Gravitational Radiation from Preheating

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Easther, Giblin, Lim: astro-ph/0612294

Easther, Giblin, Lim: arXiv/0712.2991

Outline

- Inflation and its end
- Parametric resonance
- Gravitational waves: How and Why
- Gravitational radiation from preheating
    - Methods and numerics
    - Results and parameter dependence

History / Prior Art

- Parametric resonance: Brandenberger & Traschen (1990) / Linde & co
- Khlebnikov and Tkachev: GUT scale inflation and Gravitational waves (1997)
- Yale (2005-) -- Energy Scale/Peak location correlation and lattice simulations
- Now: Garcia-Bellido et al. (Madrid), CITA group, Price and Siemens (Milwaukee)

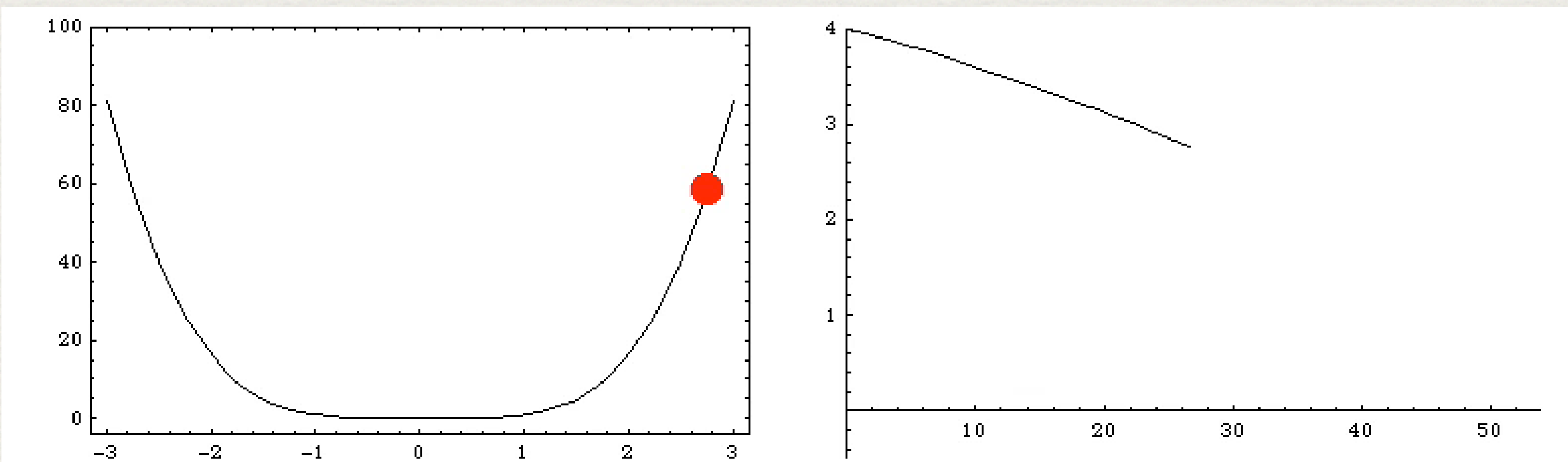
Inflation

- (Near) Exponential expansion of the universe
  - Realizable by “particle physics”
  - Energy density of the universe dominated by (scalar-field) potential
- Solves observational “issues” of Big Bang Cosmology
  - Isotropy, Homogeneity
- Quantum Fluctuations provide insight into structure formation

Inflationary Dynamics

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$H^2 = \frac{8\pi}{3m_{pl}^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right]$$



Parametric Resonance

- Consider the toy model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - g^2\phi^2\chi^2 - V(\phi)$$

- φ is the inflation and χ is a bosonic field. The mode equations for χ are

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$

where

$$\tilde{\chi}(k,t) = \int d^3x \; \chi(x,t)e^{2\pi i k\cdot x}$$

Parametric Resonance (II)

- If we assume that

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

and H=0 (non-expansion), then

$$\phi(t) = \Phi \sin(mt)$$

and

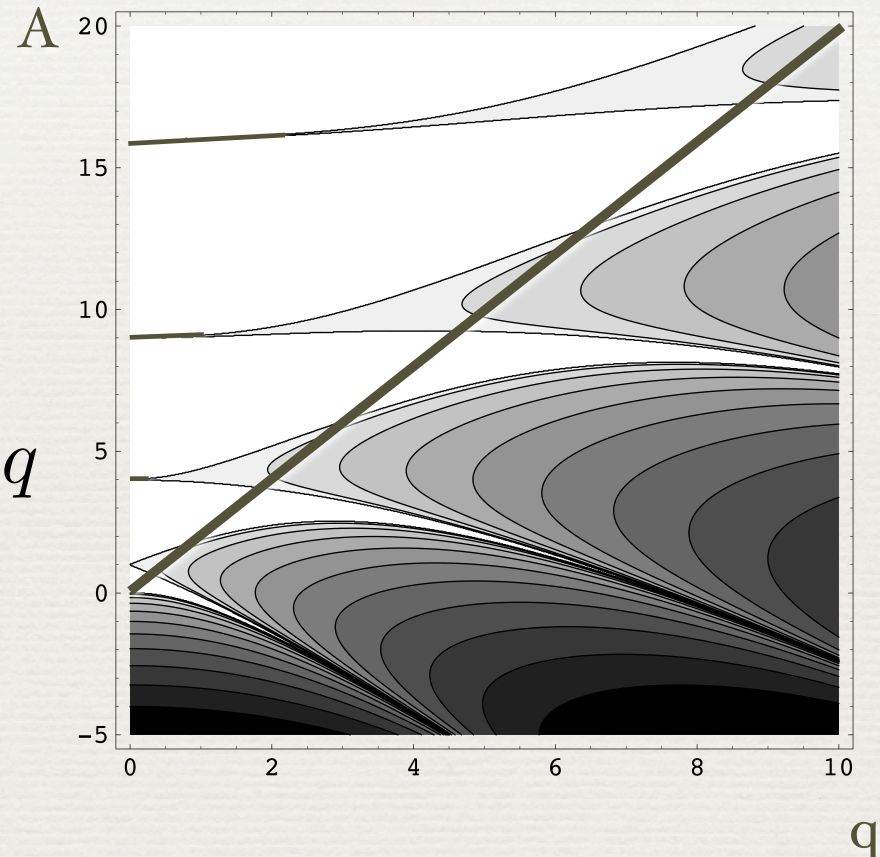
$$\ddot{\chi}_k + \left(k^2 + g^2\Phi^2 \sin^2(m_\phi t)\right) \chi_k = 0$$

Mathieu Equation

$$z = mt$$

$$q = \frac{g^2 \Phi^2}{4m^2}$$

$$A_k = \frac{k^2}{m^2} + \frac{g^2 \Phi^2}{2m^2} = \frac{k^2}{m^2} + 2q$$



Solutions are either exponential or oscillatory

$$\chi_k'' + (A_k - 2q\cos(2z))\chi_k = 0$$

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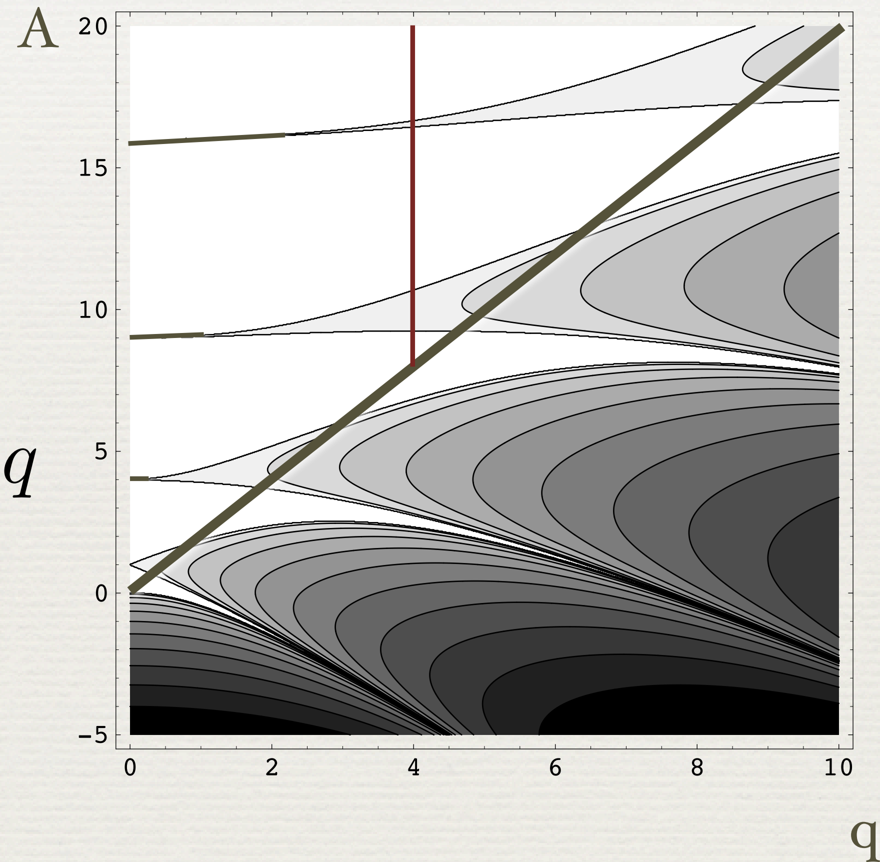

