Dynamical Cosmological Constant: A Supercondensate in Quantum Gravity

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I. The Problem and the Idea

- **Observations** indicate that the total energy density of the Universe has a dynamical uniform component with equation of state $\rho_\Lambda \approx -p_\Lambda$.
- Why
$$\Lambda = 8\pi\rho_\Lambda \sim 10^{-47}\,\mathrm{GeV}^4$$
is **so small**? Vacuum energy in standard QFT does not explain this fine tuning.

Proposal:

- Λ is **exponentially suppressed** as an effect of the topological deformation of a particular sector of **Loop Quantum Gravity**:
$$\boldsymbol{\Lambda \sim \mathrm{e}^{-|?|}}.$$
This mechanism, very similar to condensation of free fermions into Cooper pairs, is **nonperturbative, background-independent and parity-breaking**.
- Quantum gravity shows a novel behaviour reproducing Bardeen–Cooper–Schrieffer (**BCS**) **theory of superconductivity**.

II. The context: Loop quantum gravity

- **Classically**, a general-relativistic system can be described by **Ashtekar variables**, a connection $\mathbb{C}$-field $A \equiv A_\alpha dx^\alpha \equiv A^i_\alpha \tau_i dx^\alpha$ and a real triad E ($\alpha, \beta, \dots$ spatial components, $i, j, \dots$ directions in the internal gauge space, τ_i generators of the $su(2)$ gauge algebra). The **scalar**, **vector**, and **Gauss** gravitational constraints in the presence of a cosmological constant are
$$\mathcal{H} = \epsilon_{ijk} E^i \cdot E^j \times \left(B^k + \frac{\Lambda}{3}E^k\right) = 0,$$
$$\mathcal{V}_\alpha = (E_i \times B^i)_\alpha = 0\,, \qquad \mathcal{G}_i = D_\alpha E^\alpha_i = 0,$$
where B is the magnetic field generated by A.
- In the **quantum** theory, the triad becomes the operator $\hat{E}^\alpha_i = -\delta/\delta A^i_\alpha$, while $\hat{A}^i_\alpha$ is multiplicative in the naive connection representation. The constraints are annihilated by a kinematical state which we choose to be the **Chern–Simons state**
$$\Psi_{\rm CS} = \exp\left(\frac{i\vartheta}{8\pi^2}\int_{S^3} Y_{\rm CS}\right), \qquad \theta \equiv \frac{6\pi^2}{i\Lambda},$$
where $Y_{\rm CS}$ is the Chern–Simons form $Y_{\rm CS} \equiv \mathrm{tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)$.
- The CS ground state encodes degrees of freedom connecting families of spacetimes. In analogy with nonabelian gauge theories like QCD, different sectors of Euclidean gravity ($\theta \to i\theta$) are connected by unitary **large gauge transformations** which shift the **topological** phase θ by integer values.

III. Dynamical Λ

- We **deform the topological sector** as
$$\theta \to \theta(A),$$
thus **breaking large-gauge** $U(1)$ **invariance** (analogy with Peccei–Quinn invariance in QCD). Consequence: Λ is promoted to an evolving functional $\Lambda(A)$.
- **No matter introduced by hand!**
- The only sectors compatible with this and the Gauss constraint are **degenerate**: $\det E = 0$, **no metric!**
- At quantum level spacetime dimensionality is recovered.

IV. The world in one dimension

In **Jacobson sector** ($\mathrm{rk}E = 1$), the gravielectric field E lives on lines where the **classical** e.o.m. for A can be written as the $(1+1)$-dimensional Dirac equation $\gamma^0\dot\psi + \gamma^z\partial_z\psi = 0$, where
$$\psi \equiv \begin{pmatrix} iA^1_1 \\ A^1_2 \\ A^2_1 \\ iA^2_2 \end{pmatrix}.$$
One can take differently oriented gravitational lines and patch them together at their boundaries. The emerging **classical picture** is that of a $(1+1)$-geometry:

