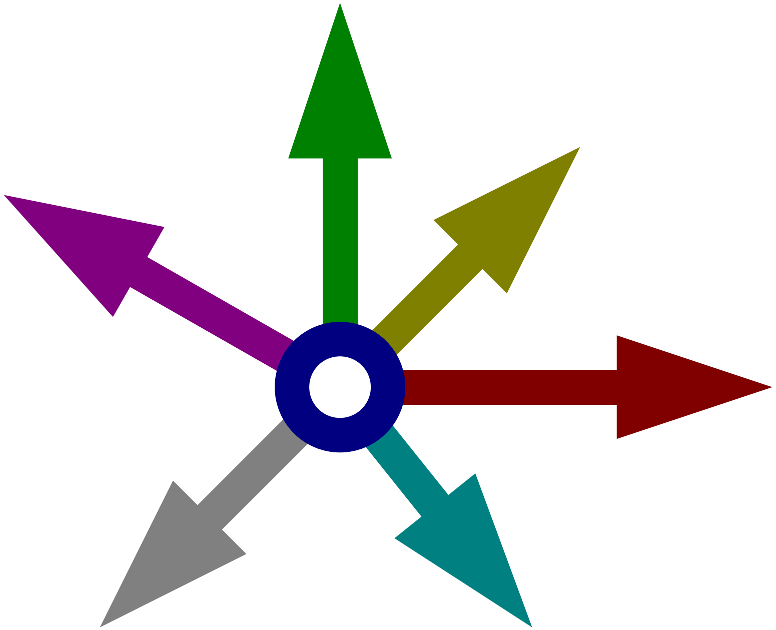
Vector Inflation



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All of the successful models of inflation use scalar fields:

- potential energy (new inflation, chaotic inflation, etc.)

$$S=\int dx^4\sqrt{-g}\left(-\frac{R}{16\pi}+\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi-V(\varphi)\right)$$

- kinetic energy (k-inflation, k-essence, DBI-inflation, etc.)

$$S=\int dx^4\sqrt{-g}\left(-\frac{R}{16\pi}+P(\nabla_\mu\varphi\nabla^\mu\varphi)\right)$$

But, scalar fields where never observed! (not yet)

Can higher spin fields drive de-Sitter expansion? Vectors?

Necessary requirements for a field to drive inflation:

A) condensate (YES)
B) isotropy (NO)
C) slow roll regime (NO) These problems were realized by Ford in 1989

B) can be resolved by a triplet of orthogonal fields [Armendariz-Picon, 2004]
C) can be resolved by fine-tuning potential and IC [Ford, 1989]

Consider a vector field non-minimally coupled to gravity:

$$S=\int dx^4\sqrt{-g}\left(-\frac{R}{16\pi}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\left(m^2+\frac{R}{6}\right)A_\mu A^\mu\right)$$

- non-minimal coupling is exactly the same as for conformal scalar
- conformal symmetry of vector field is broken by this coupling
- vector fields starts to behave as a minimally coupled scalar

Note: the slow-roll problem of a minimally coupled vector fields has the same root as the problem of slow-roll with conformal scalars

Field equations:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\sqrt{-g}\,F^{\mu\nu}+\frac{1}{2}\left(m^2+\frac{R}{6}\right)A^\mu=0$$

In Friedman universe:

$$-\frac{1}{a^2}\nabla^2 A_0+\left(m^2+\frac{R}{6}\right)A_0+\frac{1}{a^2}\partial_i\dot{A}_i=0$$

$$\ddot{A}_i+\frac{\dot{a}}{a}\dot{A}_i-\frac{1}{a^2}\nabla A_i+\left(m^2+\frac{R}{6}\right)A_i-\partial_i\dot{A}_0-\frac{\dot{a}}{a}\partial_i A_0+\frac{1}{a^2}\partial_i\partial_k A_k=0$$

Define:

$$B_i \equiv \frac{A_i}{a} = a\,A^i$$

For quasi homogeneous vector field background equations are

$$A_0 = 0 \qquad \ddot{B}_i + 3\,H\,\dot{B}_i + m^2\,B_i = 0$$

Similar to the scalar field case, when Hubble constant H > m the fields are "frozen" and quasi de Sitter expansion is possible.