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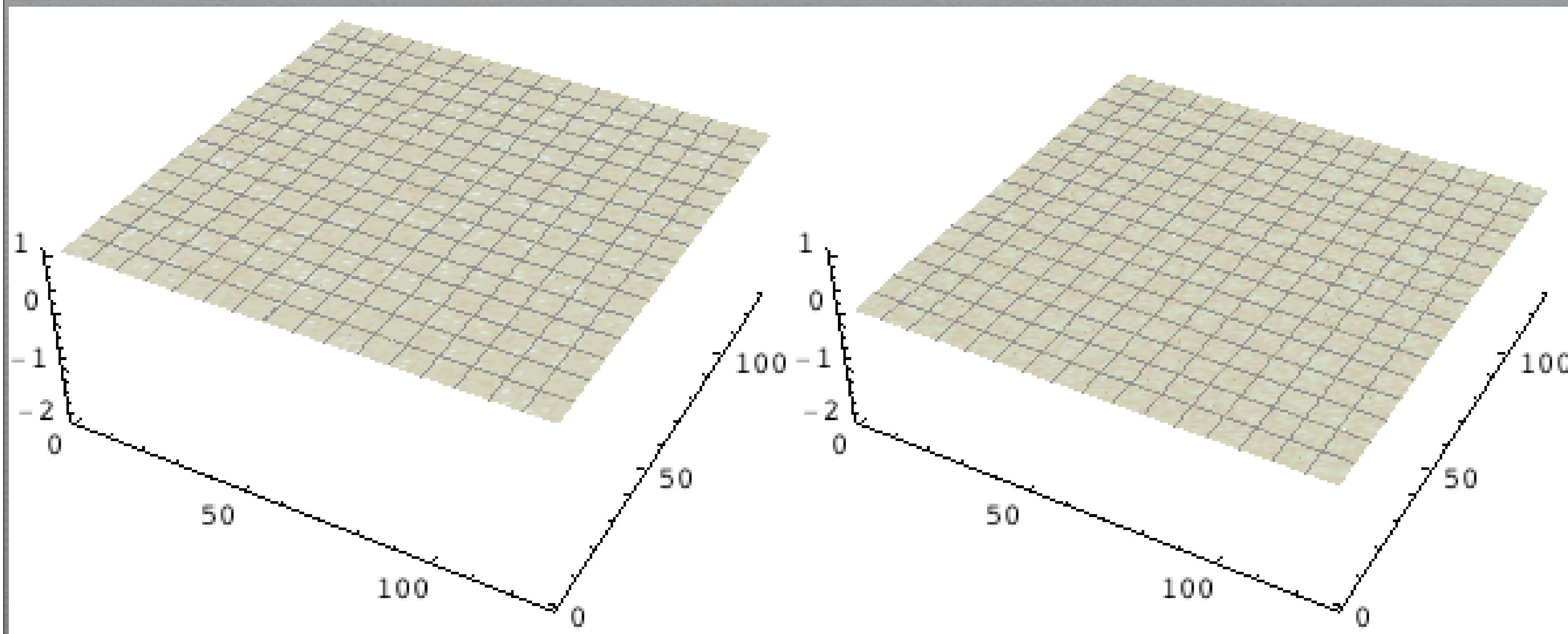
Simulation



Implications

- Certain modes are exponentially excited
  - (copious) particle production of certain momentum states
  - out-of-equilibrium universe
- Large gradient energy
- Any model whose termination oscillates will (approximately) resonate
- Inhomogeneities lead to...

Gravity Waves

- Einstein's Equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- With a metric of the form (in synchronous gauge)

$$ds^2 = dt^2 - a^2(t)\,[\delta_{ij} + h_{ij}]\,dx^i dx^j$$

- Where the perturbation is *transverse-traceless*

$$h^i_i = 0 \quad h^i_{j,i} = 0$$

Gravity Waves (II)

- We can use perturbation theory

$$\delta G_{\mu\nu}(x,t) = 8\pi G \delta T_{\mu\nu}(x,t)$$

to write equations of motion for the metric perturbations

$$h_{ij} + 3\frac{\dot{a}}{a}\dot{h}_{ij} - \frac{1}{a}\nabla^2 h_{ij} = \frac{16\pi G}{a^2}\delta S_{ij}^{TT}$$

The Source

- The source

$$S_{ij} = T_{ij} - \frac{1}{3} T^k_k \delta_{ij}$$

- must be made *transverse-traceless* by

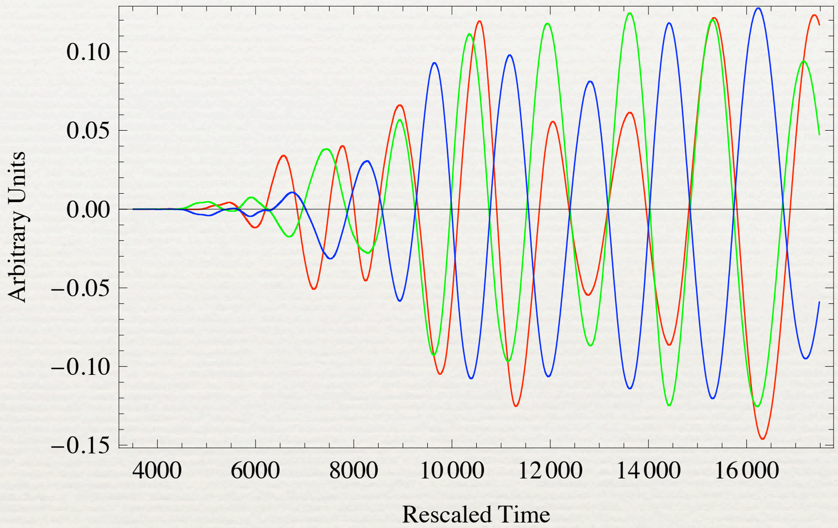
$$S^{TT}_{ij} = P_{ik} S_{kl} P_{lj} - \frac{1}{2} P_{ij} \left(P_{lm} S_{lm} \right)$$

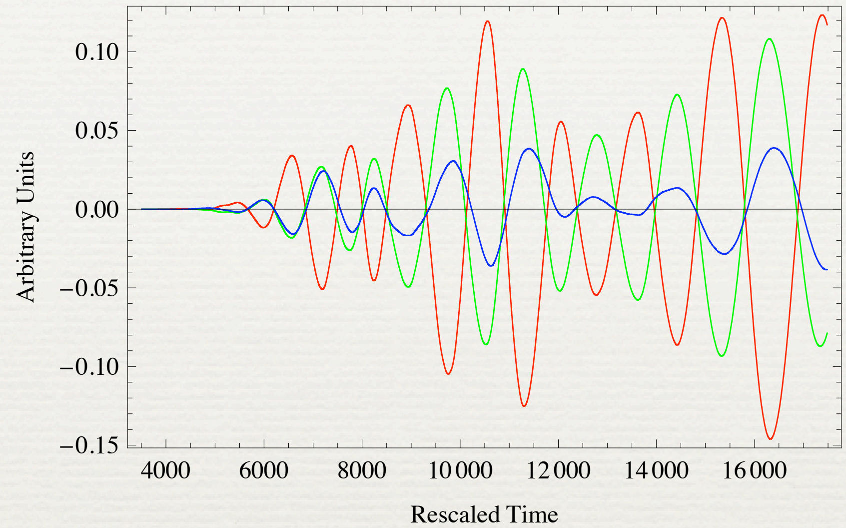
- where

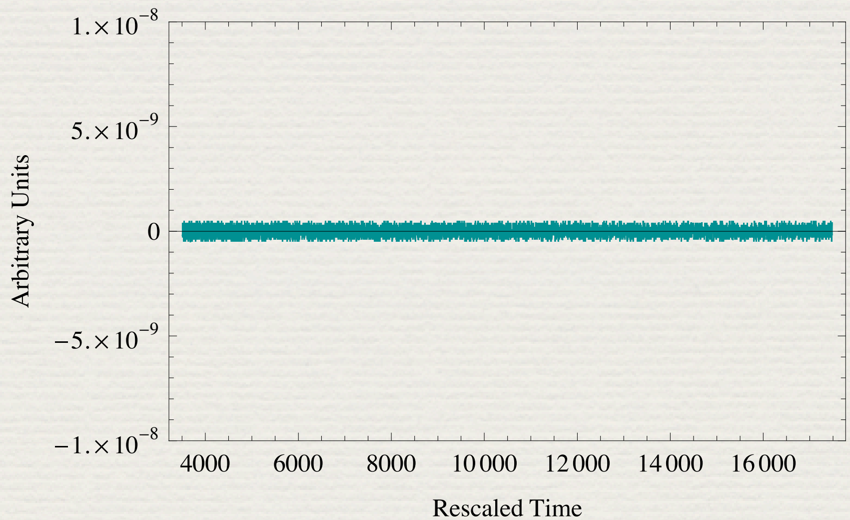
$$P_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$$

