# Reheating of the universe after inflation with $f(\phi)R$ gravity

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***Based on***

***PRD 75, 061301 (2007), [arXiv:qr-qc/0612120]***
***PRD 77, 043514 (2008), [arXiv:0711.3442]***

# Why Study Reheating?

- The universe was left cold and empty after inflation.
  - But, we need a hot Big Bang cosmology.
- **The universe must reheat after inflation.**

Successful inflation must transfer energy in inflaton to radiation, and heat the universe to at least ~1 MeV for successful nucleosynthesis.

…however, little is known about this important epoch….

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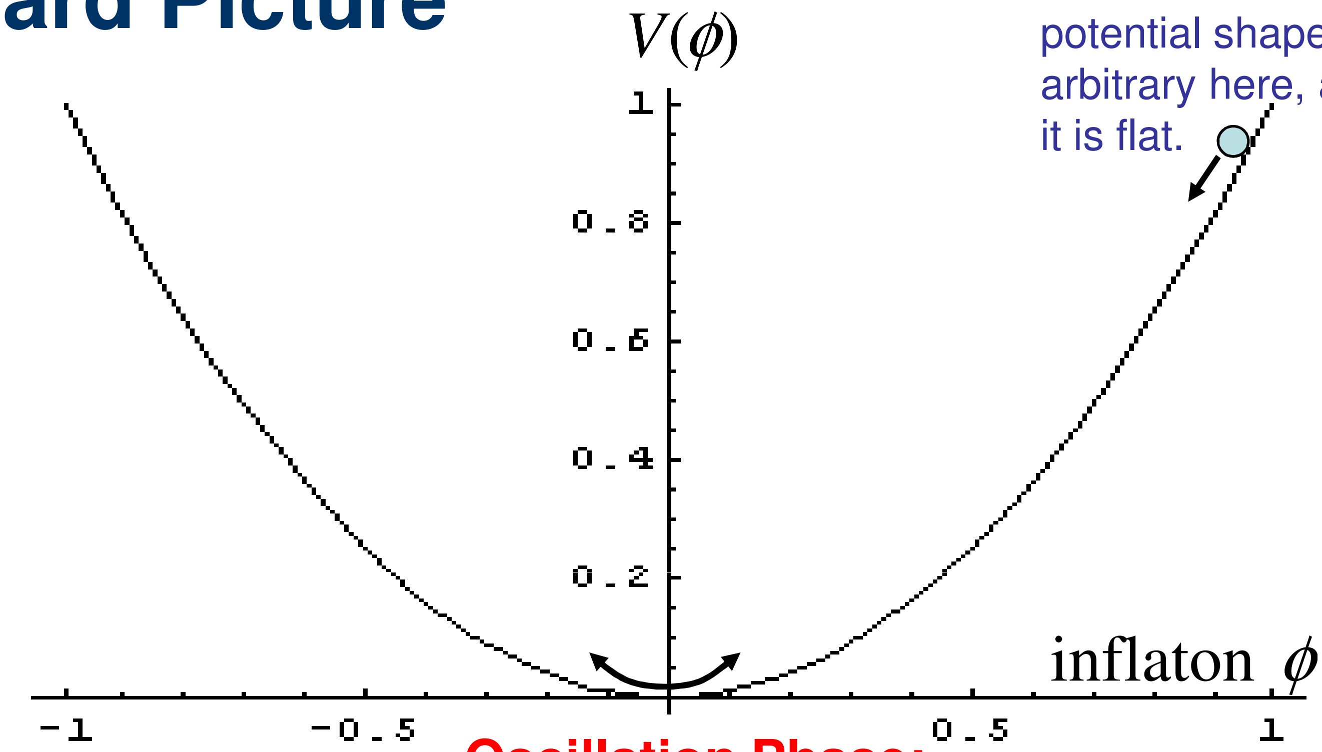
***Outstanding Questions***

- Can one reheat universe successfully/naturally?
- How much do we know about reheating?
- What can we learn from observations (if possible at all)?
- Can we use reheating to constrain inflationary models?
- Can we use inflation to constrain reheating mechanism?

