COSMO08

Staggered Multi-Field Inflation

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(submitted to JCAP)

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Why are multi-field models interesting?

Review: Wands 07

"Natural" in string theory: many moduli fields are present.

Assisted inflation effect (Liddle, Mazumdar and Schunck 98):

- possible avoidance of super-Planckian field values

- increased Hubble friction, steeper potentials work, reduced fine tuning

Many open issues (good for theorists):

- implementing concrete models in string theory (active field)

- staggered inflation effect (new, this talk)

- largely unknown theory or (p)re-heating (danger of heating hidden sectors D. Greene 08, potentially no parametric resonance D. Battefeld, S. Kawai 08; see also talk of J. Braden, …

- possibly large non-Gaussianities (model dependent, see review Wands 07)

Main Question

What are the consequences if fields drop out of multi-field inflation in a staggered fashion?

This is a generic feature in many models of multi-field inflation in string theory.

E.g. in:

- Inflation from multiple tachyons
  Majumdar, Davis 03
- Inflation from multiple M5-branes
  Becker, Becker, Krause 05
- …

Outline

1. Concrete examples of multi-field inflation within string theory.

2. Analytic formalism to compute effects due to staggered inflation
   - background (relation to warm inflation)
   - perturbations (adiabatic and isocurvature)
   - scalar spectral index
   - extensions (several follow up projects possible)

3. Application
   - inflation from tachyons

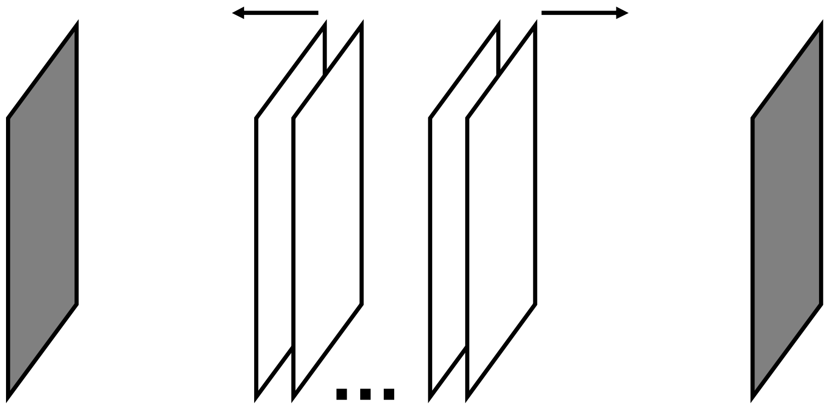
4. Conclusions

Note: $m_p^2 \equiv 1$ throughout

Example: Inflation from M5-branes (BBK setup)

Becker, Becker, Krause 05

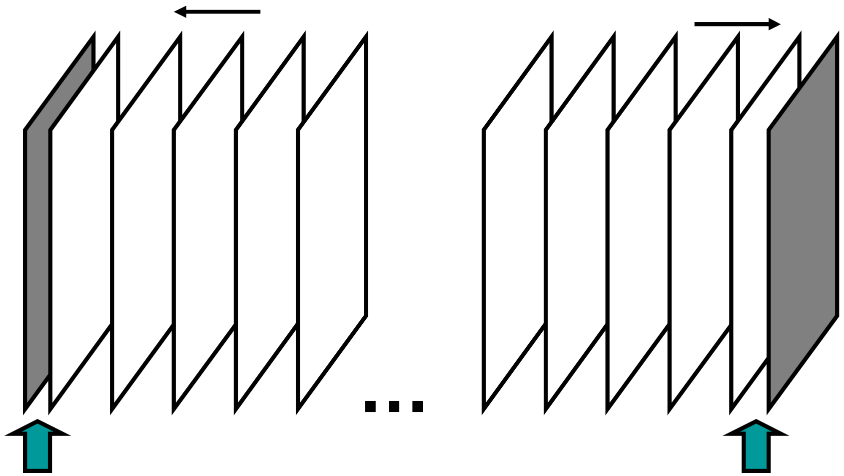
Consider Orbifold: $\mathbf{S_1/Z_2}$



Orbifold fixed planes

N M5-branes

Interbane distance $\longleftrightarrow$ inflatons;
Result: assisted inflation



Branes dissolve into Boundary branes during inflation, one after the other, fields drops out of the model; energy is converted, e.g. into radiation.