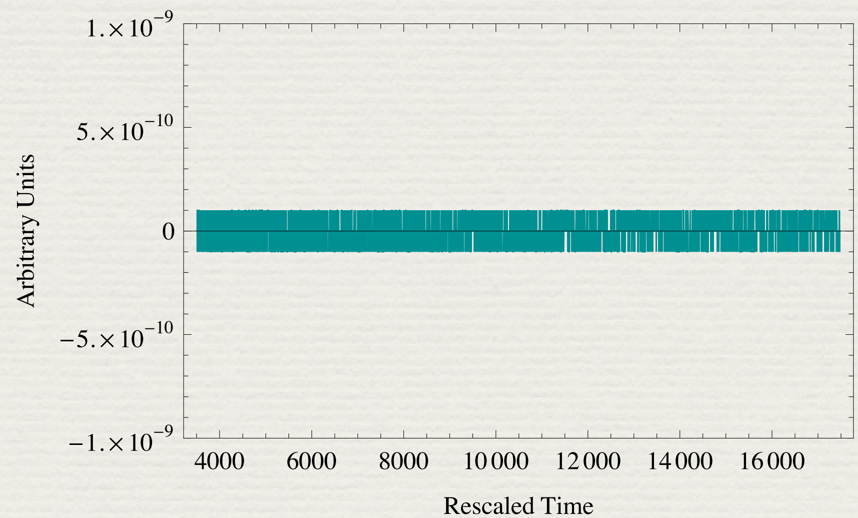
Transverse

Traceless







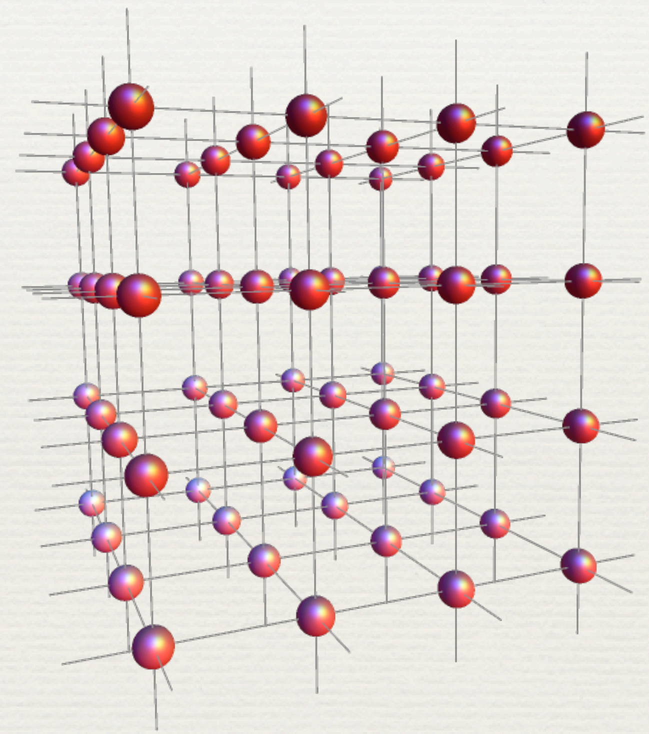


Computational Strategy

- We start by defining a 3-dimensional lattice with periodic boundary conditions and fill it with scalar fields,

$$\phi_i(\vec{x}, t)$$

- We fill the lattice with Scalar fields which obey



$$\ddot{\phi}_i + 3\frac{\dot{a}}{a}\dot{\phi}_i - \frac{1}{a^2}\nabla^2\phi_i + \frac{\partial V(\phi)}{\partial \phi_i} = 0$$

- We use LATTICEEASY to evolve fields *and* the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

- We calculate the source term of

$$\ddot{h}_{ij} + 3\frac{\dot{a}}{a}\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{16\pi G}{a^2}S_{ij}$$

in momentum space.

- Evolve the metric perturbations (in an expanding background) calculate

$$\rho_{gw} = \frac{1}{32\pi G}\left\langle h_{ij,0}h^{ij}_{,0}\right\rangle = \sum_{i,j}\frac{1}{32\pi G}\left\langle h^2_{ij,0}\right\rangle$$

Transfer Function

- specifically, we calculate

$$\frac{d\Omega_{gw}}{d\ln k} = \frac{1}{\rho_{crit}}\frac{d\rho}{d\ln k} = \frac{\pi k^3}{3H^2L^2}\sum_{i,j}|h_{ij,0}(k)|^2$$

- and transfer that to the present day using

$$\Omega_{gw}h^2 = \Omega_r h^2 \frac{d\Omega_{gw}(a_e)}{d\ln k}\left(\frac{g_0}{g_*}\right)^{1/3}$$