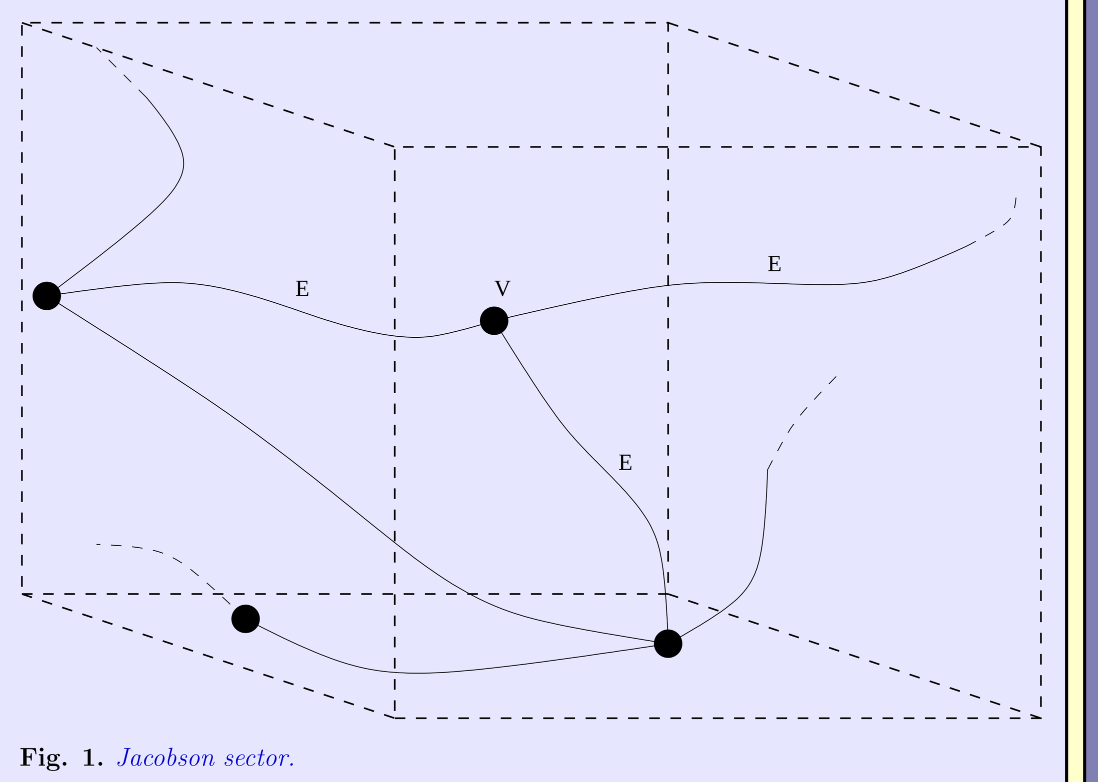

Fig. 1. *Jacobson sector.*

A model for vertex interactions and its physical interpretation **naturally emerge at the quantum level**.

V. Suppression of Λ

- The Hamiltonian requires the addition of a purely **quantum counterterm** which modifies the equation of motion for the gauge field as
$$\gamma^0\dot\psi + \gamma^z\partial_z\psi + im\psi = 0.$$
The simplest nonperturbative solution requires **an exponentially suppressed cosmological constant**
$$\boxed{\Lambda = \Lambda_0 \exp(-\bar\psi\gamma^5\gamma^z\psi)}$$
- Λ encodes the imprint of an **axial current** $j^{5\alpha}$, associated with a chiral transformation of the fermion ψ and not conserved in the presence of the mass term (nonperturbatively generated by anisotropic cross-interactions)
$$m = -2i\bar\psi\gamma^5\partial_z\psi.$$
Thus P symmetry is broken.
- **Perturbative regime** (small values of the connection): $|\langle j^{5z}\rangle| \ll 1$, $\langle\Lambda\rangle \approx \Lambda_0(1 - \langle j^{5z}\rangle) = O(1)$.
- **Nonperturbative regime** (large connection values): **Condensate** with v.e.v. $\langle j^{5z}\rangle \sim O(10^2)$, the effective mass grows important and Λ becomes exponentially small.
- **The smallness of the cosmological constant is regarded as a large-scale nonperturbative quantum mechanism** similar to quark confinement.
- After quantizing ψ as a Majorana fermion, the gravitational Hamiltonian constraint becomes proportional to the **BCS Hamiltonian** (k momentum, σ spin, $c^\dagger/c$ creation/annihilation operators)
$$\mathcal{H} \propto \mathcal{H}_{\rm BCS} = \sum_{k,\sigma}\mathcal{E}_k c^\dagger_{k\sigma}c_{k\sigma} - \sum_{k,k'} V_{kk'}c^\dagger_{k+}c^\dagger_{k-}c_{k'-}c_{k'+}.$$

