Leading Log Approximation for Inflationary QFT

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Spacetime Exp. Strengthens QFT

- Why?
  - Loops → classical physics of virtuals
  - Expansion → holds virtuals apart longer
- Maximum Effect for:
  - Inflation
  - Massless virtuals
  - NOT conformally invariant
- Two Particles
  - MMC scalars
  - Gravitons

Ehancement Manifests as "Infrared Logarithms"

- $ds^2 = -dt^2 + a^2\, dx \cdot dx$ with $a(t)=e^{Ht}$
- Eg. $\langle T_{\mu\nu} \rangle$ for MMC $\varphi + \lambda\varphi^4$
  - $\rho = \lambda(H/2\pi)^4\, [\tfrac{1}{8} \ln^2(a)] + O(\lambda^2)$
  - $p = -\lambda(H/2\pi)^4\, [\tfrac{1}{8} \ln^2(a) + {}^1/_{12} \ln(a)] + O(\lambda^2)$

Many Theories Show IR Logs

1. MMC $\varphi + \lambda\varphi^4$
2. MMC φ + Fermions (Yukawa)
3. MMC φ + EM (SQED)
4. Pure QG
5. QG + Fermions
6. QG + Scalars

IR Logs Are Physical

1. $<T_{\mu\nu}>$ with non-dynamical gravity
2. $<F_{\mu\nu}\, F_{\rho\sigma}>$ in SQED
3. QG corrections to P(k)
   - S. Weinberg: hep-th/0506236, 0605244
   - K. Chaicherdsakul: hep-th/0611352

General Form of Series

- Consider Interaction Vertex with:
  - Coupling constant K
  - N undiff. MMC φ and $h_{\mu\nu}$
  - Any other fields or diff. fields
- Each K^2 → up to N factors of ln(a)
  - $\lambda\varphi^4$ → series in $\lambda \ln^2(a)$
  - $e\varphi^*\partial\varphi A_\mu$ → series in $e^2 \ln(a)$
  - $\kappa h\partial h\partial h$ → series in $GH^2 \ln(a)$

Same for Counterterms

- $\lambda\varphi^4 \rightarrow \lambda \ln^2(a)$
  - Eg $\delta\xi\varphi^2 \rightarrow \delta\xi \ln(a) = [\#\lambda + \#\lambda^2 + \ldots] \ln(a)$
- $e^2\varphi^*\varphi A_\mu A_\nu g^{\mu\nu} \rightarrow e^2 \ln(a)$
  - $\delta\xi\varphi^*\varphi \rightarrow \delta\xi \ln(a) = [\#e^2 + \#e^4 + \ldots] \ln(a)$
- $\kappa h\partial h\partial h \rightarrow GH^2 \ln(a)$
  - $\delta\Lambda/\kappa^2 (-g)^{1/2} = \delta\Lambda/\kappa^2 [1 + \kappa h + \kappa^2 h^2 + \ldots]$

    $\rightarrow (\delta\Lambda/\kappa^2 H^4)\, GH^2 \ln(a)$

    $= [\#GH^2 + \#(GH^2)^2 + \ldots] \ln(a)$

IR Logs Are Fascinating!

1. Introduce time dependence
   - $\langle T_{\mu\nu} \rangle \neq \text{const} \times g_{\mu\nu}$
2. Compensate for small coupling const.
   - $GH^2 < 10^{-12}$ for primordial inflation
   - But ln(a) can get BIG!

