Primordial Gravitational Waves in the Anisotropic Inflationary Universe

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work w/

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PGW in Standard Inflationary Scenario

In the conventional inflation, primordial gravitational waves (PGW) are produced from quantum fluctuations and have the following properties

Statistically isotropic

Scale invariant power spectrum

No polarization

No correlation with curvature perturbation

If *H* is small as is often stated in string cosmology,
then tensor-to-scalar ratio is small $r \ll 10^{-3}$ **,**
Hence, PGW cannot be observed by any future detector!

Are these results robust? **No, not necessarily!**

What would happen if inflation is anisotropic?

There are many theoretical possibilities
of changing the predictions on PGW.

Above all, anisotropic inflation is the most interesting one because there are many vector fields in particle physics models which could produce the anisotropy of the universe.

If inflationary expansion is anisotropic,
there exists a novel mechanism to produce PGW
through a leakage from curvature perturbation.

Why has this possibility been ignored so far?

It is because that cosmic no-hair theorem tells us
the anisotropy decays exponentially fast !!

However, there is a natural way to get around the cosmic no-hair.

Anisotropic inflation

As was explained in Sugumi Kanno's talk,

we have succeeded in constructing an anisotropic inflation model by introducing a non-minimally coupled vector field as an impurity which violates an assumption in the proof of cosmic no-hair theorem.

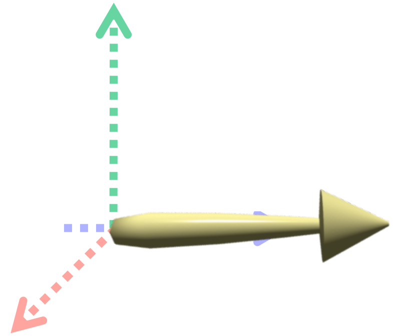
Non-minimal coupling

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2}\left(\partial_\mu \phi\right)^2 - V(\phi) - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(m^2 - \frac{R}{6} \right) A_\mu A^\mu \right]$$

Scalar Vector Mass for A_μ

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Let us assume the vector is in the *x*-direction.



In slow-roll phase $(H, \Sigma \simeq \text{const.})$, we have the metric:

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-4\Sigma t} dx^2 + e^{2\Sigma t}\left(dy^2 + dz^2 \right)\right]$$

Now, let us see how predictions are different
in this anisotropic inflationary scenario.

Statistical anisotropy

Let us consider a test scalar field, ψ in this anisotropic background metric.

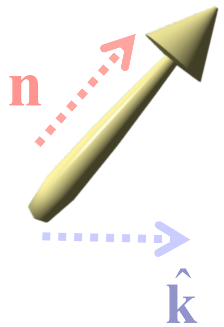
The power spectrum: Ackerman et al. (2007)

Deviation from isotropic part depends on $\hat{\mathbf{k}}$

$$P_\psi(\mathbf{k}) = P_0(k)\left[1+O\left(\frac{\Sigma}{H}\right)(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right]$$

$$\frac{\Sigma}{H} = 0.15 \pm 0.039$$ (WMAP 5-year data)

Groeneboomn & Eriksen (2008)



can be detectable 10 % with WMAP

2 % with PLANCK

Pullen & Kamionkowski (2007)

If $P_0(k)$ has a flat spectrum, the total spectrum should also be flat even though it is anisotropic.

Clearly, we can expect the same statistical anisotropy for PGW.

Gauge invariant quantity in anisotropic spacetime

Perturbed metric

$$ds^2 = -dt^2 + e^{2Ht}(1+2\zeta)\left[e^{-4\Sigma t}(1-4h_+)dx^2 + e^{2\Sigma t}(1+2h_+)\left(dy^2+dz^2\right)\right]$$

Gauge transformation in the longwavelength limit

Under the infinitesimal coordinate transformations $t \to t - \xi^0$
perturbed quantities transform as

$$\Delta\zeta = H\xi^0 \qquad\qquad \Delta h_+ = \Sigma\xi^0$$

Thus, we find the gauge invariant gravitational waves

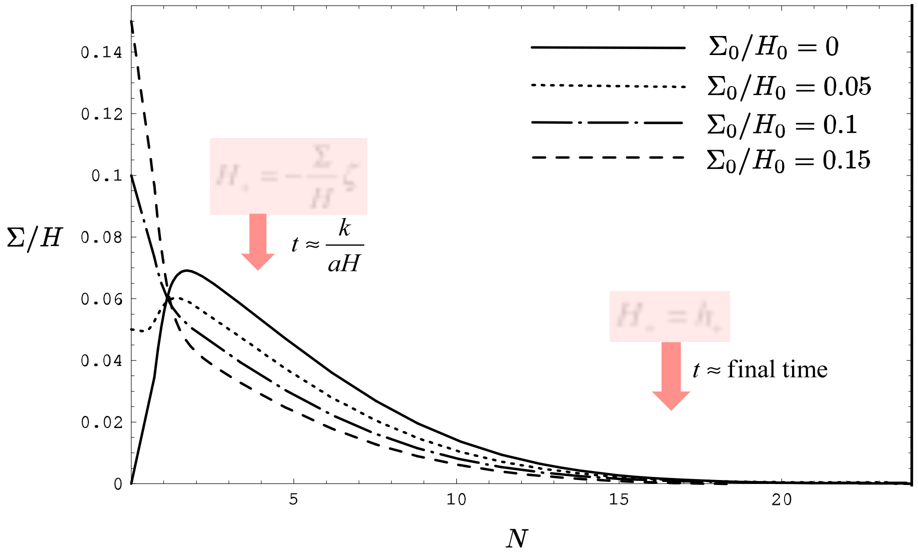
$$H_+ \equiv h_+ - \frac{\Sigma}{H}\zeta$$

