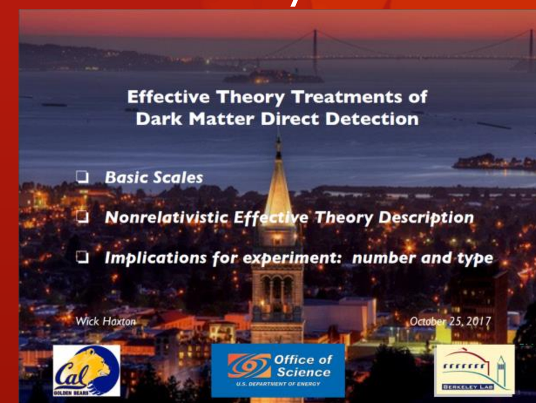
Effective Field Theory in LUX Run04

Shaun Alsum

Yes, I re-used the fall theme…
It’s a good theme…

What Is EFT?

Wick Haxton
Wednesday



http://teacher.pas.rochester.edu:8080/wiki/pub/Lux/AWGworkshop171023_Agenda/DM_LUX.pdf

For those without time machine access…

- Standard Spin Independent and Spin Dependent neglect other possible interactions
- In general, we want to account for any possible interaction that is allowed.
- There are 5 galilean and hermitian invariant quantities from which these operators can be built.
  - $\mathbf{I}$, $i\mathbf{q}$ ($i\mathbf{q}/m_N$), $\mathbf{v}^{\perp}$, $\mathbf{S}_x$, $\mathbf{S}_N$

Identity

Transferred momentum

Relative velocity ⊥ to **q**

WIMP spin

Nucleon spin

Still livin' in the past

- These invariant quantities can be combined into 16 operators up to 2nd order in momentum transfer and spin-exchange of 1 or less
- These are:

$$
\begin{aligned}
\mathcal{O}_1 &= 1_\chi 1_N \\
\mathcal{O}_2 &= (v^\perp)^2 \\
\mathcal{O}_3 &= i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right) \\
\mathcal{O}_4 &= \vec{S}_\chi \cdot \vec{S}_N \\
\mathcal{O}_5 &= i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp\right) \\
\mathcal{O}_6 &= \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right) \\
\mathcal{O}_7 &= \vec{S}_N \cdot \vec{v}^\perp \\
\mathcal{O}_8 &= \vec{S}_\chi \cdot \vec{v}^\perp \\
\mathcal{O}_9 &= i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right) \\
\mathcal{O}_{10} &= i\vec{S}_N \cdot \frac{\vec{q}}{m_N} \\
\mathcal{O}_{11} &= i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}
\end{aligned}
\qquad
\begin{aligned}
\mathcal{O}_{12} &= \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp) \\
\mathcal{O}_{13} &= i(\vec{S}_\chi \cdot \vec{v}^\perp)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right) \\
\mathcal{O}_{14} &= i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)(\vec{S}_N \cdot \vec{v}^\perp) \\
\mathcal{O}_{15} &= -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right) \\
\mathcal{O}_{16} &= -\left((\vec{S}_\chi \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right).
\end{aligned}
$$

More recap…

- These operators can be expressed as combinations of 6 nuclear responses:

$$
\begin{aligned}
&M_{JM}(q\vec{x}) \\
\Delta_{JM}(q\vec{x}) &\equiv \vec{M}^M_{JJ}(q\vec{x}) \cdot \frac{1}{q}\vec{\nabla} \\
\Sigma'_{JM}(q\vec{x}) &\equiv -i\left\{\frac{1}{q}\vec{\nabla} \times \vec{M}^M_{JJ}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{-\sqrt{J}\ \vec{M}^M_{JJ+1}(q\vec{x}) + \sqrt{J+1}\ \vec{M}^M_{JJ-1}(q\vec{x})\right\} \cdot \vec{\sigma} \\
\Sigma''_{JM}(q\vec{x}) &\equiv \left\{\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right\} \cdot \vec{\sigma} = [J]^{-1}\left\{\sqrt{J+1}\ \vec{M}^M_{JJ+1}(q\vec{x}) + \sqrt{J}\ \vec{M}^M_{JJ-1}(q\vec{x})\right\} \cdot \vec{\sigma} \\
\tilde{\Phi}'_{JM}(q\vec{x}) &\equiv \left(\frac{1}{q}\vec{\nabla} \times \vec{M}^M_{JJ}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right) + \frac{1}{2}\vec{M}^M_{JJ}(q\vec{x}) \cdot \vec{\sigma} \\
\Phi''_{JM}(q\vec{x}) &\equiv i\left(\frac{1}{q}\vec{\nabla} M_{JM}(q\vec{x})\right) \cdot \left(\vec{\sigma} \times \frac{1}{q}\vec{\nabla}\right)
\end{aligned}
$$

These are very complicated… don’t ask me to explain exactly what each is ☺

- These nuclear responses can be evaluated for a given isotope.

What does it all matter?