Perturbative Conundrum

- Pert. Theory breaks down!
  - 1 Loop QG: GH^2 [ln(a) + 1]
  - 2 Loop QG: $(GH^2)^2$ [$\ln^2$(a) + ln(a) + 1]
  - When GH^2 ln(a) ~ 1 → all loops order one
- What to do? → Sum the leading logs!
  - Cf. Renormalization Group in QFT
  - Subdominant terms perturbatively small

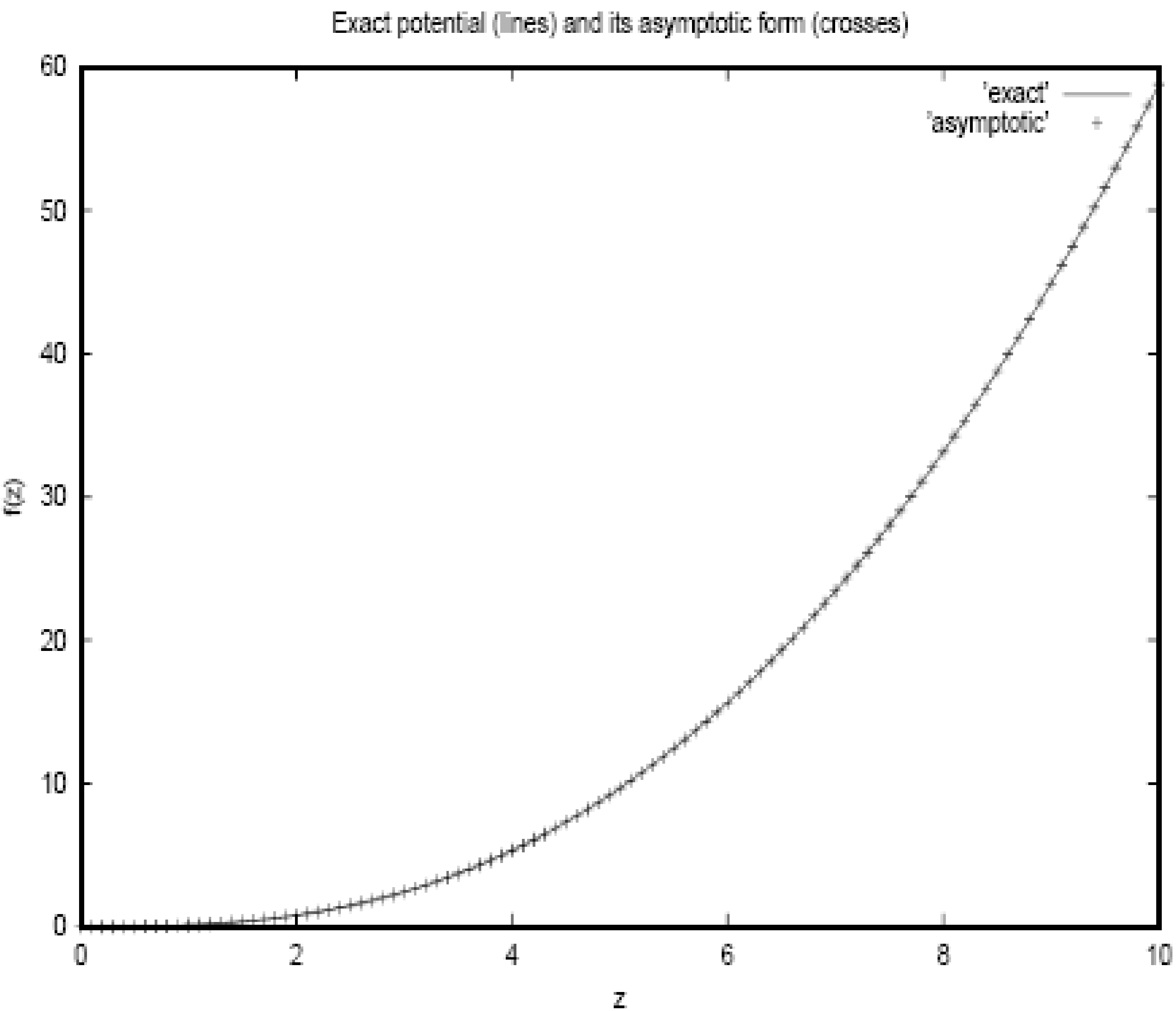
Summing the Leading IR Logs

- Method: LL QFT → simple stochastic FT
  - Starobinsky 1986
- MMC φ + V(φ)
  - Starobinsky & Yokoyama astro-ph/9407016
  - All orders proof with Tsamis gr-qc/0505115
- MMC φ + passives
  - Yukawa, with Miao, gr-qc/0602110
  - SQED, with Prokopec & Tsamis, 0707.0847
- No general result yet for derivative int's