VI. Gravity as a Fermi liquid

- $V_{kk'}$ describes a **nonlocal** many-body interaction which is determined by m.
- **Mean-field approximation** $V_{kk'}$ =const: fermions with opposite spin interact in N **Cooper pairs** at a given energy level $\mathcal{E}_k$, lower than the Fermi energy of the free-fermion sea, leading to a medium with superconducting properties. This is the **true vacuum of the theory**, separated from the perturbative vacuum by a **mass gap** $\Delta \sim e^{-1/V}$.
- **The gravitational analogues of the BCS wavefunction and mass gap are the Chern–Simons state and the cosmological constant**.
- The correspondence can be rigorously formalized in terms of a deformed WZW model.

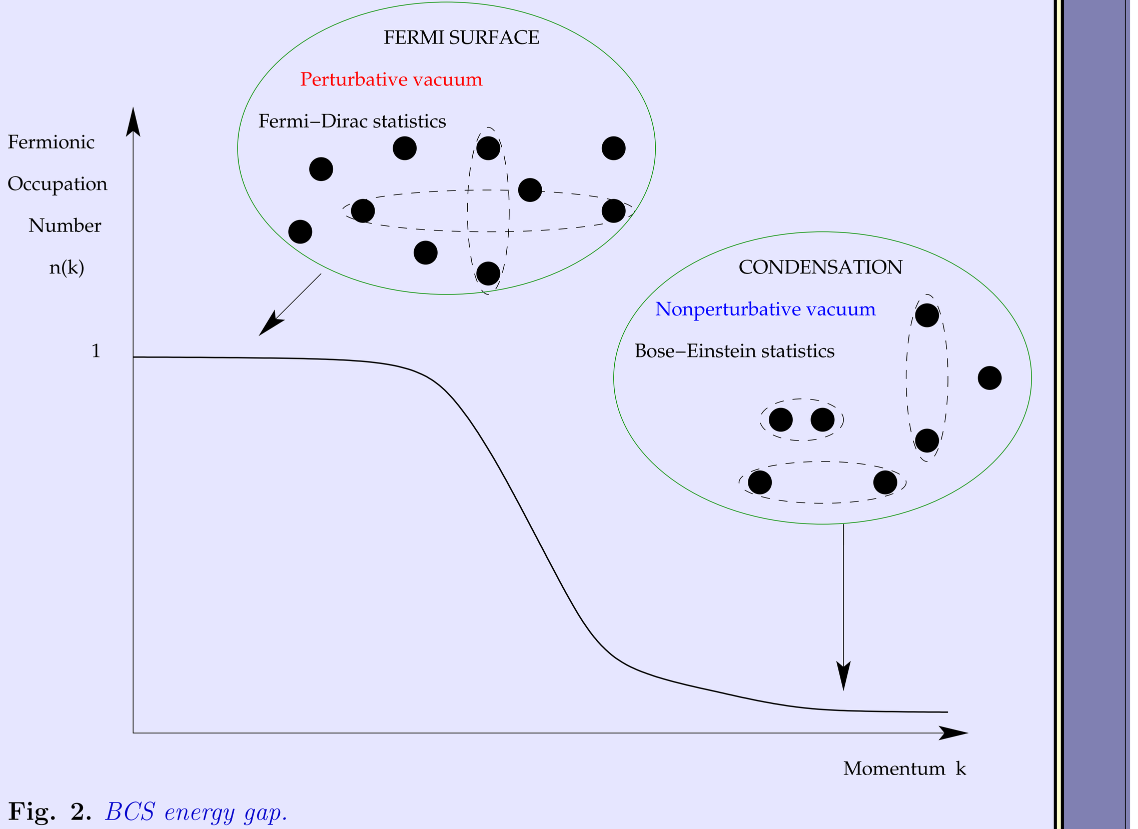

Fig. 2. *BCS energy gap.*

VII. Area measurements and dS entropy

- **Quantum area measurements correspond to counting Cooper pairs** and pair correlation can be regarded as a process of quantum decoherence.