- WIMP recoil spectrum is determined by:

$$\frac{dR}{dE_R} = \left\langle \frac{\rho_\chi m_T}{\mu_T^2 m_\chi v} \frac{d\sigma}{d\cos\theta} \right\rangle$$

- The lagrangian includes all possible interactions:

$$\mathcal{L} = \sum_{i=1}^{12} c_i^{(n)} \mathcal{O}_i^{(n)} + c_i^{(p)} \mathcal{O}_i^{(p)}$$

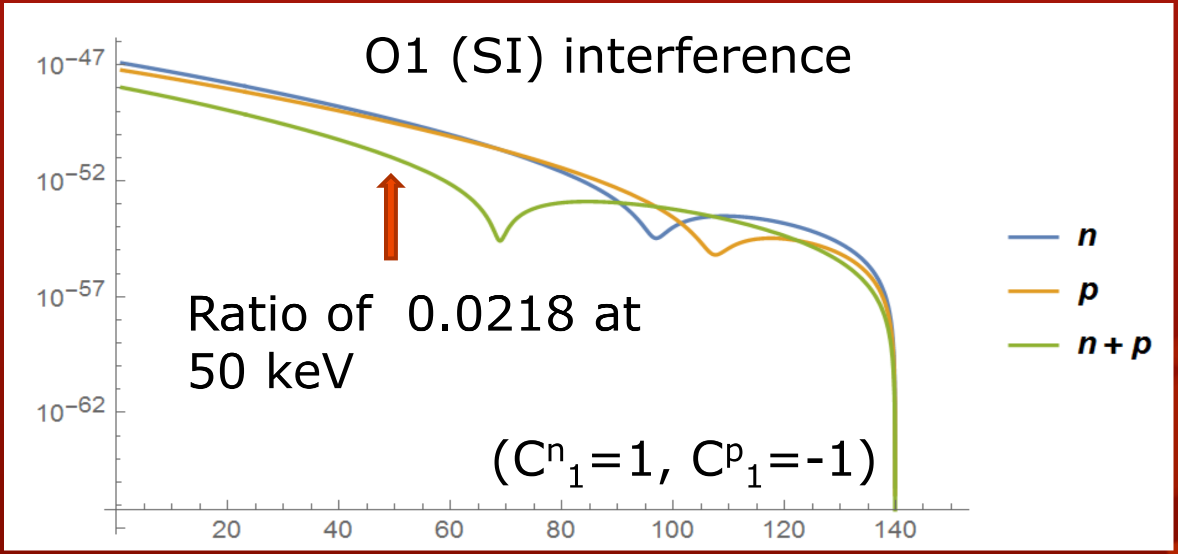
- The cross section is proportional to the matrix element squared

$$\frac{dR_D}{dE_R} = N_T \frac{\rho_\chi m_T}{32\pi m_\chi^3 m_N^2} \left\langle \frac{1}{v} \sum_{ij} \sum_{N,N'=p,n} c_i^{(N)} c_j^{(N')} F_{ij}^{(N,N')}(v^2, q^2) \right\rangle$$

  - The F_{ij} is a form factor corresponding to operators O_i and O_j, which is a known combination of the known nuclear responses referred to on the previous slide.

That’s a lot of factors! What are we to do?

- Interference can conspire to nullify almost any limits for a single parameter
  - Only certain combinations of operators can interfere because they must satisfy symmetries (parity, etc)
  - The proton and neutron couplings can interfere for any operator, however.



Still not fully sure…

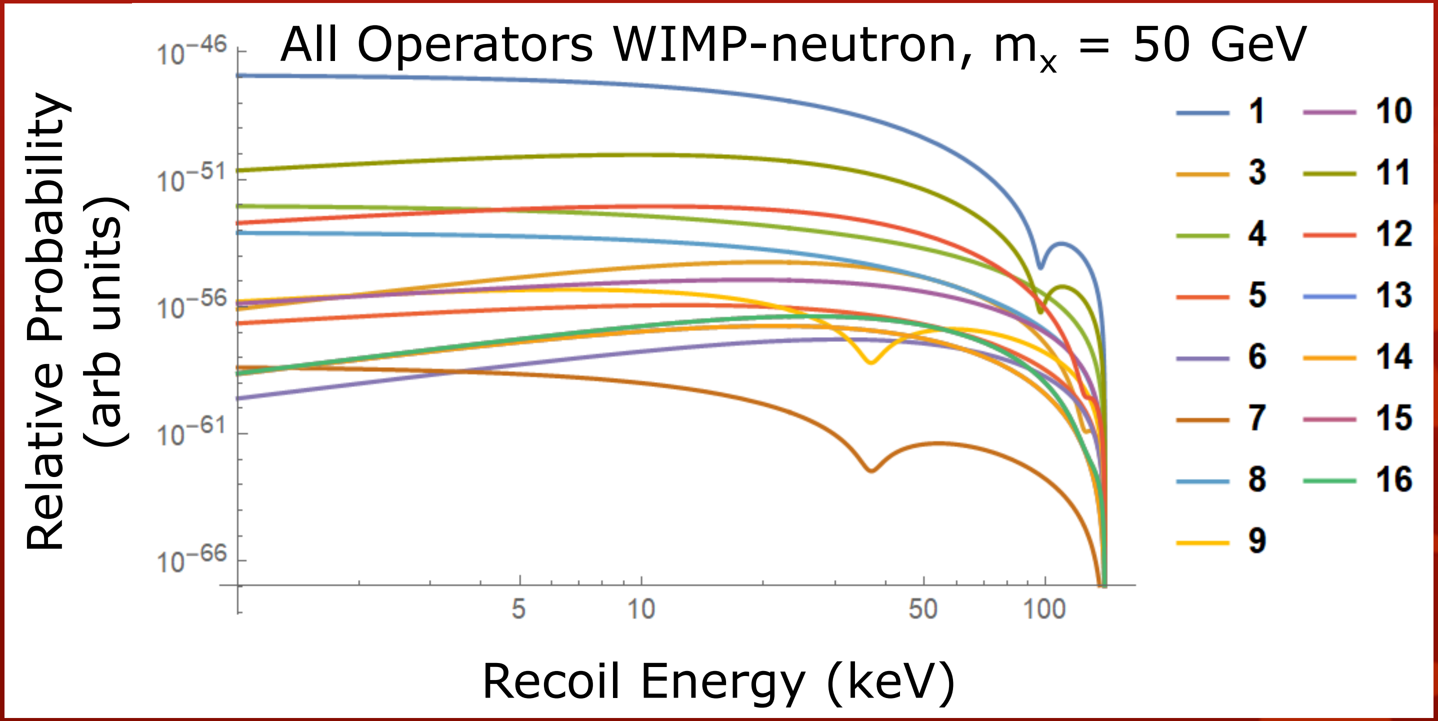
- To get a proper limit, would want to re-write