Physics of Leading Log SQED

- Inflation rips virtual scalars from vac.
- Hence $\varphi^*\varphi$ grows
- $e^2\varphi^*\varphi A_\mu A_\nu$ induces photon mass
- γ vac. energy induces V_{eff} for scalar
- $V_{eff} = 3H^4/(8\pi^2)\, f(z)$ for $z=e^2\varphi^*\varphi/H^2$

$$V_{\rm eff} = \frac{3H^4}{8\pi^2}\Bigg\{(-1+2\gamma)z + \left(-\frac{3}{2}+\gamma\right)z^2 + \int_0^z dx\,(1+x)\left[\psi\left(\frac{3}{2}+\frac{1}{2}\sqrt{1-8x}\right)+\psi\left(\frac{3}{2}-\frac{1}{2}\sqrt{1-8x}\right)\right]\Bigg\}$$



Exact potential (lines) and its asymptotic form (crosses)
'exact'
'asymptotic'
f(z)
z
0
10
20
30
40
50
60
0
2
4
6
8
10

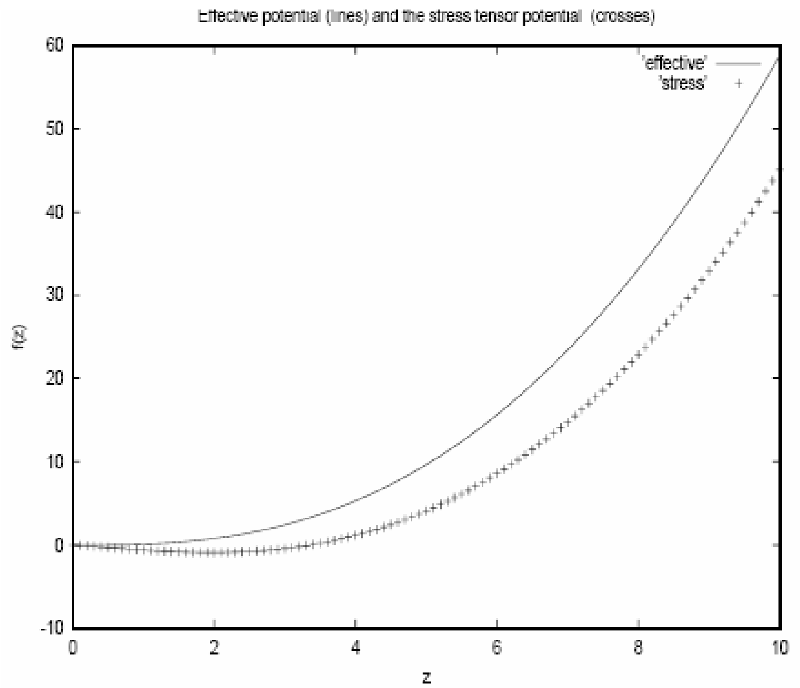
Nonpert. Results for SQED

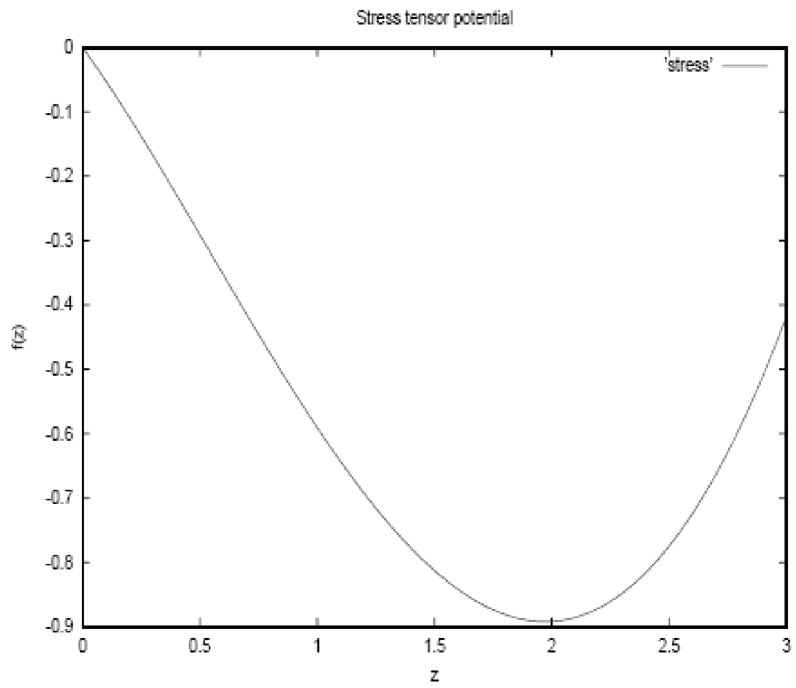
Operator	Expectation Value
$\varphi^*\varphi$	$1.6495 \times H^2/e^2$
$(\varphi^*\varphi)^2$	$3.3213 \times H^4/e^4$
$(\varphi^*\varphi)^3$	$7.6308 \times H^6/e^6$
$M_\gamma^2 \equiv 2e^2\varphi^*\varphi$	$3.2991 \times H^2$
$M_\varphi^2 \equiv V'_{\rm eff}(\varphi^*\varphi)$	$.8961 \times 3e^2H^2/8\pi^2$
$V_{\rm eff}(\varphi^*\varphi)$	$.7223 \times 3H^4/8\pi^2$
$V_s(\varphi^*\varphi)$	$-.6551 \times 3H^4/8\pi^2$
