & \Big\{ A_\mu^T & , & \begin{array}{c}\delta\Phi_1 + \delta\Phi_2 \\ \delta\Phi_3 + \delta\Phi_4\end{array} & , & \begin{array}{c}\text{Higgs} \\ A_\mu^L \\ 2 \text{ light fields}\end{array} \Big\} \\
8 & + & 2 & = & 2 & + & 4 & + & 4
\end{array}
$$

Mixing

Gauged model with 2 FD : Complete Calculation

AEG, Olive, Peloso, Sexton '08

- We include gauge field, complete computation in complete model.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi_i|^2 - V \,,$$

$$V = m^2 \left(|\Phi_1|^2 + |\Phi_2|^2 \right) + \tilde{m}^2 \left(|\Phi_3|^2 + |\Phi_4|^2 \right) + \lambda \left(\Phi_1^2 \Phi_2^2 + \text{h.c.} \right) + \tilde{\lambda} \left(\Phi_3^2 \Phi_4^2 + \text{h.c.} \right) + \frac{g^2}{8} \left(q\,|\Phi_1|^2 - q\,|\Phi_2|^2 + q'\,|\Phi_3|^2 - q'\,|\Phi_4|^2 \right)^2$$

with $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{F}{R}\, \mathrm{e}^{i\,\Sigma}\,, \quad \langle \Phi_3 \rangle = \langle \Phi_4 \rangle = \frac{G}{R}\, \mathrm{e}^{i\,\tilde{\Sigma}}$

- Degrees of freedom in unitary gauge:

$$\{\Phi_i \;,\; A_\mu\} \rightarrow \left\{ A_\mu^T \;,\; \begin{matrix} \delta\Phi_1 + \delta\Phi_2 \\ \delta\Phi_3 + \delta\Phi_4 \end{matrix} \;,\; \begin{matrix} \text{Higgs} \\ A_\mu^L \\ 2 \text{ light fields} \end{matrix} \right\}$$

$$8 + 2 = 2 + 4 + 4$$

Mixing

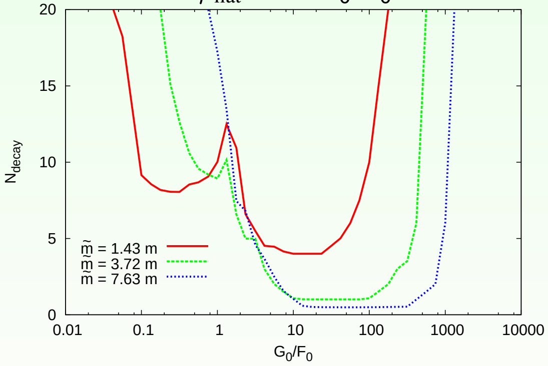
- Squared-Eigenmasses:

$$
\begin{aligned}
m_1^2 &= \frac{g^2}{4\,R^2}\left(F^2+G^2\right) , \\
m_2^2 &= \frac{g^2}{4\,R^2}\left(F^2+G^2\right) + \frac{\left(F^2\,m^2+G^2\,\tilde{m}^2\right)}{F^2+G^2} + \frac{3\left(F^2\,\Sigma'+G^2\,\tilde{\Sigma}'\right)^2}{R^2\,\left(F^2+G^2\right)^2} , \\
m_3^2 &= \frac{\left(F^2\,\tilde{m}^2+G^2\,m^2\right)}{F^2+G^2} + \frac{3\,\left(F\,G'-G\,F'\right)^2}{R^2\,\left(F^2+G^2\right)^2} + \frac{3\,F^2\,G^2\,\left(\Sigma'-\tilde{\Sigma}'\right)^2}{R^2\,\left(F^2+G^2\right)^2} , \\
m_4^2 &= \frac{\left(F^2\,\tilde{m}^2+G^2\,m^2\right)}{F^2+G^2} .
\end{aligned}
$$

- $m_1, m_2 \sim M_{\rm GUT} - M_p$ (Higgs + Longitudinal vector field)
- $m_3, m_4 \sim {\rm TeV}$ (Light fields)
- $m'_{\rm light} > m^2_{\rm light} \rightarrow$ Also have non-adiabatic eigenvalue evolution.

- Need to control both scales in numerical analysis. Fortunately, the ratio of energy densities have the scaling property

$$\frac{\rho_{\rm prod}}{\rho_{\rm flat}} \simeq C\,\frac{m\,\tilde{m}}{F_0\,G_0}\,10^{\sigma\,N_{\rm rot}} \qquad C,\sigma = f\left(\frac{G_0}{F_0},\frac{\tilde{m}}{m}\right)$$



$$m = \mathrm{TeV}$$

$$\sqrt{F_0\,G_0} = 10^{-2} M_p$$

- Production when intantaneous vevs comparable, not the initial vevs. $\Leftarrow$ Elliptic orbits.

Conclusions

- MSSM Flat directions deserve study: Generally present, can be populated during inflation.
- It is often assumed that the decay of F.D. perturbative: decay after $\sim 10^{11}$ rotations.
- For a model with 2 F.D. and for a range of initial ratios of VEVs, found quick decay (after $\mathcal{O}(10)$ rotations).
- Need to study non-linear effects using lattice simulations.