Result: N(t) decreasing during inflation.

Branes collide and dissolve, one after the other: staggered inflation (cascade inflation).

Numerical treatment:
Ashoorioon, Krause,06

Example: Inflation from multiple tachyons

Majumdar, Davis 03

N D-brane/antibrane pairs give rise to U(N)xU(N) sym.:
N^2 coupled tachyons,

M.&D. focus on abelian part:
N, uncoupled fields.

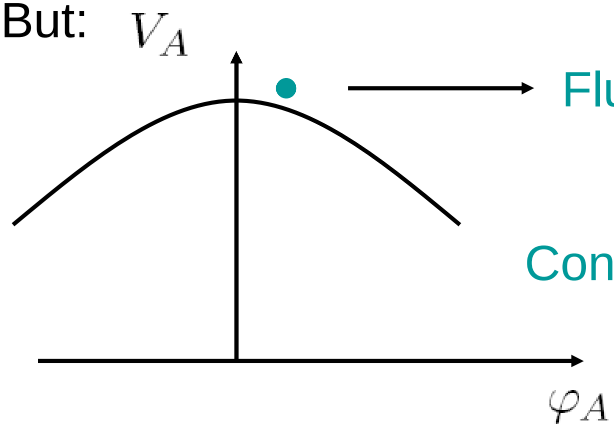
Result: assisted inflation

Potential (trustworthy near origin):

$$W = \sum_{A=1}^{\mathcal{N}} V_A$$
$$= \mathcal{N}\tau_p - c_1 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^2 + c_2 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^4 + \mathcal{O}\left(|\varphi_A|^6\right)$$

Review: K. Ohmori 01 $\quad c_1 \approx 0.87 \quad c_2 \approx 0.21$

But:



Fluctuation (e.g. QM)

↓

Condensation (sudden): field drops out of model,
its energy is converted, e.g. into radiation.

One field after the other drops out, N(t) decreasing during inflation:
Staggered Inflation

D.Battefeld, T.Battefeld, A.C.Davis 08

Analytic Formalism to deal with fields dropping out:

Smooth out N(t) and introduce a continuous decay rate:

$$\Gamma(t) \equiv -\dot{\mathcal{N}}/\mathcal{N}.$$

The rate is model dependent and can depend on time. For this smoothing to be a good approximation, we need that in any given Hubble time of interest several fields drop out.

Further simplifying assumptions (not crucial):

- uncoupled fields
- identical potentials
- identical initial conditions

Effects we recover:

- additional decrease of energy that drives inflation
- additional, leading order contributions to observables (e.g. scalar spectral index)

Effects we do not recover:

- sharp steps in observables such as the scalar spectral index
- ringing in the spectrum

Background evolution

Introduce effective single field: $\varphi \equiv \sqrt{\mathcal{N}}\varphi_A$

$$W(\varphi) = \mathcal{N}V(\varphi/\sqrt{\mathcal{N}})$$

Energy transfer to additional component:

$$\dot\rho_\varphi = -3H(\rho_\varphi + p_\varphi) + \dot{\mathcal{N}}V$$
$$\dot\rho_r = -3H(\rho_r + p_r) - \dot{\mathcal{N}}V$$

See Watson, Perry, Kane, Adams 06 for related work on a relaxing CC.