$$f = 6\times 10^{10}\frac{k}{\sqrt{M_p H_e}}\ \mathrm{Hz}$$

The First Model

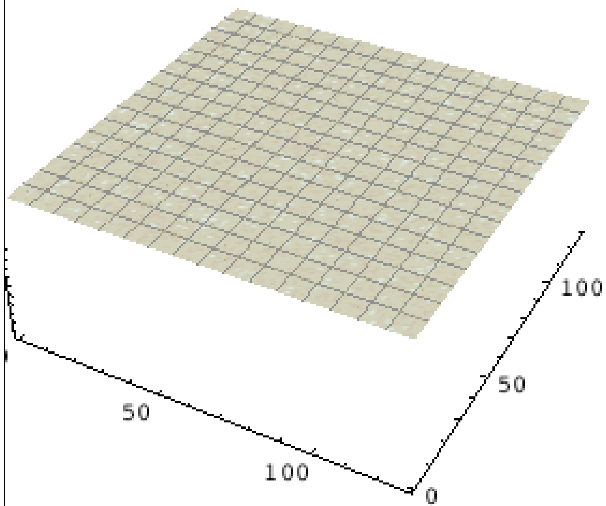
- We start by looking at

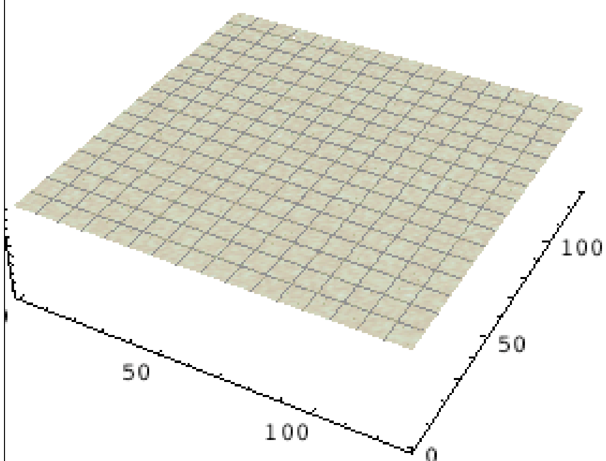
$$V(\phi) = \frac{1}{4}\lambda\phi^4$$

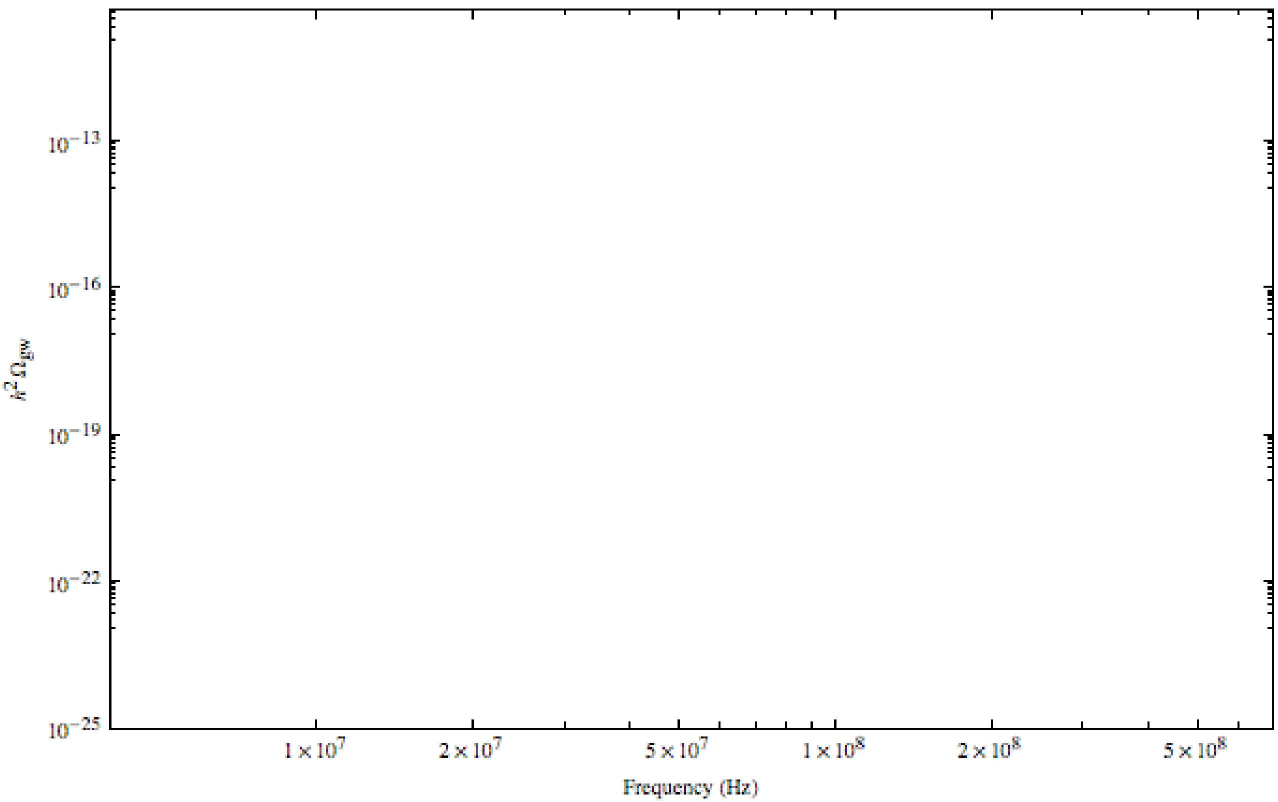
- First used by Khlebnikov and Tkachev

hep-ph/9701423

- Useful comparison to other work



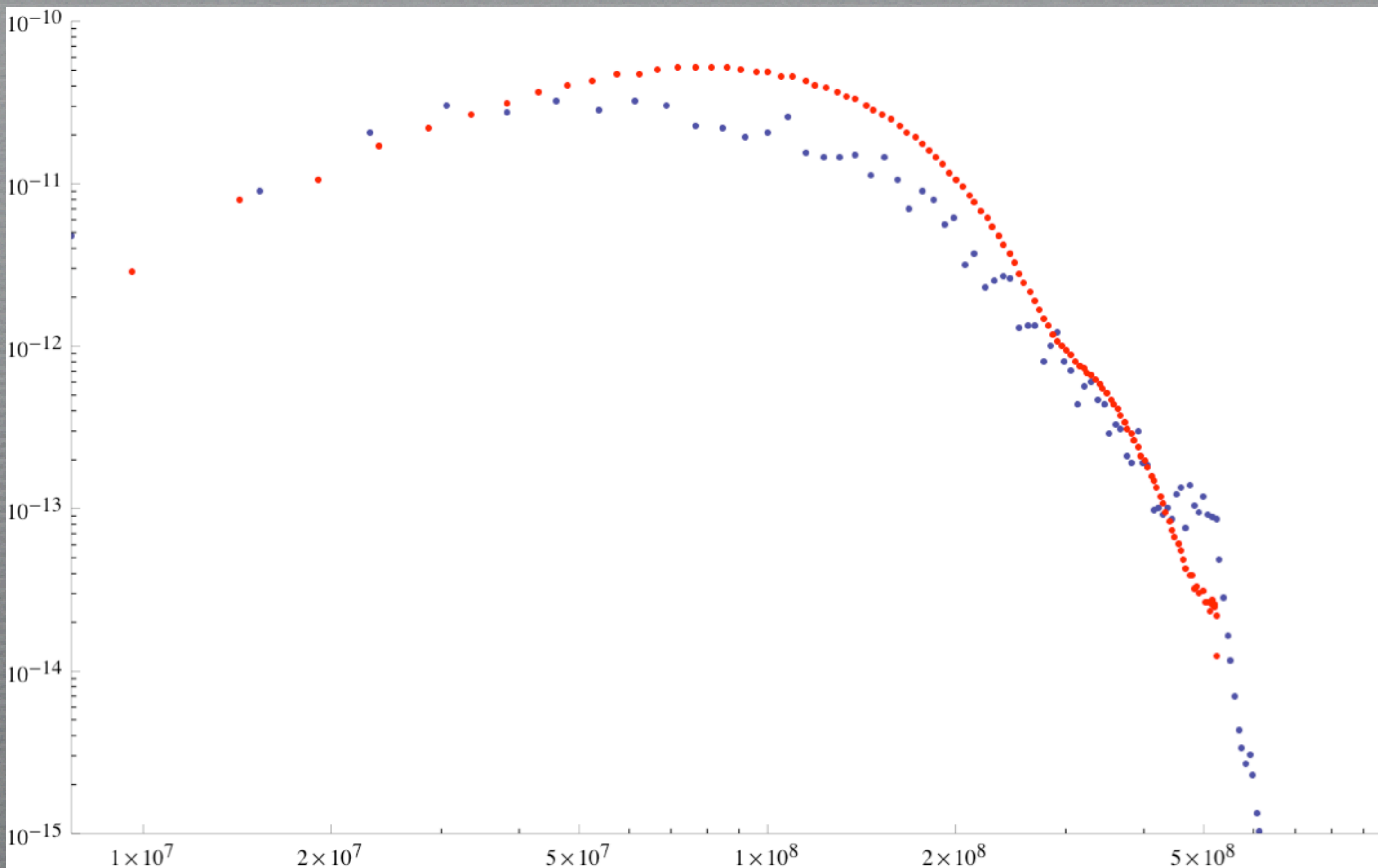




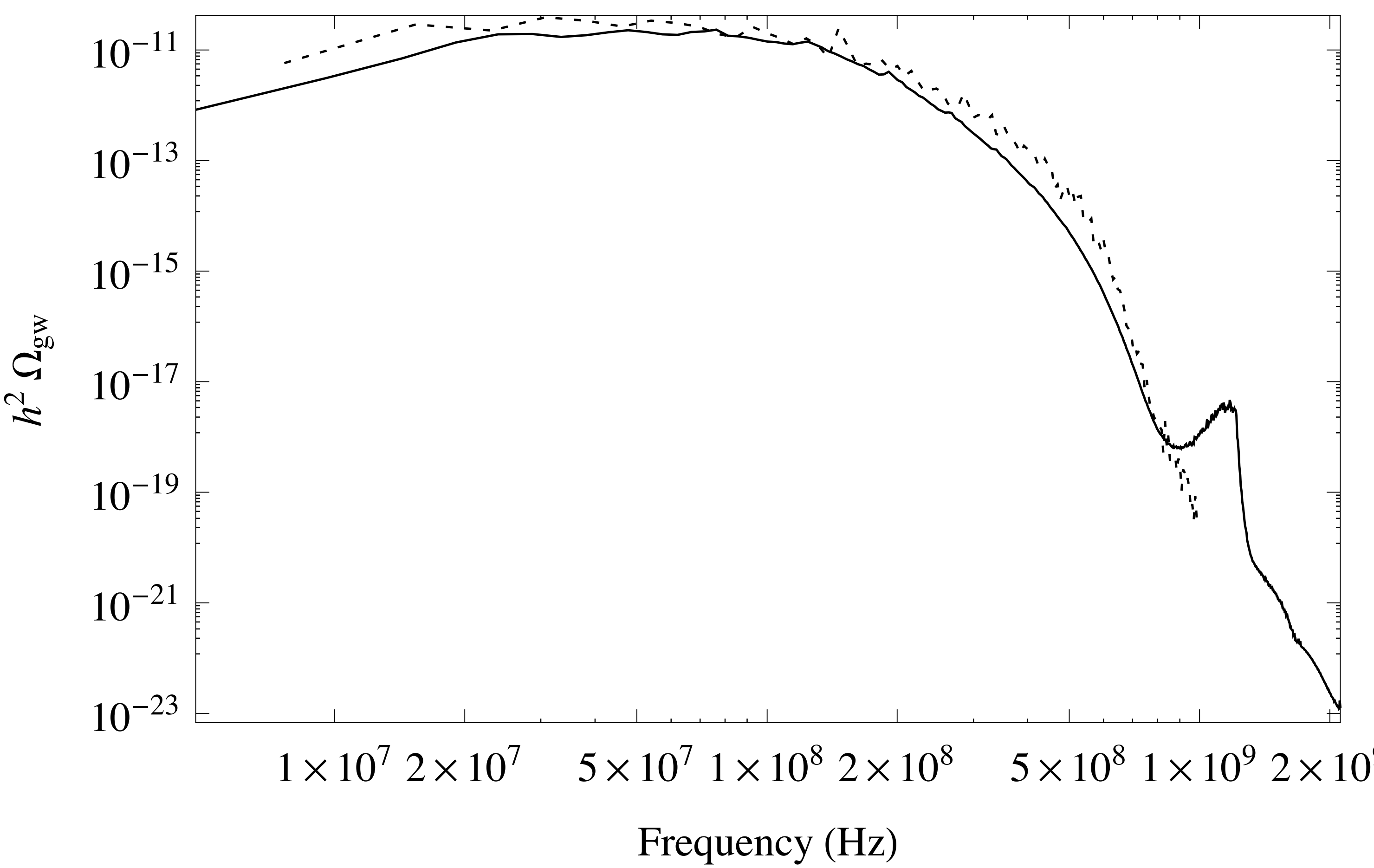
Gravitational Wave Production

State of Affairs

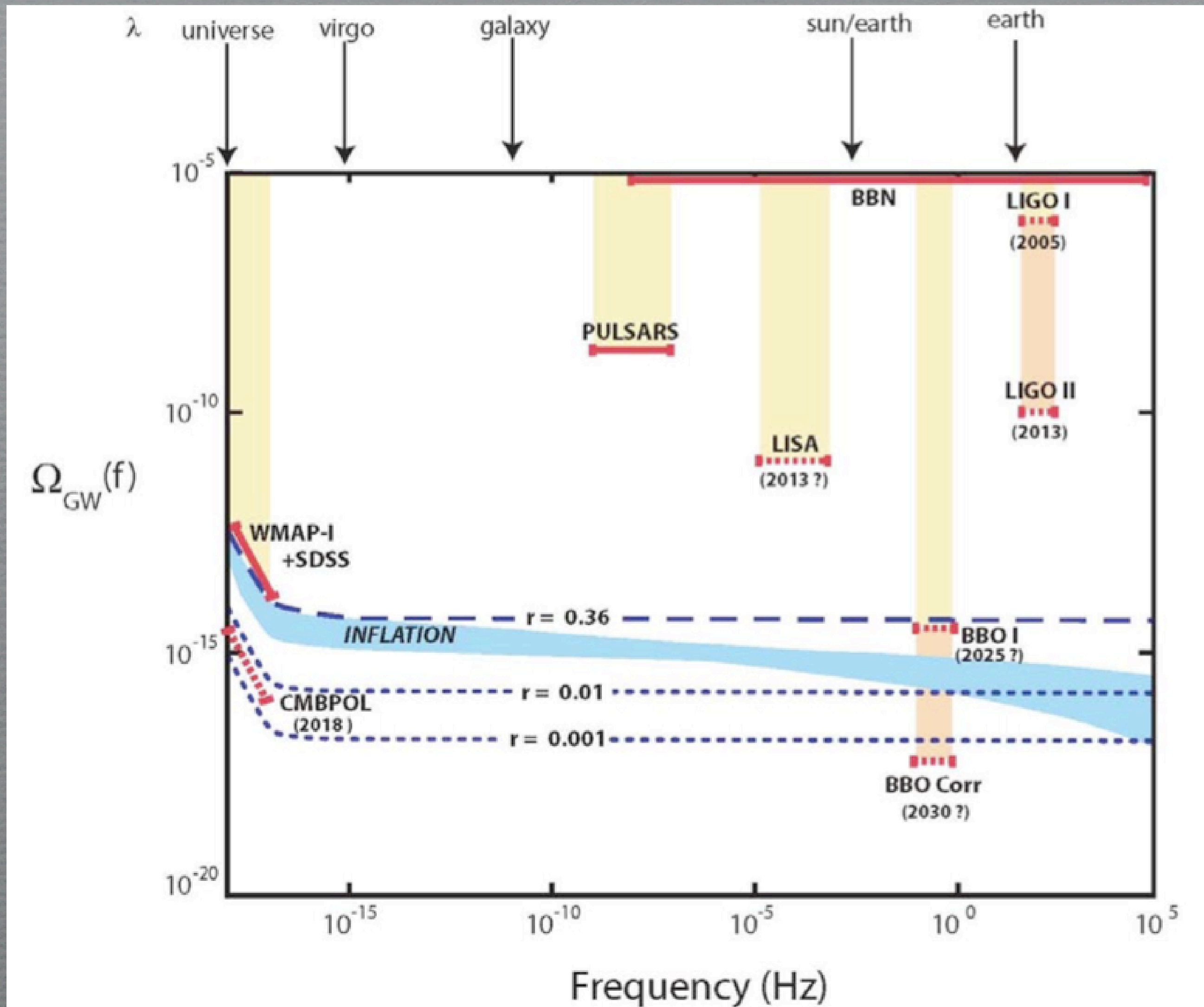
Group	Yale	CITA	Madrid	Milwaukee
Algorithm	Compute tensor source, solve in momentum space	Green's function method	Compute tensor source, solve in position space	Green's function method
Scalar Dynamics	Staggered Leapfrog	Staggered Leapfrog	Staggered Leapfrog	Staggered Leapfrog
Approximations	No backreaction Numerical noise?	<< 1/Hubble "Sample" of k-modes.	Low Scale models have non-expanding background	Expansion is radiation or matter dominated
Comments	Few approximations; noisy sources.	Orthogonal to our approach	Same method as "Yale group" but (sometimes) no expansion	Exact method in radiation era



- Our Current Simulations
- CITA



Yale (solid) and Milwaukee (dotted)



The gravitational wave search

Scaling Argument

- Assuming that the largest possible wavelength to resonate corresponds to (approximately) the Hubble horizon, 1/H, where