# Standard Picture

**Slow-roll Inflation:**
potential shape is arbitrary here, as long as it is flat.




**Oscillation Phase:**
around the potential minimum at the end of inflation

## Energetics:

$$\rho_{rad} \sim T_{rh}^4$$

$$\sim g^4 V_{\inf}(\phi) \sim g^4 M_{Pl}^2 H_{\inf}^2$$

$$\Rightarrow T_{rh} \sim \sqrt{g^2 M_{Pl} H_{\inf}}$$

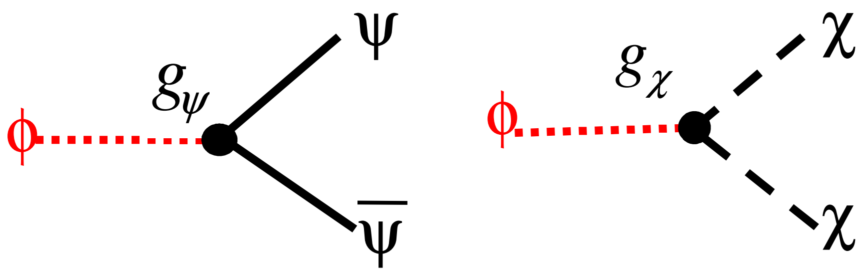
**What determines *"energy-conversion efficiency factor", g?***

# Perturbative Reheating

**Dolgov & Linde (1982); Abbott, Farhi & Wise (1982); Albrecht et al. (1982)**

Inflaton decays and thermalizes through the tree-level interactions like:

$$L_{\text{int}} = -\left( g_\psi \phi \bar{\psi} \psi + g_\chi \phi \chi^2 + \frac{1}{4} \lambda \phi^4 + \cdots \right)$$




Inflaton can decay if allowed kinematically with the widths given by

$$\Gamma_{\phi\bar{\psi}\psi} = \frac{g_\psi^2 m_\phi}{8\pi} \left(1 - \frac{4m_\psi^2}{m_\phi^2}\right)^{3/2} \tanh\left(\frac{m_\phi}{4T}\right) C_\psi$$

$$\Gamma_{\phi\chi\chi} = \frac{g_\chi^2}{8\pi m_\phi} \left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^{1/2} \coth\left(\frac{m_\phi}{4T}\right) C_\chi,$$

Pauli blocking

Bose condensate

Thermal medium effect

## Reheating Temperature from Energetics

$$\ddot{\phi} + (3H + \Gamma_{tot})\dot{\phi} + m_\phi^2 \phi = 0 \qquad H_{\inf} \gg H_{osc} \propto a^{-3/2}$$

$3H_{osc} > \Gamma_{tot} \Rightarrow$ Inflaton dominates the energy density.

$3H_{osc} < \Gamma_{tot} \Rightarrow$ Decay products dominate the energy density.

$$\rho_{rad}(t_{rh}) = 3M_{Pl}^2 H_{osc}^2 = \frac{M_{Pl}^2 \Gamma_{tot}^2}{3} = \frac{\pi^2}{30} g_*(T_{rh}) T_{rh}^4$$

$$T_{rh} = \frac{\sqrt{M_{Pl}\Gamma_{tot}}}{(10\pi^2)^{1/4}} \left( \frac{g_*(T_{rh})}{100} \right)^{-1/4}$$

Coupling constants determine the decay width, $\Gamma$.

**But, what determines coupling constants?**

## What are coupling constants?
## Problem: arbitrariness of the nature of inflaton fields

- Inflation works very well as a concept, but we do not understand the nature (including interaction properties) of inflaton.

  e.g. Higgs-like scalar fields, Axion-like fields, Flat directions, RH sneutrino, Moduli fields, Distances between branes, and many more…

- Arbitrariness of inflaton = Arbitrariness of couplings
- Can we say anything generic about reheating? Universal reheating? **Universal coupling?**

*Gravitational coupling is universal*

➢ too weak to cause reheating with GR.

In the early universe, however, GR would be modified.

⟹ **What happens to "gravitational decay channel", when GR is modified?**

# Conventional Einstein gravity during inflation

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right] + \sqrt{-g}L_{matt}$$

Einstein-Hilbert term generates GR.
Inflaton minimally couples to gravity.

$$L_{\rm int} = -\left(g_\psi \phi \bar{\psi}\psi + g_\chi \phi \chi^2 + \lambda \phi^2 \chi^2 + \cdots\right)$$

Conventionally one introduced explicit couplings between inflaton and matter.