$\sum_i \ c_i^n O_i + c_i^p O_i^p$ as $O_a^n + \frac{c_a^p}{c_a^n} O_a^p + \sum_{i \neq a} \frac{c_i^n}{c_a^n} O_i^n + \frac{c_i^p}{c_a^n} O_i^p$ for an arbitrary choice of c_a^n for each interference class. Then minimize the above to get the worst-case value for each $\frac{c_i^{(N)}}{c_a^n}$. Then use the PLR to determine a limit on c_a^n which would determine a worst-case limit on each other parameter.

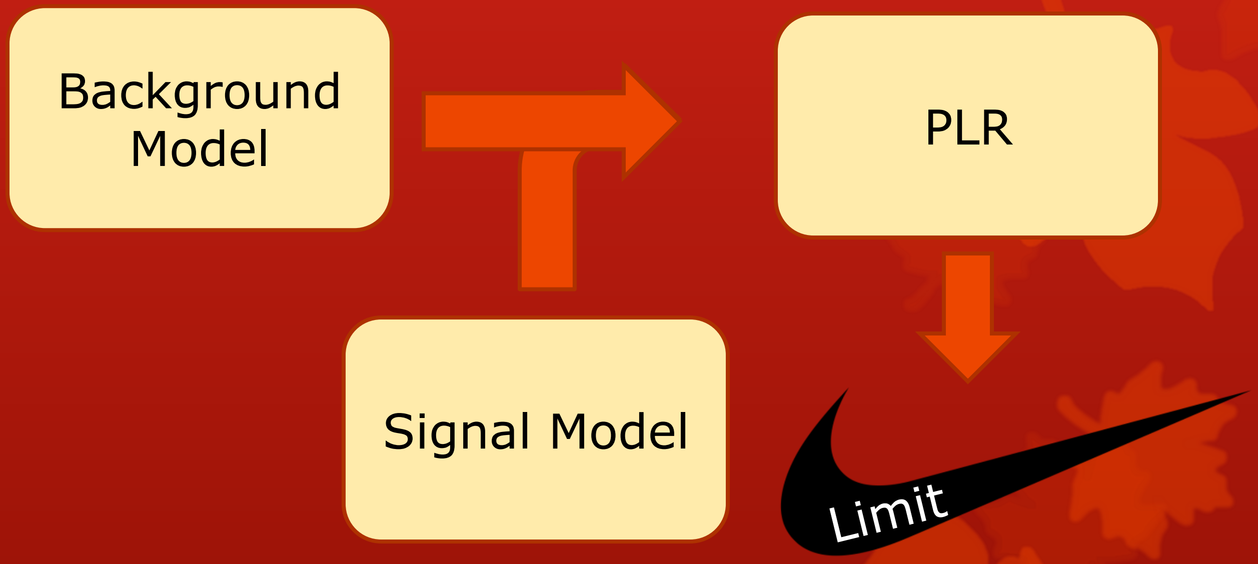
- This is hard, so for now we just work on 1 parameter at a time while setting all others to 0

It's all about the recoil spectrum

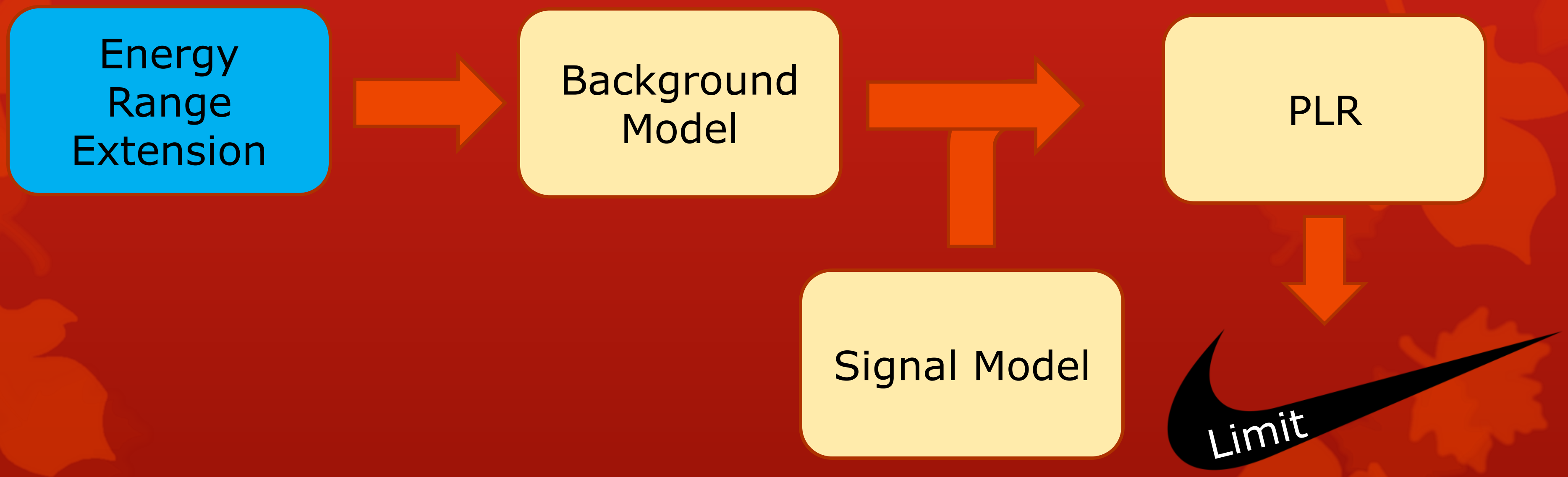
- Different combinations, then, have different recoil spectra as expected.
- Here are all of them for a 50 GeV WIMP coupling only to neutrons



What makes a limit?