$$H \sim \frac{\sqrt{V_e}}{m_{pl}}$$

- we can use our transfer function to calculate

$$f = 6 \times 10^{10} \frac{H_e}{\sqrt{m_{pl} H_e}} \propto V^{1/4}$$

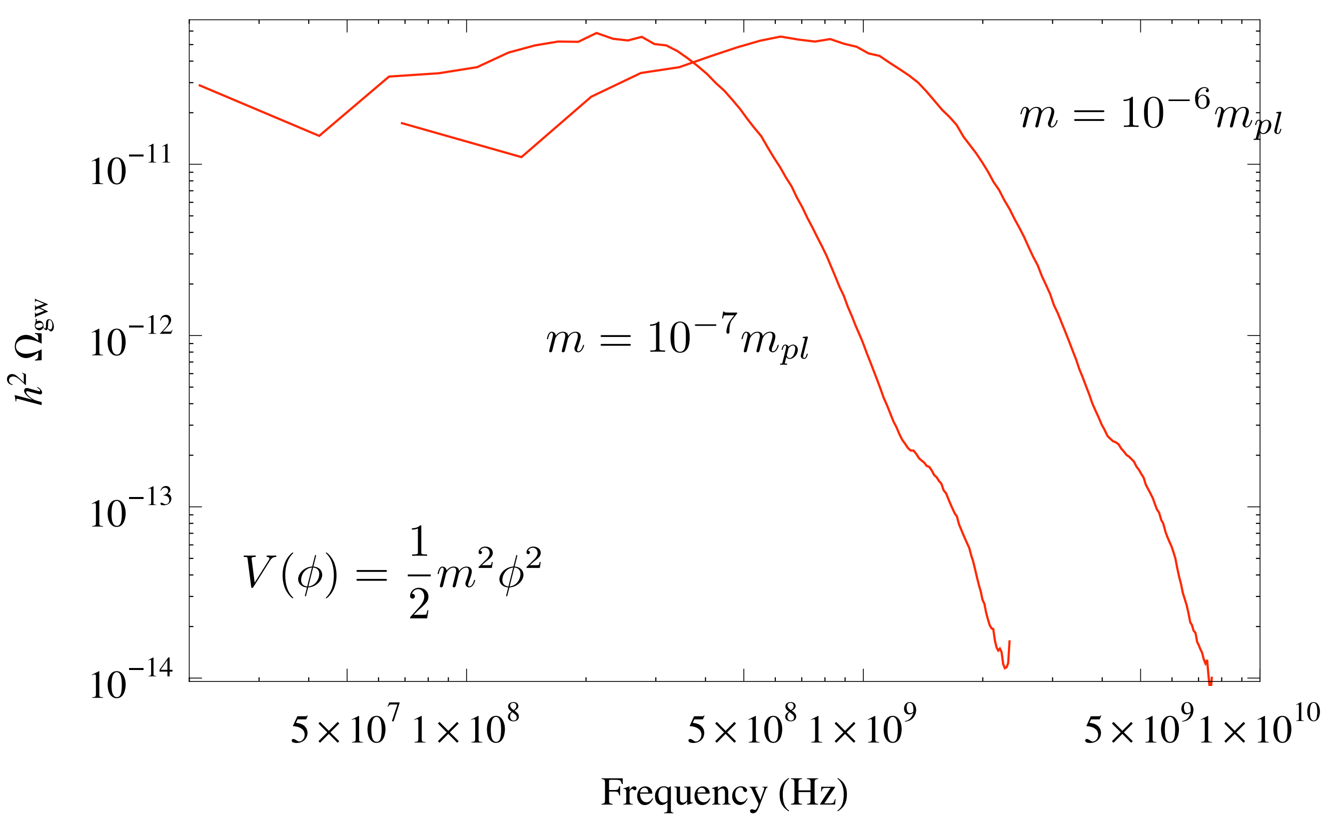
Other Models

- Consider

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

- For single field inflation, the CMB places a bound that

$$m \approx 10^{-6} m_{pl}$$

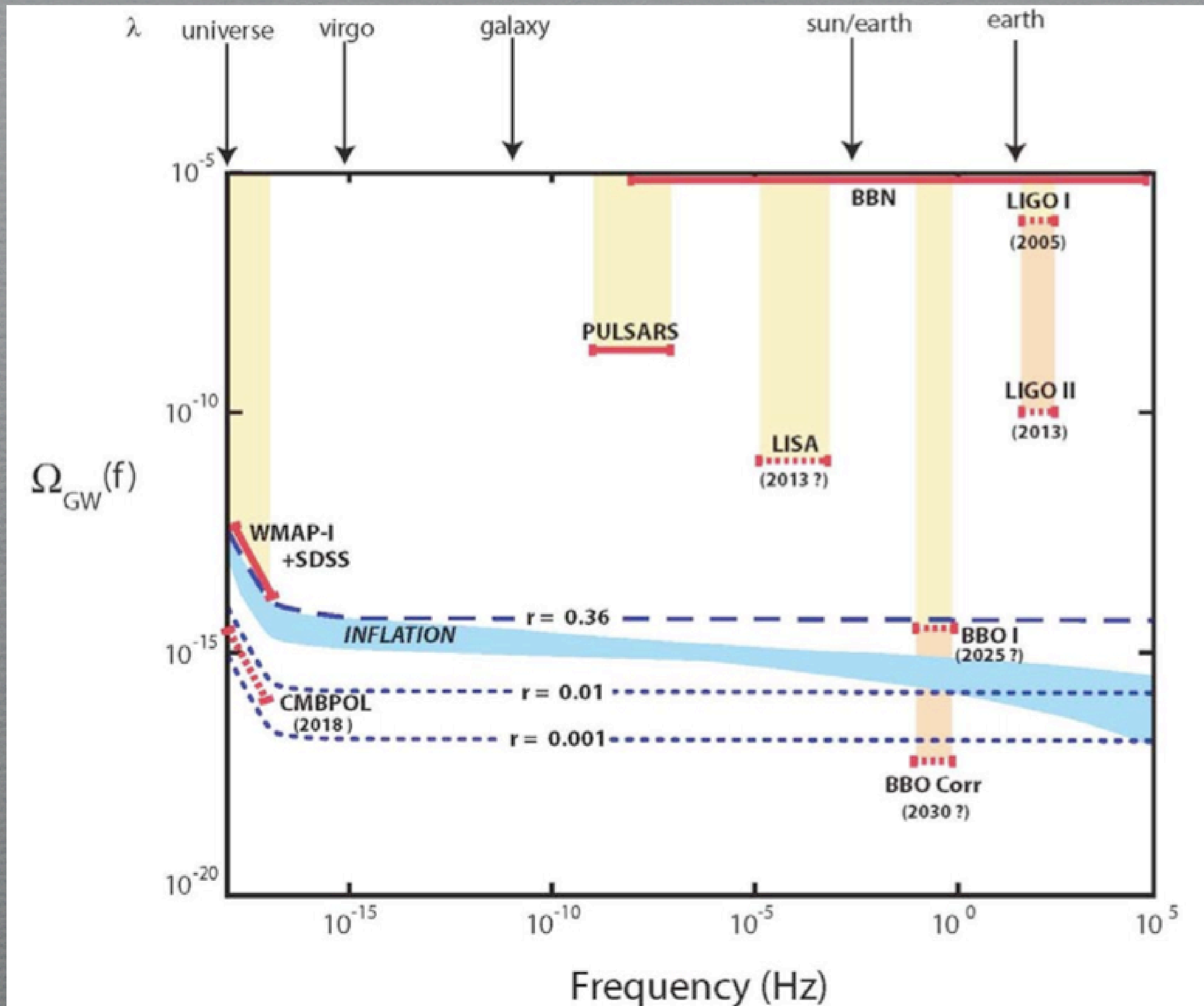


$m = 10^{-6} m_{pl}$
$m = 10^{-7} m_{pl}$
$V(\phi) = \frac{1}{2} m^2 \phi^2$
10^{-11}
10^{-12}
10^{-13}
10^{-14}
$h^2\,\Omega_{\rm gw}$
5×10^7 1×10^8 5×10^8 1×10^9 5×10^9 1×10^{10}
Frequency (Hz)

GUT Scale

Chaotic Inflation

- Amplitude is FIXED
- Scale depends on the parameter, *m*, and hence on the energy scale of inflation
- Observation at MHz and GHz seems (at least now) unrealistic
- More realistic observations need a lower energy scale



The gravitational wave search

Hybrid Inflation

- Let's add a field:

$$V = \frac{(M^2 - \lambda\sigma^2)^2}{4\lambda} + \frac{m^2}{2}\phi^2 + \frac{h^2}{2}\phi^2\sigma^2$$

- For large values of φ, σ=0 is a stable point, however as φ decreases, σ is drawn to a minimum at

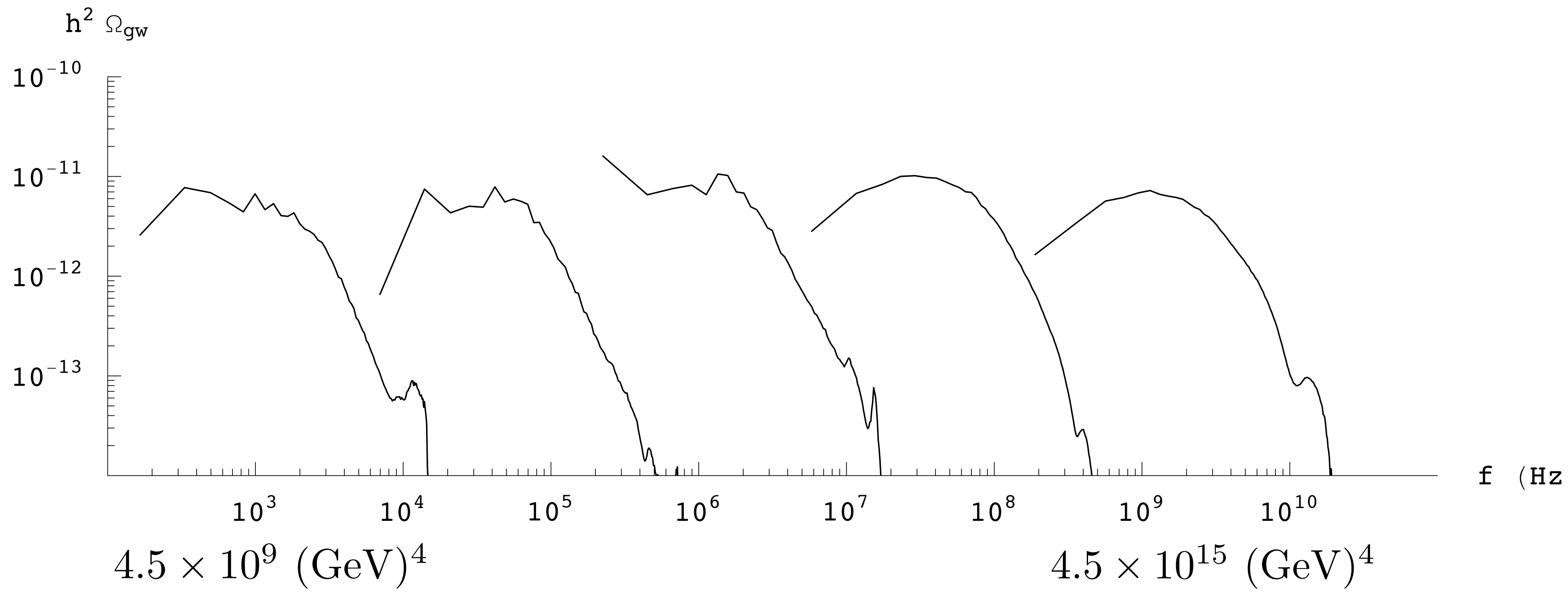
$$\sigma = \langle\sigma\rangle = \frac{M}{\sqrt{\lambda}}$$

Hybrid Inflation (II)

- Assume no fluctuations in σ
- We get an effective potential

$$V(\phi) = \frac{1}{2}\left(m^2 + \frac{h^2 M^2}{\lambda}\right)\phi^2$$

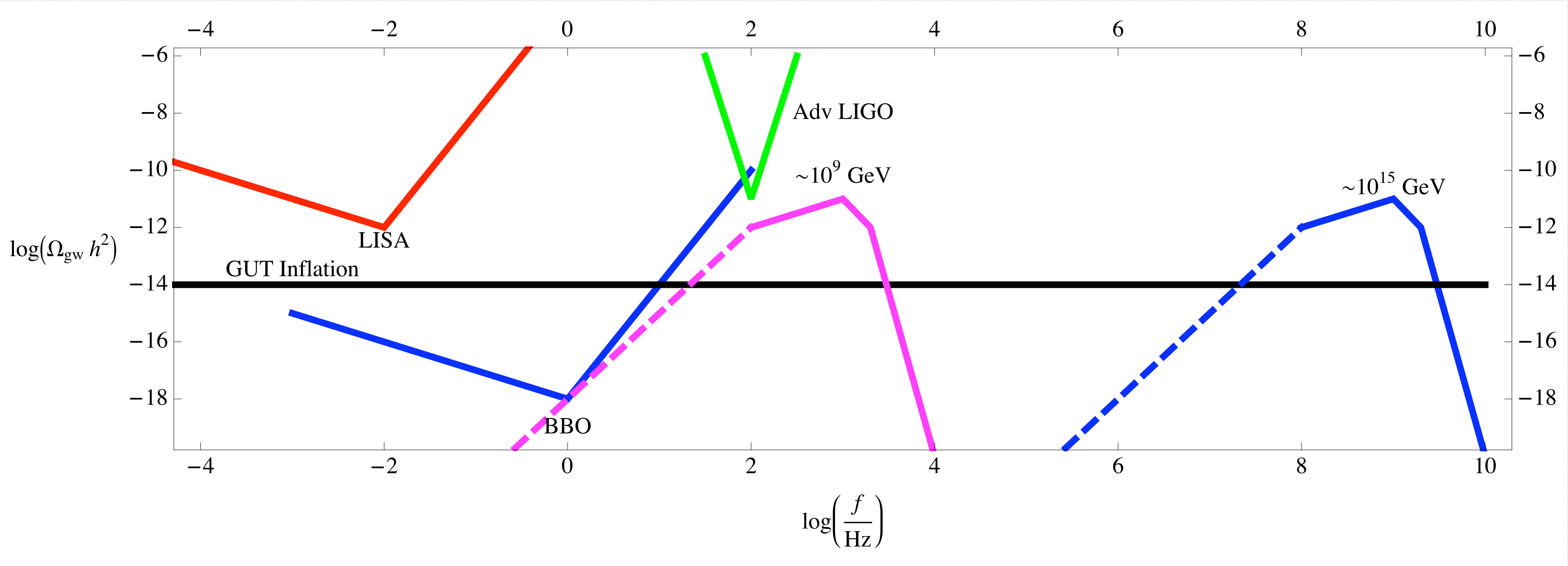
which is (dynamically) identical to chaotic inflation,



$h^2\ \Omega_{gw}$
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10}
f (Hz)
$4.5 \times 10^9\ (\mathrm{GeV})^4$
$4.5 \times 10^{15}\ (\mathrm{GeV})^4$

Low Scale

Where this fits in...