$(F_{\mu\nu}F_{\rho\sigma})_{\rm fin}$	$-9.5246 \times H^4/8\pi^2\ (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$

Curious Case of $<T_{\mu\nu}>$

- At Lead. Log: $<T_{\mu\nu}> = -g_{\mu\nu}<V_s(\varphi^*\varphi)>$
- But $V_s \neq V_{eff}$!
- Why not?
  - $V_{eff} = 3H^4/(8\pi^2)\ f(z)$ for $z = e^2\varphi^*\varphi/H^2$
  - Most factors of $H^2 \rightarrow {}^1/_{12}\ R$ for gen. $g_{\mu\nu}$
  - Varying $-V_{eff}\,(-g)^{1/2}$ wrt $g_{\mu\nu}$ gives V_s
- So SQED gives a precise f(R) model!

$V_s \neq V_{eff}$ for SQED





Conclusions I

- Undiff. MMC φ's and $h_{\mu\nu}$'s cause IR logs
- They have been seen in many models
  - $\lambda\varphi^4$ → series in $\lambda \ln^2(a)$
  - SQED → series in $e^2 \ln(a)$
  - Yukawa → series in $f^2 \ln(a)$
  - Q. Grav. → series in $GH^2 \ln(a)$
- They affect physical quantities
- They introduce time dependence
- They compensate for small coupling const's

Conclusions II

- Long inflation → pert. theory breaks
- Sum leading logs for
  - MMC φ + V(φ)
  - MMC φ + A_μ and ψ
  - Not yet for quantum gravity
- Nonperturbative results for SQED
  - $\varphi^*\varphi \sim H^2/e^2$
  - $M_\gamma \sim H$ and $M_\varphi \sim e H$
  - Vacuum energy ~ $-H^4$
  - Precise f(R) model induced