Changes to the Background?



Background Model Modification

- Initial analysis only extended to 50 keVnr.
- EFT signals can possibly extend much further than this for some heavier WIMP candidates.
- ^{83m}Kr, which is not a huge problem for traditional energy ranges, may be an issue.

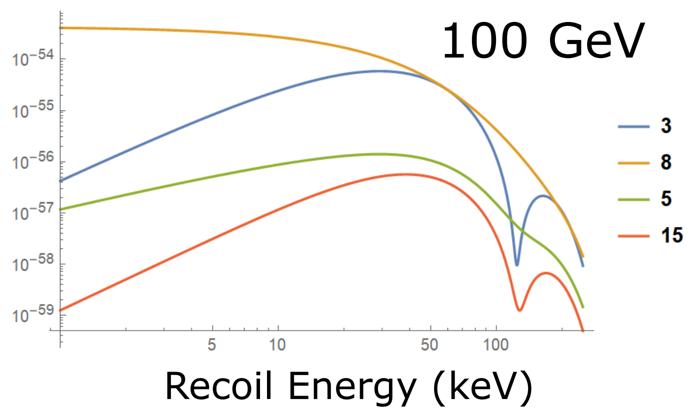
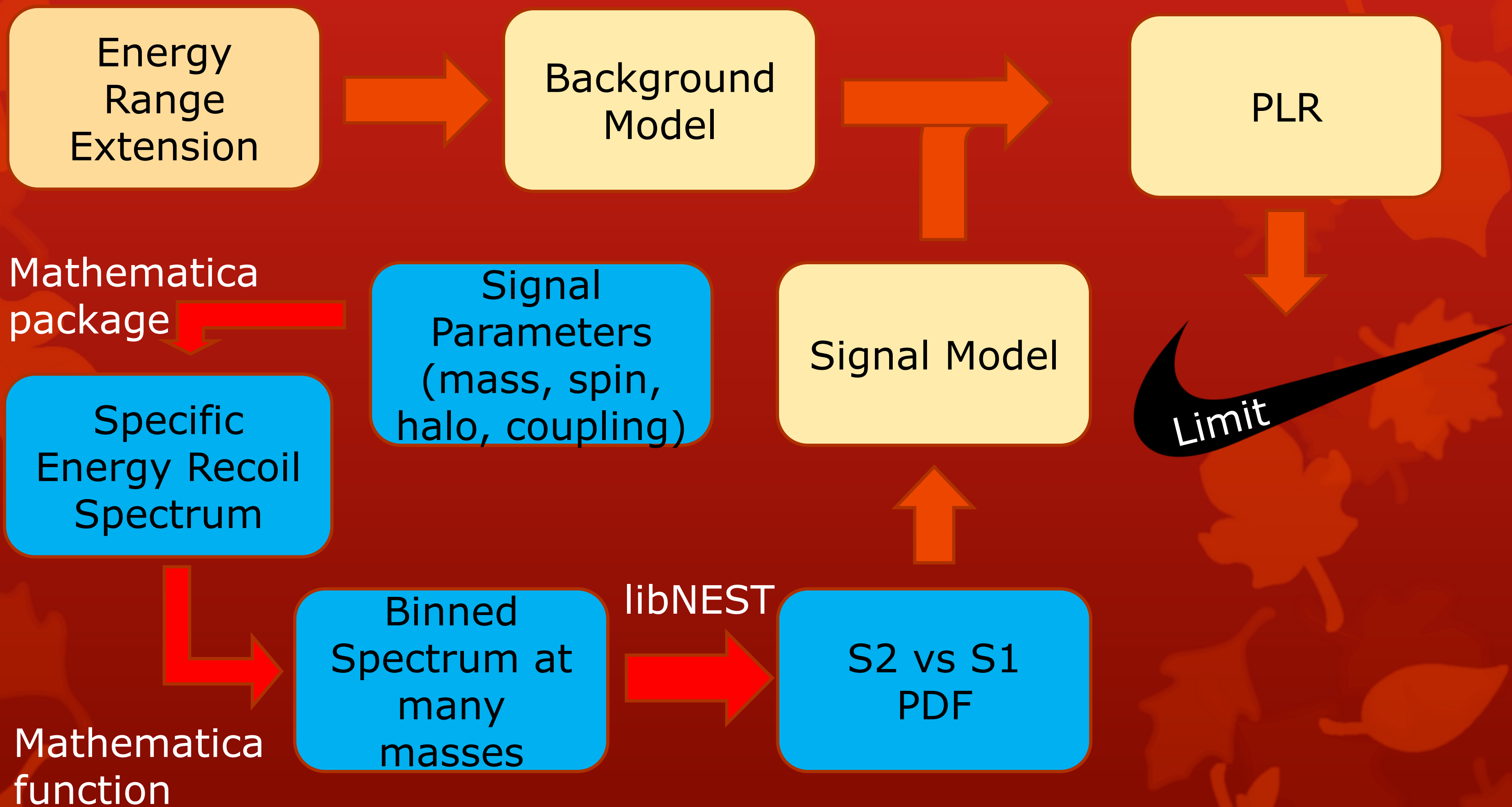