Different from Primordial Spectrum

Primordial	Preheating
Quantum source	Classical source
Scale Invariant	Peaked near observable range
Low H : Low amplitude	Low H : redder peak
Always generated	Strongly model dependent
Amplitude bounded by CMB	Amplitude possibly large
$\Omega_{gw,inf}\, h^2 < 10^{-14}$	$\Omega_{gw}\, h^2 \lesssim 10^{-10}$

Conclusions

- Direct detection of gravitational waves "inevitable"?
- Preheating provides a frequency-dependent, constant amplitude probe
- Detection could serve as a "model" selector
- Preheating a new window on inflation
  - Particularly for low scales (<10^9GeV)

Fin

Changing the Sign

What if we look at a more general case...

$$V(\phi, \chi) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4 + \frac{g}{2}\phi^2\chi^2,$$

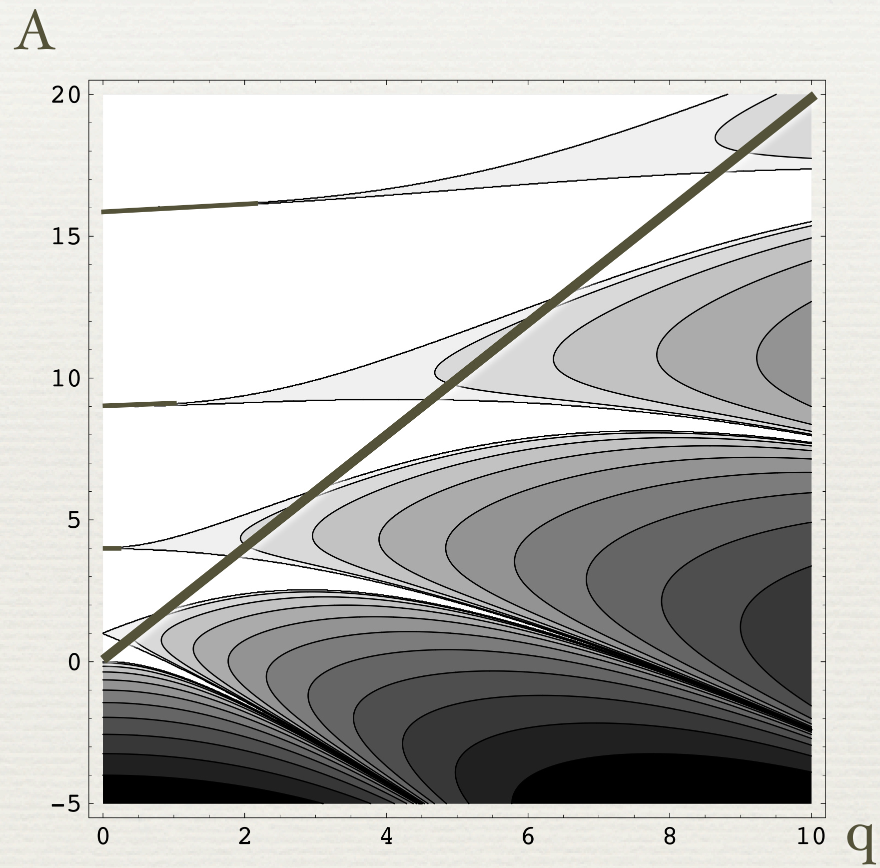
where

$$\frac{\lambda_\phi \lambda_\chi}{g^2} > 1$$

so that the energy potential is always bounded from below

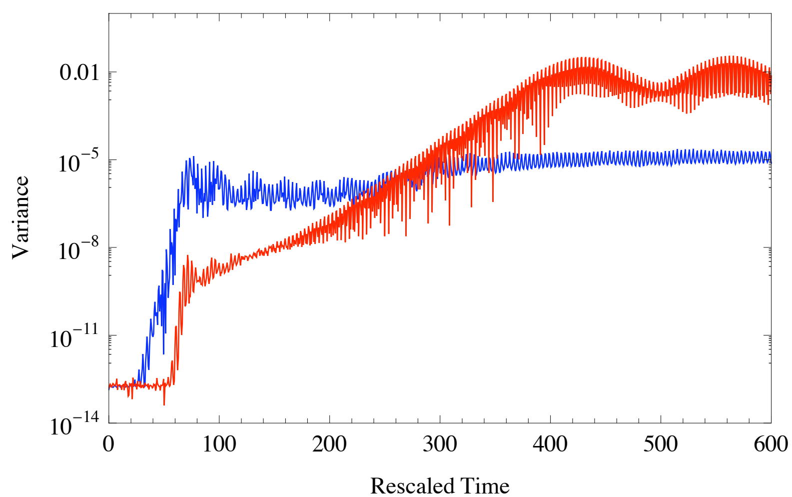
Resonance, Reprise

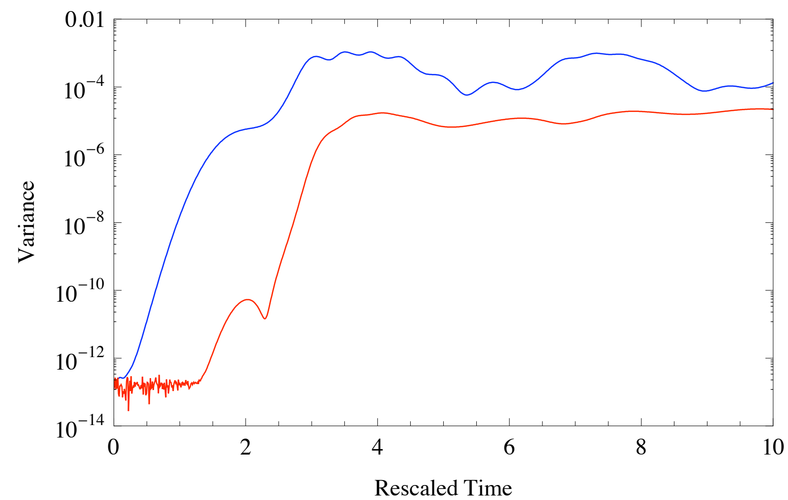
- If we allow g<0 we begin to see the region of the stability chart *below* the A=2q line
- Corresponds to very broad resonance



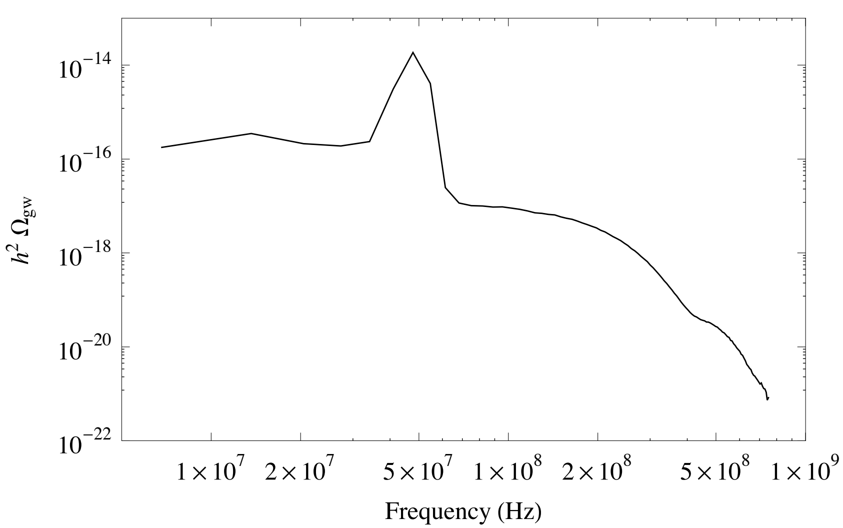
Simple Comparison

- g>0
- g<0

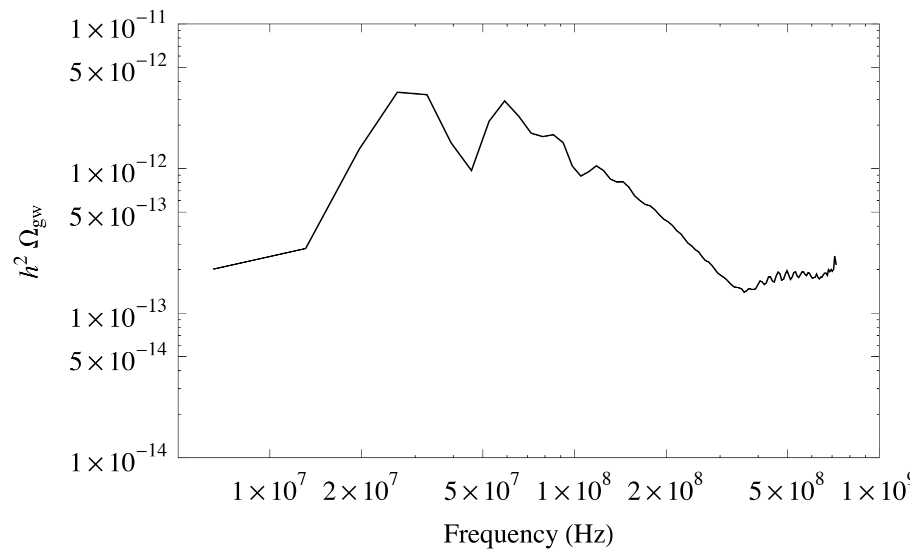




- Two-stage resonance
- Fast broad resonance



10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
$h^2\,\Omega_{gw}$
1×10^7 2×10^7 5×10^7 1×10^8 2×10^8 5×10^8 1×10^9
Frequency (Hz)

