Gravity and the cosmological constant as superconducting phenomena

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PENNSTATE
1855

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Based on

1. S. Alexander, G.C., *Superconducting loop quantum gravity and the cosmological constant* [0806.4382].
2. S. Alexander, G.C., *Quantum gravity as a Fermi liquid* [0807.0225].

Aims of the talk

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- To throw a rock.

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- Not to hide the hand.

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- Result 2: Gravity behaves as a Fermi liquid, in particular BCS.

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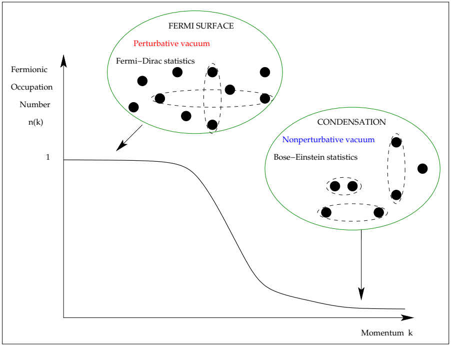

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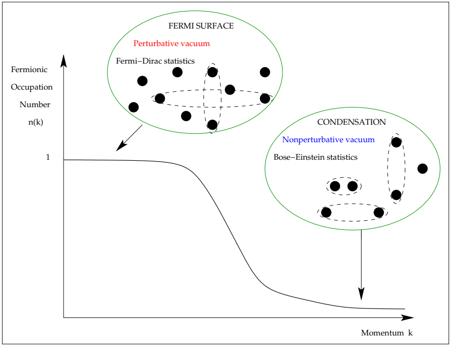

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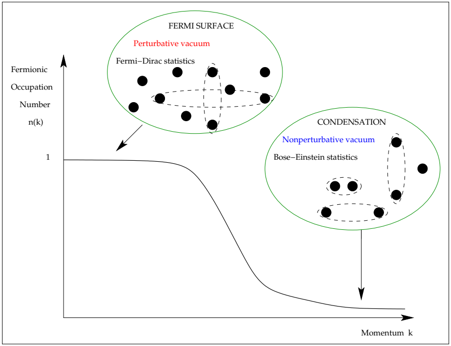

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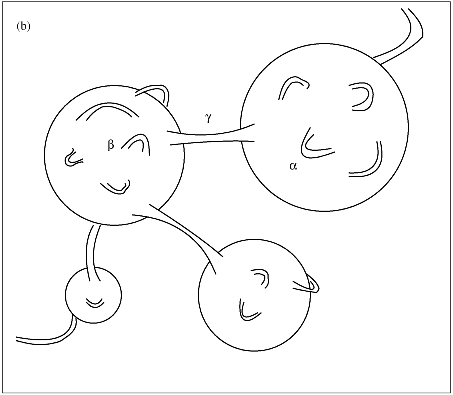
5. Matter is ‘hidden’ in gravity?

Nonlocal degrees of freedom on dS horizons



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- Different sectors of Euclidean gravity ($\theta \to i\theta$) connected by **large gauge transformations.**

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- The only sectors compatible with this step and the Gauss constraint are **degenerate**: $\det E = 0$, no metric!

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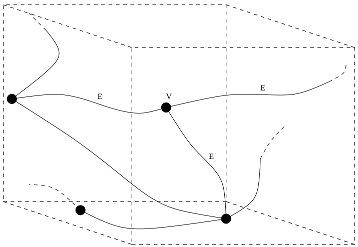

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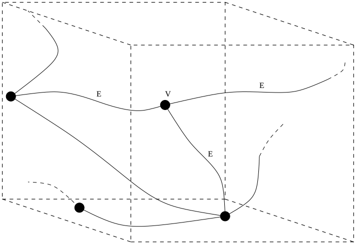

A model for V interactions and physical interpretation naturally emerge at quantum level.

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- Quantizing ψ as a Majorana fermion,

$$\mathcal{H} \propto \mathcal{H}_{\mathrm{BCS}} = \sum_{k,\sigma} \mathcal{E}_k c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_{k,k'} V_{kk'} c^{\dagger}_{k+} c^{\dagger}_{k-} c_{k'-} c_{k'+}.$$

- Correspondence made rigorous using a deformed CFT (WZW model at critical level).