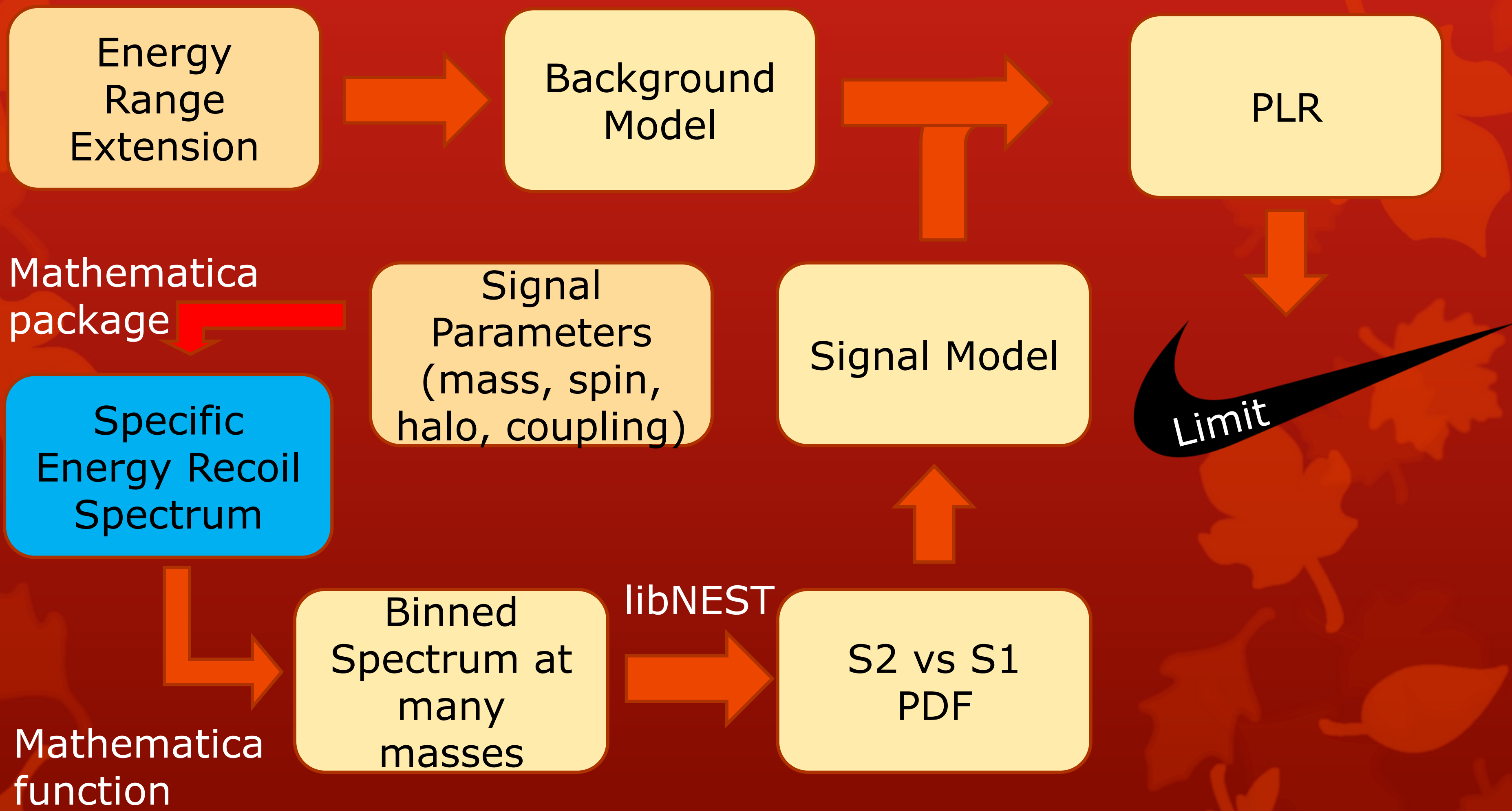
TABLE I. The upper energy threshold E_{max} (in $\mathrm{keV_{nr}}$) for each of the effective field theory operators, such that an energy window from 0 to E_{max} captures either 50% or 90% of WIMP-neutron recoil events for the given operator and WIMP mass.

Operator	50-GeV WIMP $E^{50\%}_{max}$ ($\mathrm{keV_{nr}}$)	50-GeV WIMP $E^{90\%}_{max}$ ($\mathrm{keV_{nr}}$)	500-GeV WIMP $E^{50\%}_{max}$ ($\mathrm{keV_{nr}}$)	500-GeV WIMP $E^{90\%}_{max}$ ($\mathrm{keV_{nr}}$)
SI	10.8	27.3	16.6	44.7
$\mathcal{O}_1$	6.8	21.7	11.8	43.8
$\mathcal{O}_3$	26.4	49.1	148.1	344.4
SD	8.6	21.6	11.9	37.5
$\mathcal{O}_4$	7.0	24.0	32.8	299.6
$\mathcal{O}_5$	16.2	38.6	65.5	328.9
$\mathcal{O}_6$	33.6	64.0	267.3	433.7
$\mathcal{O}_7$	5.0	16.2	25.2	279.9
$\mathcal{O}_8$	6.8	22.2	14.5	64.8
$\mathcal{O}_9$	13.7	37.2	276.7	464.7
$\mathcal{O}_{10}$	21.7	48.6	112.6	340.4
$\mathcal{O}_{11}$	15.5	34.4	39.0	279.9
$\mathcal{O}_{12}$	17.4	38.1	34.8	176.5
$\mathcal{O}_{13}$	28.2	53.2	54.5	219.7
$\mathcal{O}_{14}$	11.9	27.9	240.9	400.0
$\mathcal{O}_{15}$	34.3	59.1	261.2	433.7

Changes to the Background?