## Modifying Einstein gravity during inflation

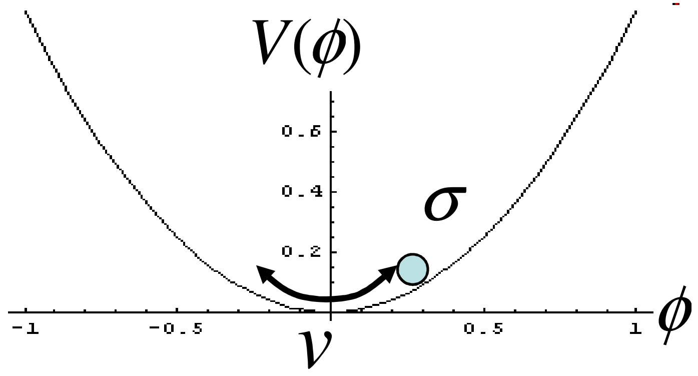
Instead of introducing explicit coupling by hand,

$$L_{\rm int} = -\left(g_\psi \phi \bar{\psi}\psi + g_\chi \phi \chi^2 + \lambda \phi^2 \chi^2 + \cdots\right)$$

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2} f(\phi) R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi)\right] + \sqrt{-g} L_{matt}$$

Non-minimal gravitational coupling: common in effective Lagrangian from extra dimensional theories

In order to ensure GR after inflation, $f(v) = M_{Pl}^2$




Matter (everything but gravity and inflaton) completely decouples from inflaton and minimally coupled to gravity as usual.

# New decay channel through "scalar gravity waves"

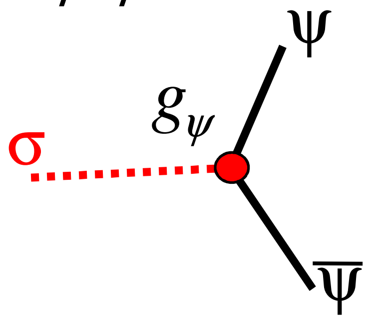
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu} - \frac{F(v)}{M_{Pl}^2}\bar{g}_{\mu\nu}\sigma, \qquad F(v) = f'(v)\left(1 + \frac{3[f'(v)]^2}{2M_{Pl}^2}\right)^{-1/2}$$

## Fermionic (spinor) matter field:

Yukawa interaction

$$L_\psi = -\bar{e}\,\bar{\psi}[\bar{e}^{\mu\alpha}\gamma_\alpha D_\mu + m_\psi]\psi + \cdots + \bar{e}\frac{F(v)m_\psi}{2M_{Pl}^2}\sigma\bar{\psi}\psi$$

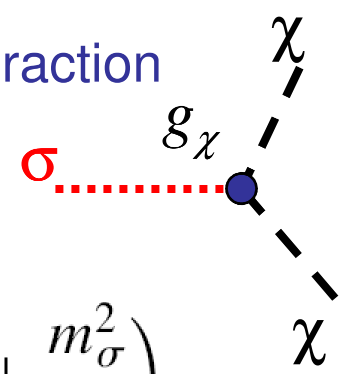
$$\frac{F(v)m_\psi}{2M_{Pl}^2} = g_\psi$$




## Bosonic (scalar) matter field:

$$L_\chi = -\bar{e}\left[\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + U(\chi) + \cdots - \frac{F(v)}{2M_{Pl}^2}\sigma\left(4U(\chi) + \bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right)\right]$$

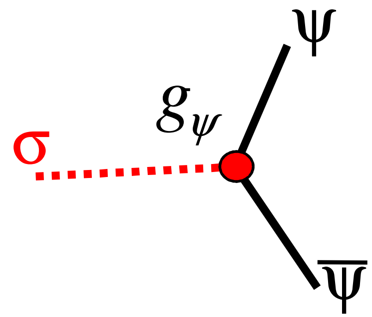
Trilinear interaction




$$U(\chi) = m_\chi^2\chi^2/2$$

$$g_\chi \equiv \frac{F(v)}{2M_{\rm Pl}^2}\left(m_\chi^2 + \frac{m_\sigma^2}{2}\right)$$

