Conservation and evolution of the curvature perturbation

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Based on

- D. J. H. Chung and JG, in preparation
- K.-Y. Choi, JG and D. Jeong, in preparation

Outline

1. Introduction
   - Curvature perturbation?
2. Single field inflation
   - Equation of motion
   - Subtleties
3. Multi field inflation
   - Single and multi field inflation
   - Evolution after multi field inflation
4. Conclusions

Curvature perturbation$\cdots$ Err?

Gauge invariant curvature perturbation ζ

$$-\zeta = \psi + H\frac{\delta\rho}{\dot{\rho}}$$

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Generic hypersurface → the one with uniform energy density

1. Wiggles in the spatial curvature
2. Strongly constrained by observations

$$\mathcal{P}_\zeta \sim 10^{-5}\,, \qquad n \sim 0.96\,, \qquad \left|\frac{dn}{d\log k}\right| \lesssim 0.01\,, \cdots$$

(The only?) Window to the early universe

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Q: What do we know about ζ?

Equation of motion of ζ

From perturbed Klein-Gordon equation / Einstein equation / variation principle we can obtain

$$\ddot{\zeta} + \left(\frac{2\ddot{\phi}}{\dot{\phi}} - \frac{2\dot{H}}{H} + 3H \right) \dot{\zeta} - \frac{\nabla^2}{a^2} \zeta = 0$$

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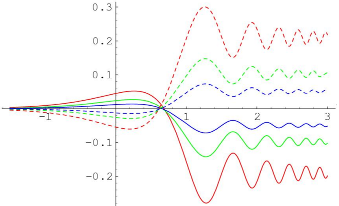

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Conservation of ζ: Slow-roll is necessary even for single field case

$$\ddot{\zeta} + \mathcal{O}(H)\dot{\zeta} = 0$$

Case of particle production (1/2)

Conservation of $\zeta \longleftrightarrow$ conservation of energy

$$\Delta\mathscr{L} = -\frac{1}{2}g^2\phi^2 \chi^2$$

At $\phi_\star = m_\chi/g$, χ particles are resonantly produced

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$$\chi = \chi_0 + \delta\chi$$

χ field is **quantum**: How to treat it?

Case of particle production (2/2)

- With $Q = \delta\phi + (\dot\phi/H)\psi = -(\dot\phi/H)\zeta$,

$$\ddot{Q}_k = -3H\dot{Q}_k - \left\{ \frac{k^2}{a^2} + V'' + g^2\mathcal{N}\left\langle\chi^2\right\rangle + \frac{1}{m_{\rm Pl}^2 H}\left[\left(3H + \frac{\dot H}{H}\right)\dot\phi + 2V' - g\mathcal{N}(m_\chi - g\phi)\left\langle\chi^2\right\rangle\right]\right\} Q_k$$

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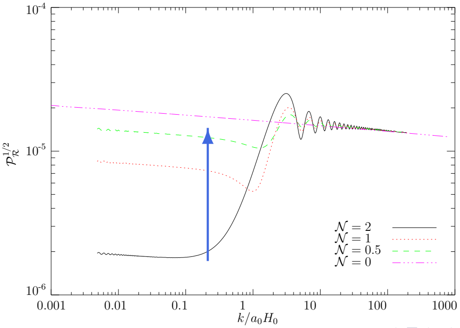

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  From conservation equation: $\dot{\mathcal{P}}_\zeta = -\dfrac{\rho_\chi^2 H}{6(\rho+p)^2}\mathcal{P}_\chi$

  Many subtleties

  - How to “turn on” χ?
  - What is the range of wavenumber and time?

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We are trying to find a new perspective on ζ regarding its conservation

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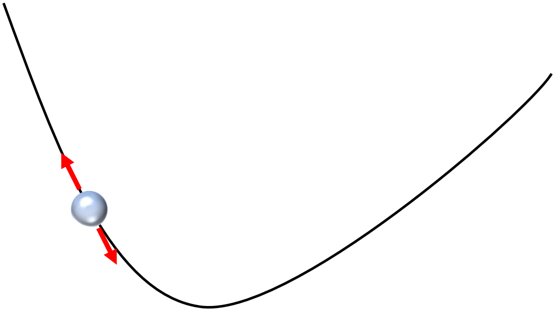

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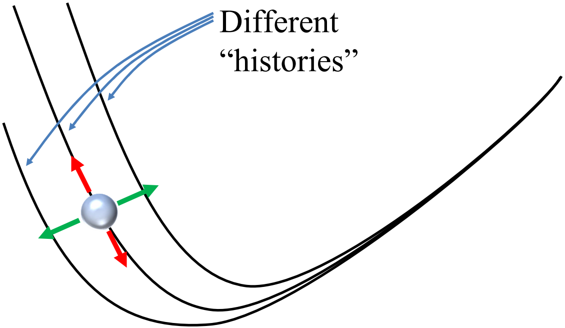