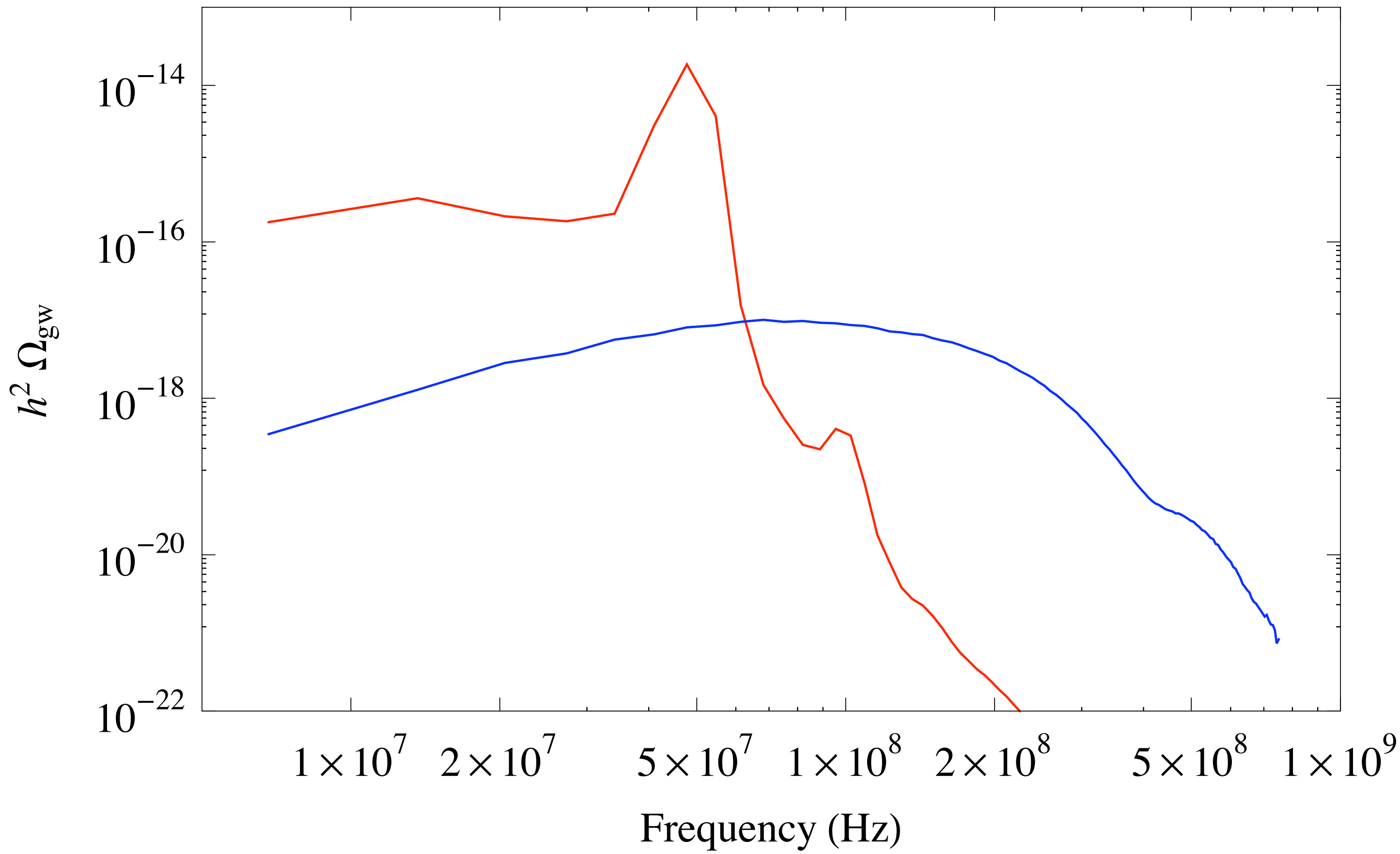


1×10^{-11}
5×10^{-12}
1×10^{-12}
5×10^{-13}
1×10^{-13}
5×10^{-14}
1×10^{-14}
$h^2\,\Omega_{gw}$
1×10^7 2×10^7 5×10^7 1×10^8 2×10^8 5×10^8 1×10^9
Frequency (Hz)

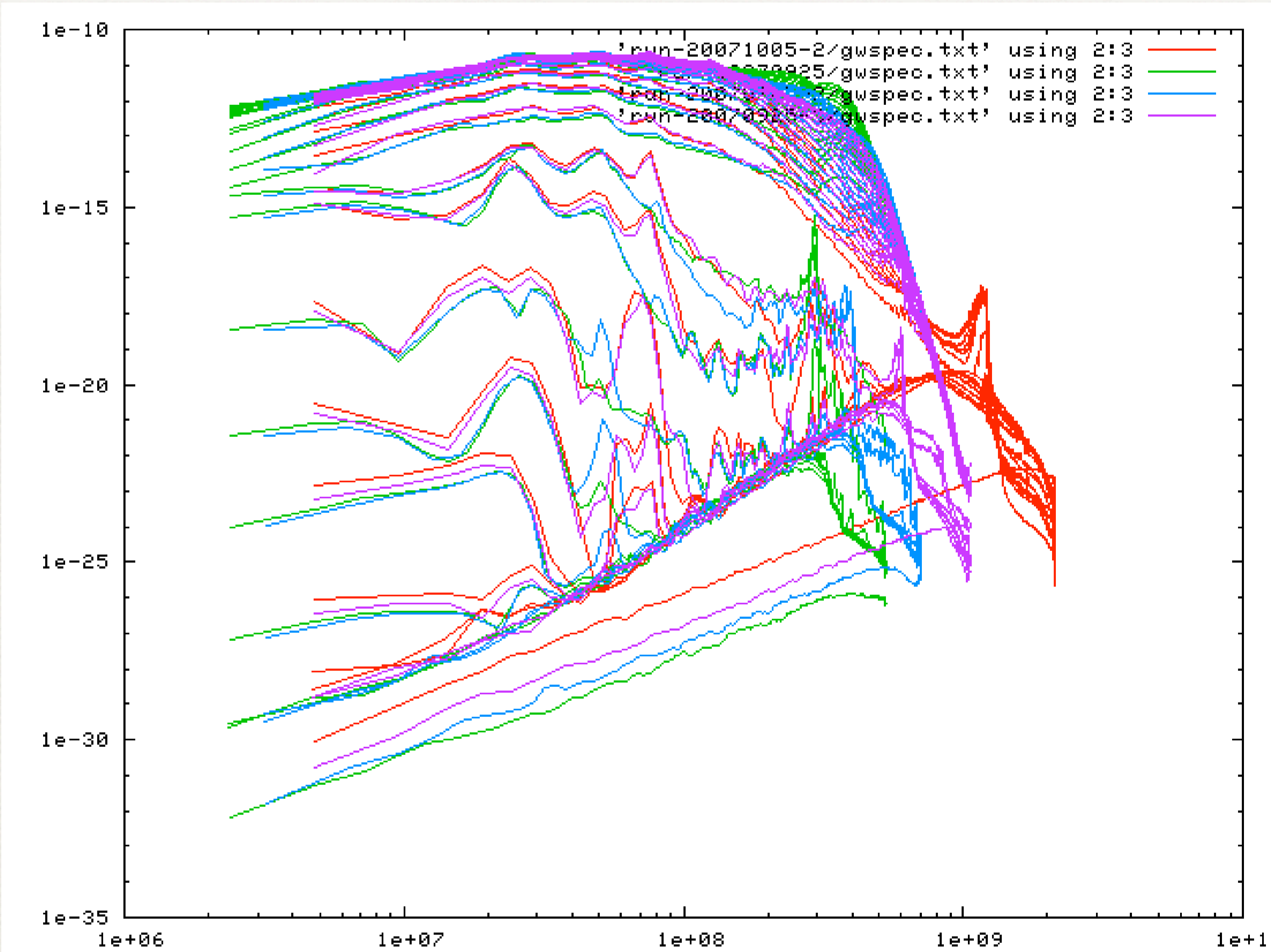
g>0

g<0

parts of the positive spectrum



“new” runs



'run-20071005-2/gwspec.txt' using 2:3
gwspec.txt' using 2:3
gwspec.txt' using 2:3
gwspec.txt' using 2:3
1e-10
1e-15
1e-20
1e-25
1e-30
1e-35
1e+06
1e+07
1e+08
1e+09
1e+10