## Magnitude of Yukawa coupling




$$g_\psi \equiv \frac{F(v)\, m_\psi}{2M_{\rm Pl}^2}$$

- For $f(\phi) = \xi\phi^2$, $g_\psi = \xi(1+6\xi)^{-1/2}(v/M_{pl})(m_\psi/M_{pl})$
  - Natural to obtain a small Yukawa coupling, $g_\psi \sim 10^{-7}$ , for e.g., $m_\psi \sim 10^{-7} M_{pl}$
- The induced Yukawa coupling vanishes for massless fermions: conformal invariance of massless fermions.

- Massless, minimally-coupled scalar fields are not conformally invariant. Therefore, the three-legged interaction does not vanish even for massless scalar fields:

$$g_\chi \equiv \frac{F(v)}{2M_{\rm Pl}^2}\left(\cancel{m_\chi^2} + \frac{m_\sigma^2}{2}\right)$$

# Decay Width Summary

(Kinematical and thermal factors are neglected.)

## Fermions

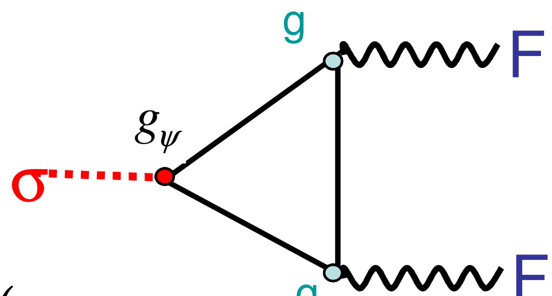
$$\Gamma = \frac{[f'(v)]^2 m_\psi^2 m_\sigma}{32\pi\ M_{pl}^4}\left(1+\frac{3[f'(v)]^2}{2M_{pl}^2}\right)^{-1}$$

## Scalar Bosons

$$\Gamma = \frac{[f'(v)]^2 m_\sigma^3}{128\pi\ M_{pl}^4}\left(1+\frac{3[f'(v)]^2}{2M_{pl}^2}\right)^{-1}$$

← Probably the most dominant decay channel

## Gauge Bosons




$$\Gamma = \frac{\alpha^2[f'(v)]^2 m_\sigma^3}{256\pi^3 M_{pl}^4}\left(1+\frac{3[f'(v)]^2}{2M_{pl}^2}\right)^{-1} \times (\#\ \text{of internal fermions etc})$$

# Constraint on f(ϕ)R gravity from reheating

$$T_{rh} = \frac{\sqrt{M_{Pl}\Gamma_{tot}}}{(10\pi^2)^{1/4}} \left( \frac{g_*(T_{rh})}{100} \right)^{-1/4}$$

$$\Gamma_{\rm tot} > \Gamma_{\sigma\bar{\psi}\psi} + \Gamma_{\sigma\chi\chi} + \Gamma_{\sigma FF}$$

$$|f'(v)| \left( 1 + \frac{3[f'(v)]^2}{M_{pl}^2} \right)^{-1/2} < 8\pi (40)^{1/4} T_{rh} \left( \frac{M_{pl}}{m_\sigma} \right)^{3/2} \left( \frac{g_*(T_{rh})}{100} \right)^{1/4}$$

e.g. $f(\phi) = M^2 + \xi\phi^2$

$$|\xi| \left( 1 + \frac{6\xi^2 v^2}{M_{pl}^2} \right)^{-1/2} < 4\pi (40)^{1/4} \frac{T_{rh}}{v} \left( \frac{M_{pl}}{m_\sigma} \right)^{3/2} \left( \frac{g_*(T_{rh})}{100} \right)^{1/4}$$

(c.f.) Constraints from chaotic inflation

$\xi > -10^{-3}$

Futamase & Maeda(1989)