Introduce small parameters:

$$\bar\varepsilon \equiv \frac{3}{2}(1+w_r)\frac{\rho_r}{\rho_\varphi + \rho_r}$$

$$\varepsilon_{\mathcal{N}} \equiv -\frac{\dot{\mathcal{N}}}{\mathcal{N}}\frac{1}{2H} = \frac{\Gamma}{2H}$$

$$\hat\varepsilon \equiv -\frac{\dot H}{H^2}$$

$$\simeq \varepsilon + \bar\varepsilon$$

$$\varepsilon_A \equiv \frac{1}{2}\left(\frac{V_A'}{W}\right)^2 \ll 1 \quad , \quad \varepsilon \equiv \frac{1}{2}\left(\frac{W'}{W}\right)^2 \ll 1\,,$$

$$\eta_A \equiv \frac{V_A''}{W} \quad , \quad |\eta_A| \ll 1\,,$$

$$\eta \equiv \frac{W''}{W} \quad , \quad |\eta| \ll 1\,,$$

Can show that within inflationary models of interest to first order in small parameters:

$$\varepsilon_{\mathcal{N}} \simeq \bar\varepsilon$$

Background evolution

We get

$$\dot{\rho}_\varphi \simeq -2H(\varepsilon_{\mathcal{N}} + \varepsilon)\rho_\varphi$$
$$\dot{\rho}_r \simeq 2H(\varepsilon_{\mathcal{N}} - \bar{\varepsilon})\rho_\varphi$$

⇦ This leads to a scaling solution during inflation.

Then the effective single field evolves according to

$$3H\dot{\varphi} \simeq -W'\gamma$$
$$\gamma \equiv 1 + \varepsilon_{\mathcal{N}}\varphi\frac{W}{W'}$$

Resembles warm Inflation (Barera 95),
but

- the radiation-bath originates from transferring potential energy, not kinetic
- model can arise in string theory
- avoids many problems of regular warm inflation

Next, perturbations; new effects due to

- presence of ρ_r and perturbations therein
- additional decrease of W due to the decay rate $\Gamma \neq 0$

Perturbations (straightforward)

We can show that isocurvature/entropy perturbations are suppressed (follow Malik, Wands, Ungarelli 03, …), so we can focus on adiabatic perturbations.

Use the Mukhanov variable, satisfying $$v_k'' + \left(k^2 c_s^2 - \frac{z''}{z}\right) v_k = 0$$

Where $$z \equiv \frac{1}{\theta c_s}, \quad \theta^2 \equiv \frac{1}{3a^2(1+w)} \qquad w = p/\rho$$

$$p = p_\varphi + p_r \qquad \rho = \rho_\varphi + \rho_r$$

Focus on large scales, so

$$c_s^2 \approx \dot{p}/\dot{\rho}$$

Imposing QM initial conditions and using the background solutions, we can compute the curvature perturbation

$$v_k = z\zeta_k$$

and the power-spectrum: $$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}|\zeta_k|^2$$

Perturbations

The scalar spectral index $n_s - 1 = \frac{d\ln \mathcal{P}_\zeta}{d\ln k}$

becomes

$$n_s - 1 \simeq -2(\varepsilon + \varepsilon_{\mathcal{N}}) - \frac{2}{\varepsilon\gamma^2 + \varepsilon_{\mathcal{N}}}\left[\varepsilon\gamma^2(2\varepsilon - \varepsilon_{\mathcal{N}} - \eta) + (\varepsilon + \varepsilon_{\mathcal{N}})(1-\delta)(\varepsilon\gamma(\gamma - 1) + \frac{\varepsilon_{\mathcal{N}}}{2})\right]$$

Recover known limits:

$$\delta \equiv \frac{\dot{\Gamma} H}{\Gamma \dot{H}}$$

- Usual slow roll: $\left.\begin{array}{l}\Gamma = 0\\ \varepsilon_{\mathcal{N}} = 0\end{array}\right\}$ $n_s^{SR} - 1 \simeq -6\varepsilon + 2\eta$

- Dynamically relaxing CC (Inflation without Inflatons):

$$\left.\begin{array}{l}\varepsilon_{\mathcal{N}} = const\\ \delta = 1\\ \varepsilon = \eta = 0\end{array}\right\} \quad n_s^{relax.CC} - 1 = -2\varepsilon_{\mathcal{N}}$$

Watson, Perry, Kane, Adams 06 .