Changes to the Background?



DMFormFactor - Mathematica

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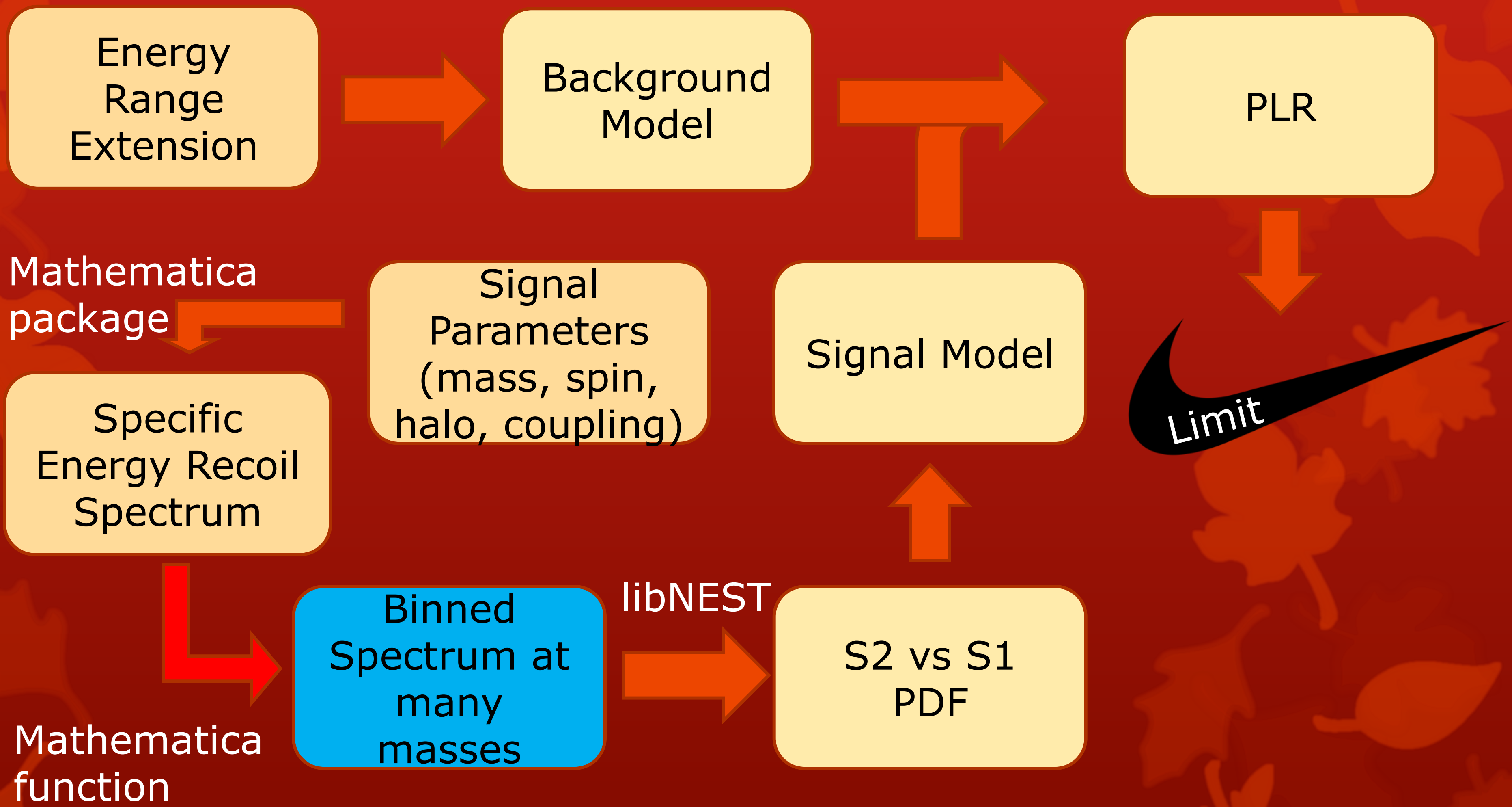
Welcome to

  DMFormFactor version 1.1.