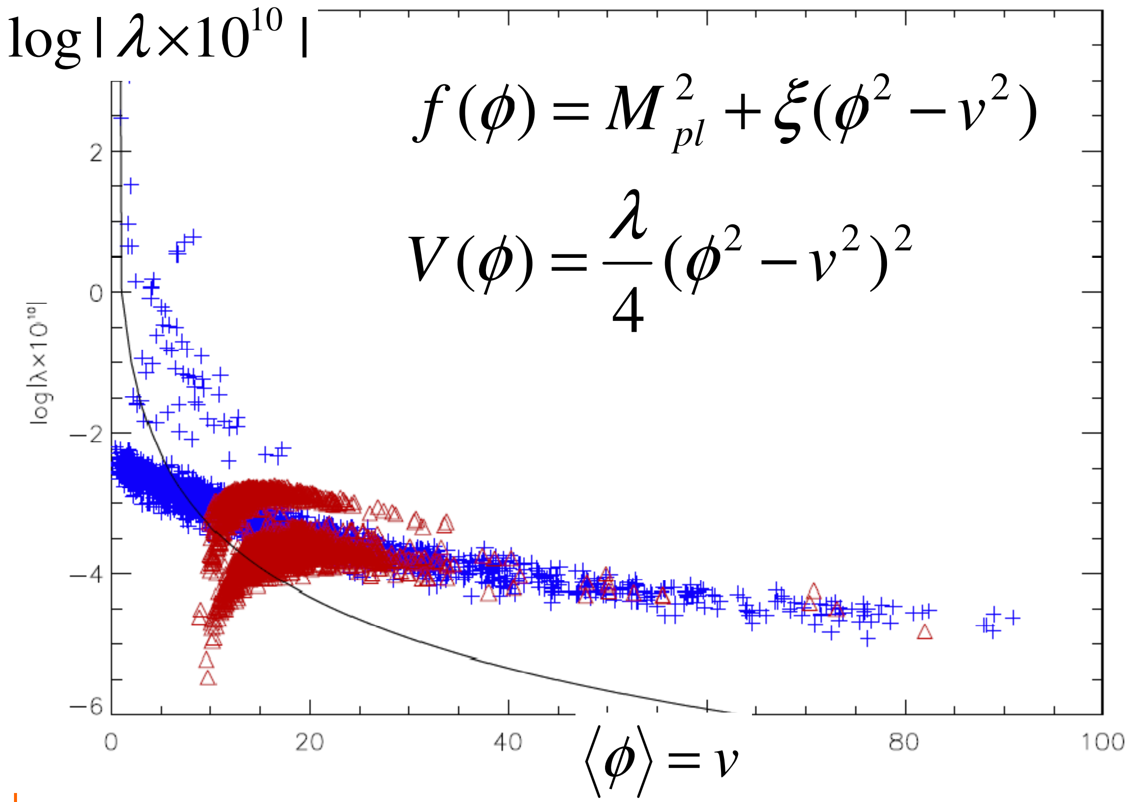
$|\xi| > 5 \times 10^4 \sqrt{\lambda}$

Komatsu & Futamase(1999)

# Worked Example: WMAP 3-yr data with Ginzburg-Landau potential

$T_{rh} < 0.1 M_{Pl}$ excludes the region above the black line if $\xi \sim O(1)$:

$$\lambda < \left(\frac{T_{rh}}{0.1 M_{Pl}}\right)^{4/3} \left(\frac{v}{M_{Pl}}\right)^{-10/3} |\xi|^{-4/3} \left(1 + 6\xi^2 \frac{v^2}{M_{pl}^2}\right)^{2/3} \left(\frac{g_*}{100}\right)^{1/3}$$




The theoretical parameter space allowed by the WMAP 3-yr data.
Each point (red: large-field i.c., blue: small-field i.c.) shows a solution of slow-roll equations given a point of data ($r$, $n_s$, $\Delta_R^2$).

# Conclusions

- A natural mechanism for reheating after inflation with $f(\phi)R$ gravity: *Why natural?*
  - Inflaton quanta decay spontaneously into any matter fields (spin-0, ½, 1) without explicit interactions in the original Lagrangian
  - Conformal invariance must be broken at the tree-level or by loops
  - Reheating spontaneously occurs in any theories with $f(\phi)R$ gravity

- A constraint on $f(\phi)$ from the reheat temperature can be found.
  - A possible limit on the reheat temperature can constrain the form of $f(\phi)$, or vice versa.
  - These constraints on $f(\phi)$ are totally independent of the other constraints from inflation and density fluctuations.

Further study in progress…

- Preheating? $F(\phi,X,R)$ gravity?