Summary of the Formalism

The expressions for the scalar spectral index (and running) are general, but rely on the smoothing (time averaging) of N(t), assuming slow roll and for simplicity identical, uncoupled field potentials and identical initial field values.

Application: inflation from tachyons

Majumdar, Davis 03

Consider

$$W = \mathcal{N}\tau_p - c_1 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^2 + c_2 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^4 + \mathcal{O}\left(|\varphi_A|^6\right)$$

$$c_1 \approx 0.87$$

$$c_2 \approx 0.21$$

Cases:

- Exponential decrease of N:

$$\Gamma = const$$

- Serial condensation: $\mathcal{N}(t) = \mathcal{N}_0(1 - t/\tilde{t})$

$$\Gamma = 1/(\tilde{t} - t)$$

- Condensation all at once (unlikely):

$$\Gamma = 0$$

Discuss these cases only, assuming negligible slow roll contributions, that is assuming that the fields are very close to the origin – this is actually the best motivated case.

(for inclusion of slow roll contributions and application to different setups, see arxiv:0806.1953)

Application: inflation from tachyons

1. Exponential condensation: $\Gamma = const$

$$\mathcal{N}(t) = \mathcal{N}_0 e^{-\Gamma t}$$

We need around N=60 e-folds of inflation: $N = \int_{t_{ini}}^{t_{end}} H\, dt$

Inflation ends once $\mathcal{N} \sim 1$

so that $$N \approx \frac{2}{\Gamma}\left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}\left(1 - \frac{1}{\sqrt{\mathcal{N}_0}}\right) \approx \frac{2}{\Gamma}\left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}$$

Further $$\varepsilon_{\mathcal{N}} = \frac{\Gamma}{2}\left(\frac{3}{\tau_p \mathcal{N}_0}\right)^{1/2} \approx \frac{1}{N}$$

Applying the general formula for the scalar spectral index:

$$\boxed{n_s - 1 \simeq -3\varepsilon_{\mathcal{N}} \approx -\frac{3}{N}}$$

⇦ Within 1 sigma of WMAP5.

If we set $\tau_p \equiv c_1^2/(4c_2) \approx 0.90$ we need $\frac{\mathcal{N}_0}{\Gamma^2} \approx \frac{3N^2 c_2}{c_1^2} \approx 3000$

Application: inflation from tachyons

2. serial condensation: $\Gamma = (\tilde{t} - t)^{-1}$

$$\mathcal{N}(t) = \mathcal{N}_0 \left(1 - \frac{t}{\tilde{t}}\right)$$

We need around 60 e-folds of inflation: $N = \int_{t_{ini}}^{t_{end}} H \, dt$

Inflation ends once $\mathcal{N} \sim 1$

so that $N \approx \frac{2\tilde{t}}{3} \left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2} \left(1 - \frac{1}{\mathcal{N}_0^3}\right) \approx \frac{2\tilde{t}}{3} \left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}$

Further $\varepsilon_{\mathcal{N}} \approx 1/(3N)$

$$\delta \simeq -2\varepsilon_{\mathcal{N}}/\hat{\varepsilon} \simeq -2$$

Applying the general formula for the scalar spectral index:

$$\boxed{n_s - 1 \simeq -5\varepsilon_{\mathcal{N}} \approx -\frac{5}{3N}}$$

⬅ Close to 1 sigma boundary of WMAP5.

If we set $\tau_p \equiv c_1^2/(4c_2) \approx 0.90$ we need $\mathcal{N}_0 \tilde{t}^2 \approx \frac{27 N^2 c_2}{c_1^2} \approx 27000$

Conclusions

Staggered inflation occurs naturally in several multi-field models within string theory.

We developed the formalism to compute effects on some observables (scalar spectral index, running).

Many possible follow up projects

- relax the simplifying assumptions (initial conditions, type of potential, …)
- compute tensor modes, Non-Gaussianities, …
- revisit other existing models
- compare with full numerical studies
- construct new models based on staggered inflation effect (remark: hilltop potentials are feasible as long as rolling of the hill is possible classically).

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