Functions are SetCoeffsNonrel, SetCoeffsRel, SetCoeffsNucl, ZeroCoeffs,
  SetJChi, SetMchi, SetIsotope, SetHALO, SetHelm, TransitionProbability,
  ResponseNucl, DiffCrossSection, ApproxTotalCrossSection, and EventRate.
```

- Mathematica package written by Nikhil Anand, A. Liam Fitzpatrick, and W. C. Haxton.
- Returns the recoil energy (really momentum) spectrum in the form of a mathematica function given model inputs.
- Notable inputs:
  - Halo type, earth velocity, local DM mean velocity, escape vel
    - (set to typical values)
  - WIMP spin
    - Set to 1/2
  - WIMP mass
  - Target isotope
  - Operator coefficients (for each op and proton vs neutron coupling)

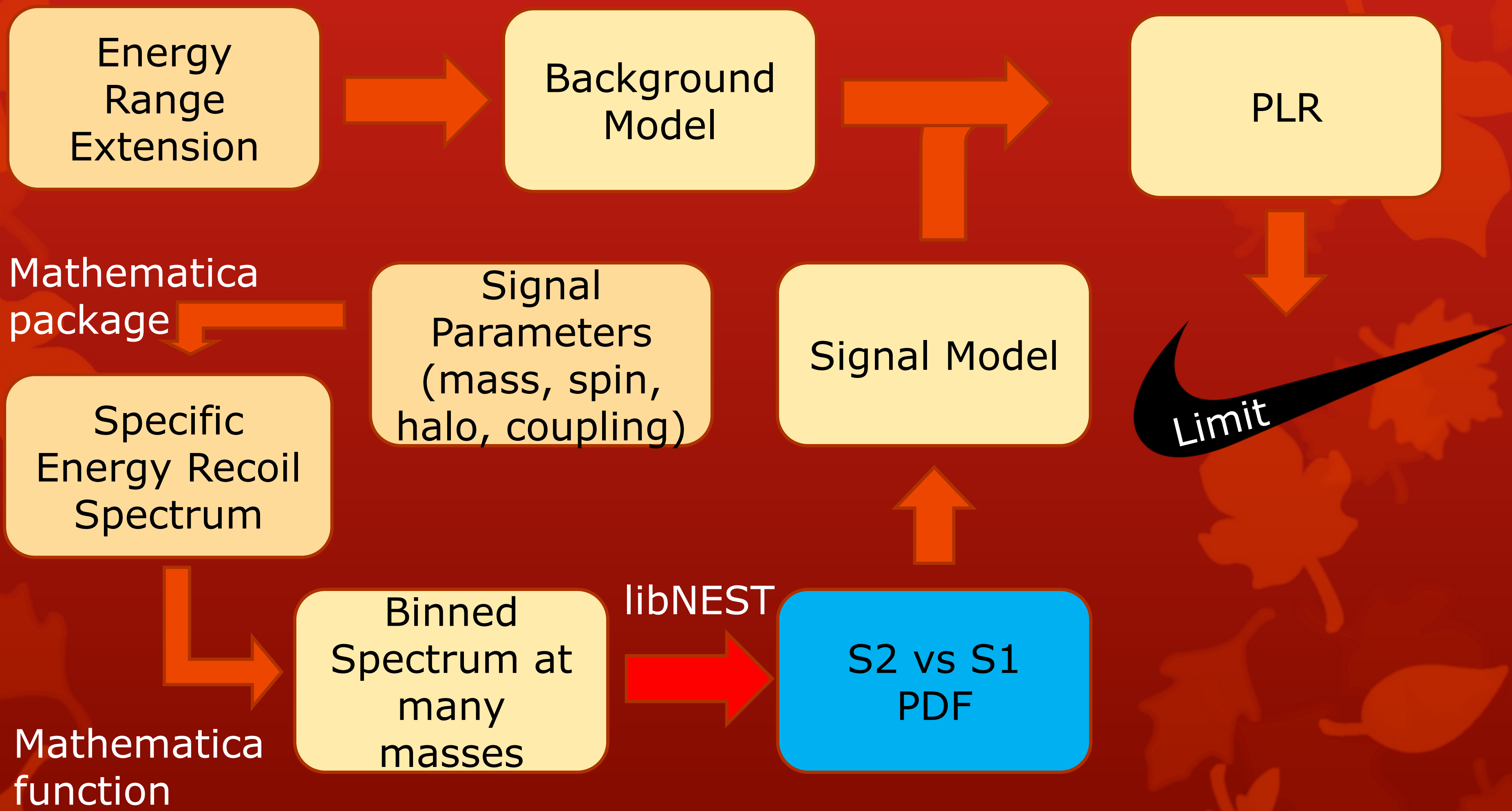
Changes to the Background?



Spectra generation

- Mathematica function calls DMFormFactor for different values of mass and different isotopes in an array.
- Resulting analytical spectra are integrated into bins used by the PLR analysis.
- Isotope Spectra are weighted by their abundance and then added together.