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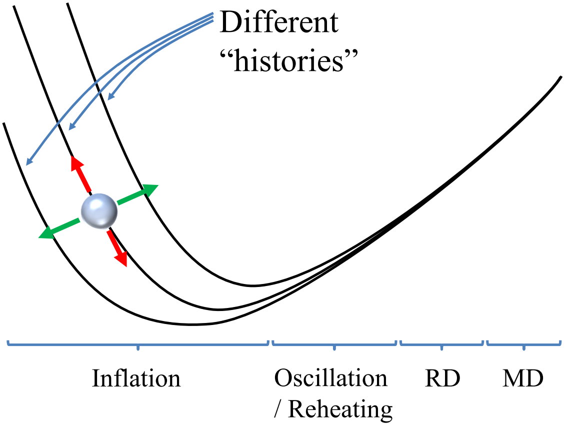

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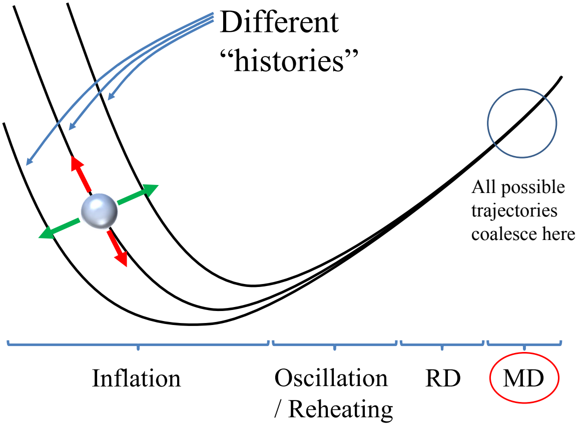

Multi field inflation and afterwards

For multi field inflation, **inflationary estimates are not enough** and the evolution after inflation should be taken into account

e.g. curvaton σ should satisfy

1. flat potential: $m_\sigma \ll H$
2. non-zero amplitude: $\sigma \gtrsim 10^{-8} m_{\text{Pl}}$
3. small energy fraction: $V_\sigma \ll V_{\text{tot}}$

Easily satisfied by individual inflaton field after multi field inflation

inflaton = curvaton

Evolution of ζ in multi field inflation

Multiple chaotic inflation: $V = \frac{1}{2}\sum_i m_i^2\phi_i^2$ with $\phi_i \to \Gamma_\gamma^{(i)}, \Gamma_m^{(i)}$

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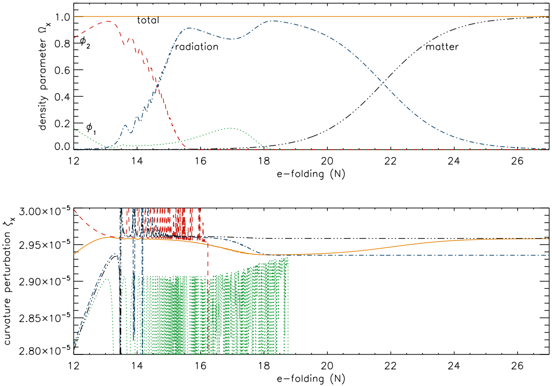

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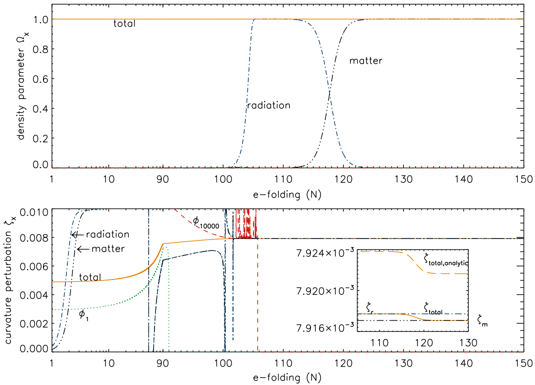

Matter-radiation isocurvature perturbation

$$S_{\alpha\beta} = 3\left(\zeta_\alpha - \zeta_\beta\right) \Rightarrow S_{m\gamma} = \frac{\delta\rho_m}{\rho_m} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}$$

1. $S_{m\gamma} = 0$ in single field inflation
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We can find

1. $S_{m\gamma} \to 0$ with large e-folds: $\zeta_i =$ adiabatic + ~~non-adiabatic~~
2. ζ does change after inflation: non-zero δp_{nad} **DO** exist
3. $\mathscr{P}_\zeta$ and $\mathscr{P}_S$ have slightly different scale dependence

Conclusions

Conservation of the curvature perturbation is not as simple as a piece of cake

1. Single field inflation
   - Slow-roll is *required* to ensure the conservation of ζ
   - But many subtleties regarding the conservation of ζ
2. Multi field inflation
   - ζ varies throughout the whole evolution of the universe
   - Inflationary estimates *may not* work
   - Possibly non-zero, detectable $S_{m\gamma}$