Energy momentum tensor:

$$T^\alpha_\beta = \frac{1}{4} F^{\gamma\delta} F_{\gamma\delta}\,\delta^\alpha_\beta - F^{\alpha\gamma} F_{\beta\gamma} + \left(m^2 + \frac{R}{6}\right) A^\alpha A_\beta - \frac{1}{2} m^2 A^\gamma A_\gamma\,\delta^\alpha_\beta$$

$$+ \frac{1}{6}\left(R^\alpha_\beta - \frac{1}{2}\delta^\alpha_\beta R\right) A^\gamma A_\gamma + \frac{1}{6}\left(\delta^\alpha_\beta \nabla^\delta \nabla_\delta - \nabla^\alpha \nabla_\beta\right) A^\gamma A_\gamma$$

For a homogeneous vector field in a flat Friedman universe:

$$T^0_0=\frac{1}{2}\left(\dot{B}_k^2+m^2 B_k^2\right)$$

$$T^i_j=\left[\frac{-5}{6}\left(\dot{B}_k^2-m^2 B_k^2\right)-\frac{2}{3}H\dot{B}_k B_k-\frac{1}{3}\left(\dot{H}+3H^2\right)B_k^2\right]\delta^i_j$$

$$+\dot{B}_i\dot{B}_j+H\left(\dot{B}_i B_j+\dot{B}_j B_i\right)+\left(\dot{H}+3H^2-m^2\right)B_i B_j$$

For a vector triad (three mutually orthogonal vector fields)

$$T^0_0=\varepsilon=\frac{3}{2}\left(\dot{B}_k^2+m^2 B_k^2\right) \qquad T^i_j=-p\,\delta^i_j=-\frac{3}{2}\left(\dot{B}_k^2-m^2 B_k^2\right)\delta^i_j$$

and the components of any field from the triplet satisfy

$\ddot{B}_i+3H\dot{B}_i+m^2 B_i=0$ where $H^2=4\pi\left(\dot{B}_k^2+m^2 B_k^2\right)$

As for the scalar field $\varepsilon\approx-p$ in the slow roll regime $B>1$

Another way to achieve isotropy is to consider a large number N of randomly oriented vector fields. For simplicity take all masses to be the same.

It follows that

$$T^0_0 = \varepsilon \approx \frac{N}{2}\left(\dot{B}_k^2 + m^2 B_k^2\right)$$

and for the spatial components of energy-momentum tensor we note

$$\sum_{a=1}^{N} B_i^{(a)} B_j^{(a)} \approx \frac{N}{3} B_k^2 \delta^i_j + O(1)\sqrt{N}\, B_k^2$$

where summation over k is assumed.

Corrections of order $\sqrt{N}$ are due to random distribution of directions, which do not vanish for $i \neq j$.

Typical value of the off-diagonal spatial components is

$$T^i_j \sim H^2 \sqrt{N}\, B^2$$

Isotropic inflationary solution is self-consistent only if

$$T^i_j < T^i_i \sim T^0_0 \sim H^2$$

Hence, the quasi de Sitter solution is valid only if:

$$B < N^{-\frac{1}{4}}$$

On the other hand the vector fields are in the slow-roll only if the effective friction exceeds their mass, and the inflation is over when the Hubble constant is of the order of the mass:

$$H^2 > \frac{8\pi}{3}\varepsilon \approx \frac{4\pi}{3} N m^2 B^2$$

Therefore

$$B > N^{-\frac{1}{2}}$$

When the field drops below this value its starts to oscillate.
The number of e-foldings can be estimated as

$$\frac{a_f}{a_i} \approx \exp\left(2\pi N B_{init}^2\right)$$

The maximum number of e-foldings of isotropic inflation is $2\pi\sqrt{N}$

One hundred vector field is enough to explain observed homogeneity, but global anisotropy of order $\frac{1}{\sqrt{N}}$ can survive until the end of inflation

Conclusions

- Vector inflation is possible!

- Slow-roll condition is satisfied for non-minimally coupled vector fields.

- Isotropicity can be achieved by:
  a) three mutually orthogonal vector fields
  b) symmetric configurations with four or more fields
  c) a large number of randomly oriented vector fields

- Anisotropicity of the order of $\frac{1}{\sqrt{N}}$ remains until after the end of inflation.

- The lightest vector fields might also force the late time acceleration of the Universe.