- The resulting array of varying masses and bins is stored in a text file whose name identifies the non-zero coupling constant used.

Changes to the Background?

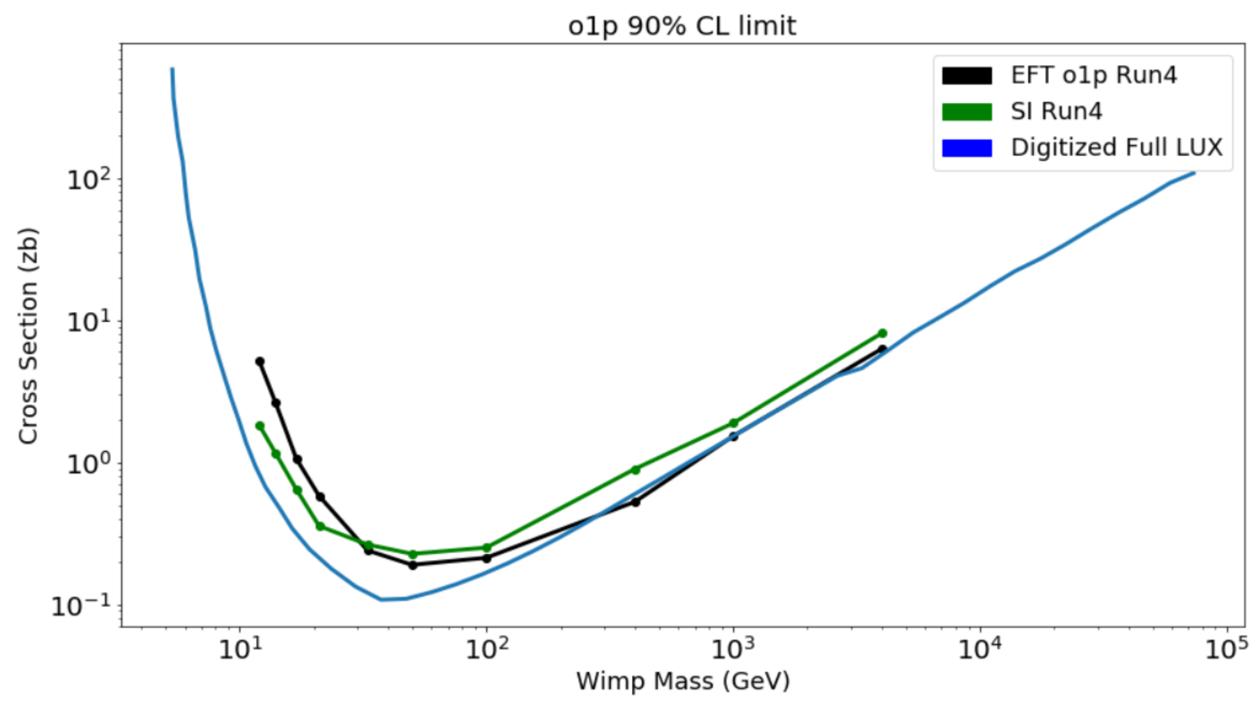


PDF creation

- libNEST is called for each mass in question.
- libNEST imports the recoil spectrum then simulates the S2 and S1 response given energies sampled from the imported spectrum.
- The result is stored as a root histogram to be used by the limit code.

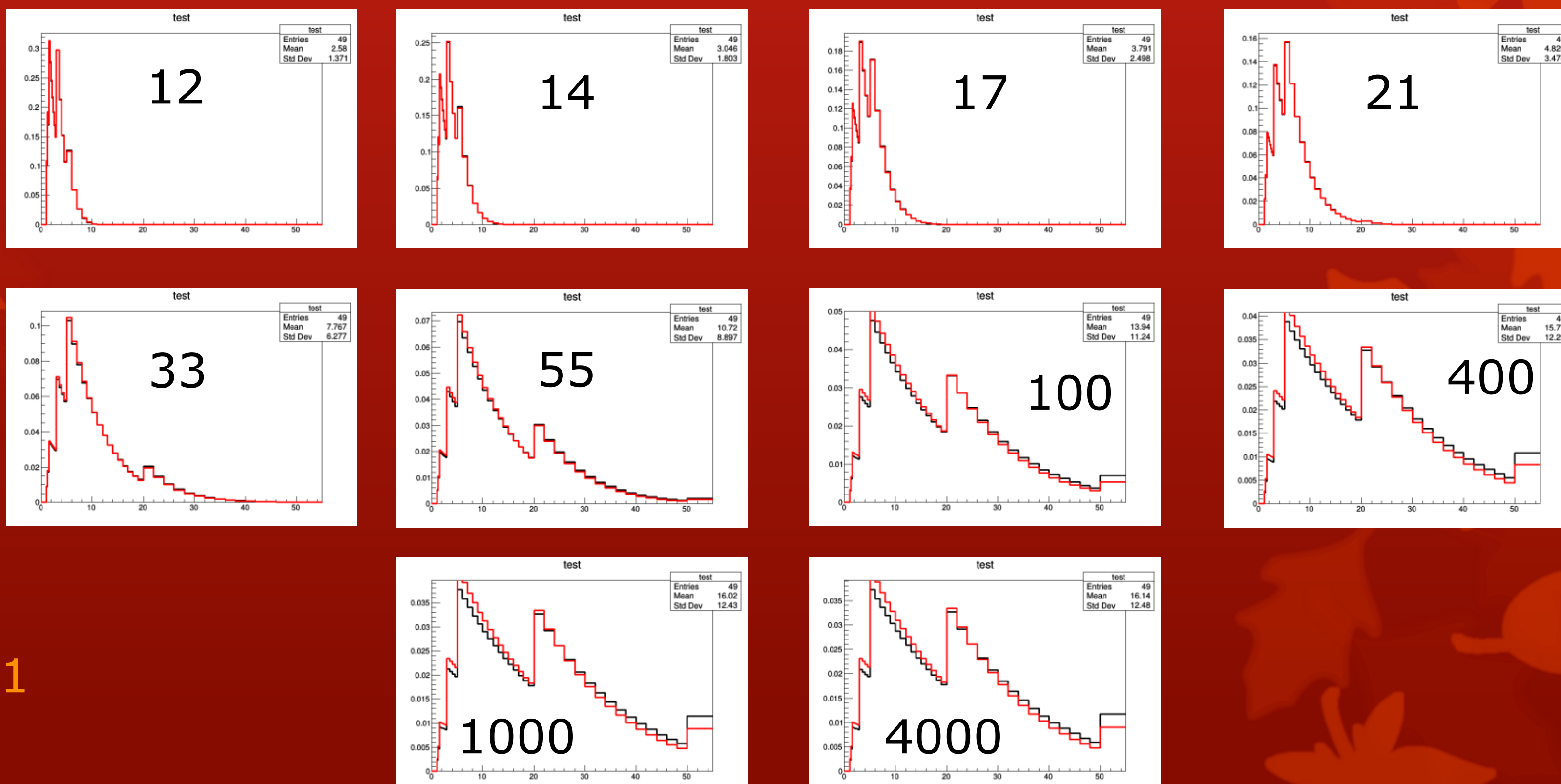
How do we do?

- Operator 1 should closely match the SI result.
- We attempted to validate our process by comparing the limit on the WIMP-proton coupled operator 1 to our SI result.



Not great, but why?

- Low statistics (1000 per nSig, mass point)?
- Model mismatch?



Conclusion

- Getting there…
- Process is in place
- Background model must be validated and incorporated
- Difference between WIMP proton operator 1 limit and SI results must be understood
- After single-parameter limits, other intersting things could be done.
  - Proton-neutron interference looked into?
  - Joint PLR with experiments with other detector mediums?