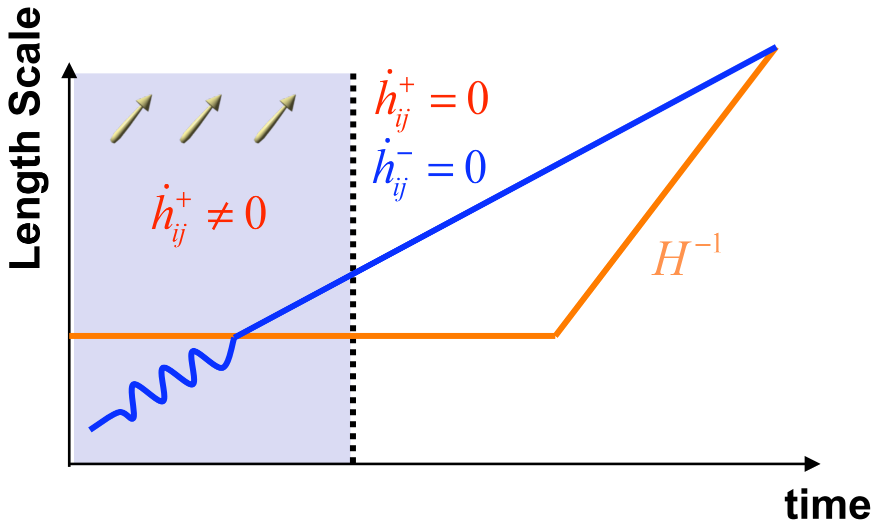
Gauge inv. Tensor Scalar

Note that anisotropic perturbation h_+ is not gauge invariant in anisotropic universe.

PGW from Curvature Perturbations

$$\frac{dH_+}{dt} = 0$$ ⬅ Homogeneous solution
= long wavelength limit





$$h_+\Big|_{final} = -\frac{\Sigma}{H}\zeta\Big|_{k \approx aH}$$

It gives rise to a chance of observing PGW even in the low scale inflation.

Tensor-to-scalar ratio

Let us estimate the tensor-to-scalar ratio

$$h_+ = O\left(\frac{\Sigma}{H}\right)\zeta$$

$$\longrightarrow \quad r = \frac{P_{h^+}}{P_\zeta} = O\left(\left(\frac{\Sigma}{H}\right)^2\right)$$

Assume that the result $\frac{\Sigma}{H} = 0.15 \pm 0.039$ (WMAP 5-year data) **is correct**

Groeneboomn & Eriksen (2008)

and it stems from anisotropic inflation, we obtain the tensor-to-scalar ratio

$$\frac{\Sigma}{H} \sim 0.15$$

$$\longrightarrow \quad r \sim 0.017$$

cf. $r \leq 0.43$ (WMAP)

$r \leq 0.1$ (Planck)

$r \leq 0.05$ (Clover, EBEX, Spider)

$r \leq 10^{-6}$ (BBO or DECIGO)

Correlation between Curvature and Tensor perturbation

Since the tensor perturbation comes from curvature perturbations, we have the correlation

$$\langle \zeta \, h^{+} \rangle \sim O\left(\frac{\Sigma}{H} \right) \langle \zeta \, \zeta \rangle$$

For CMB

Curvature perturbation → CMB temperature anisotropy
Tensor perturbation → Polarization (B-mode)

$$\rightarrow \quad \langle TB \rangle \sim O\left(\frac{\Sigma}{H} \right) \langle TT \rangle$$

We expect 15% correlation between temperature and B-modes.

Linear Polarization in PGW

After inflation, we have the PGW of the form

$$ds^2 = -dt^2 + e^{2Ht}\left[(1-4h_+)\,dx^2 + (1+2h_+)\left(dy^2 + dz^2\right)\right]$$

which implies 100% linear polarization in primordial gravitational waves.

This should be detected by CMB or DECIGO!

Conclusion

We have proposed an anisotropic inflationary scenario.

Based on our scenario and the observed anisotropy $\frac{\Sigma}{H} = 0.15 \pm 0.039$

Groeneboomn & Eriksen (2008)

we have predicted the following numbers

the tensor-to-scalar ratio	**r = 0.017**
correlation between T and B	**15%**
linear polarization of GW	**100%**

If these are detected, they would be strong evidences for the anisotropic inflation.