- A spin network is the gravity counterpart of an unexcited Fermi sea, and **as soon as an area measure is performed on the state, the system relaxes to a lower-energy vacuum** corresponding to the selection of one of the area eigenstates in a wave superposition.
- Degenerate sectors can evolve to a nondegenerate metric even at the classical level; at quantum level the job is believed to be done by the embedding of a spin network.
- The entropy of the corresponding classical spacetime (de Sitter) is explained in terms of the **microscopic degrees of freedom at the boundary**, which are nothing but Cooper pairs or, in the language of gravity, oppositely charged pairs of singularities (wormholes) opening and closing on the boundary.

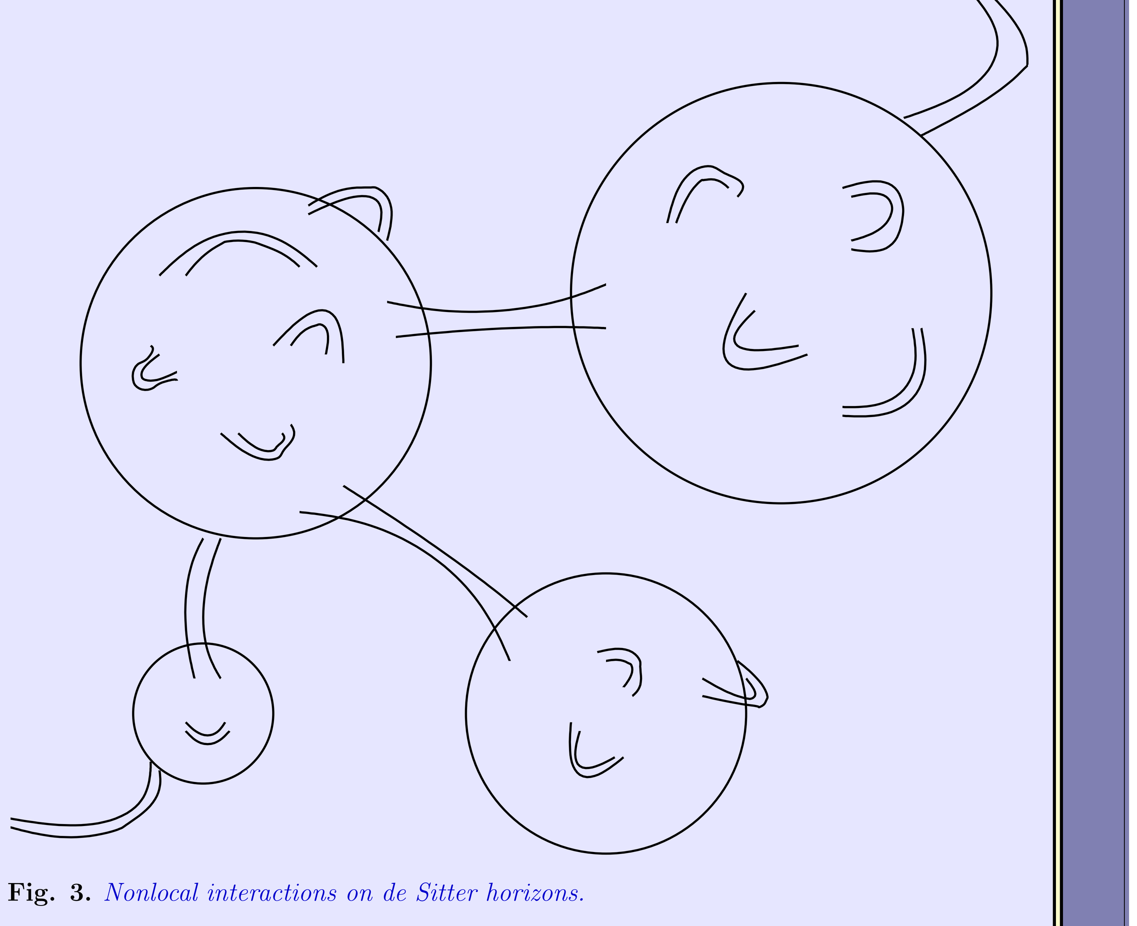

Fig. 3. *Nonlocal interactions on de Sitter horizons.*

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