Simulating Disordered Spin Systems for Quantum Computing Using HTCondor

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HTCondor Week 2018

or: How I Learned to Stop Worrying and Love the Bomb (HTCondor)

bomb

1. (before 1997) **Something** really **bad**; a failure

2 (after 1997) Something considered **excellent** and/or the best (uses modifier "the")

*1. I **hated** that movie! I'm not surprised that it was a total bomb at the box **office**.*

*2. I loved that movie! It was **the bomb**!*

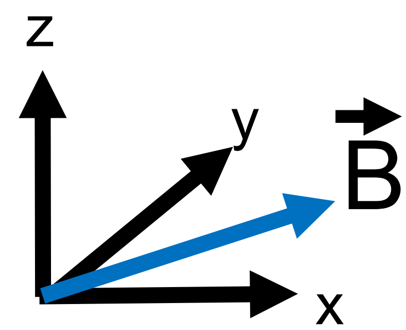
by Bill M. July 27, 2004

source: urbandictionary ☺

based on a true story
by
A.B. Özgüler, R. Joynt, M.G. Vavilov
Steering Random Spin Systems to Speed up the Quantum Adiabatic Algorithm

Spin ½ (2-level system, qubit)

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Hamiltonian

$$H(t) = \vec{B}(t).\vec{\sigma}$$

Eigenstates and eigenvalues

$$H(t)\,|n(t)\rangle = E_n(t)\,|n(t)\rangle$$

Schrödinger equation

$$H(t)\,|\psi(t)\rangle = i\hbar\,\frac{\partial|\psi(t)\rangle}{\partial t}$$

Probability

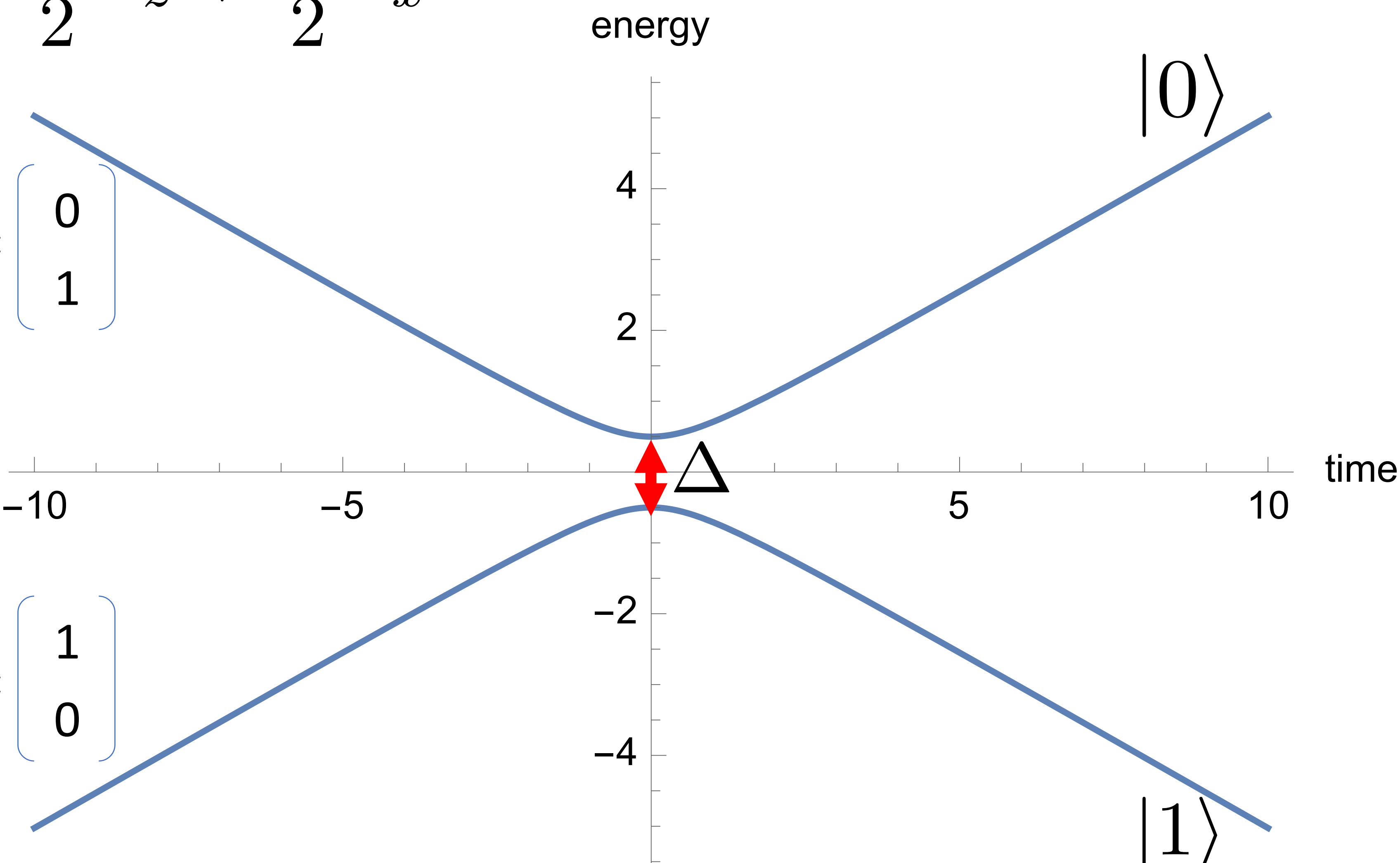
$$P_n = |\langle\psi(t)|n(t)\rangle|^2$$

Landau-Zener-Stueckelberg-Majorana Tunneling

$$H = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x$$

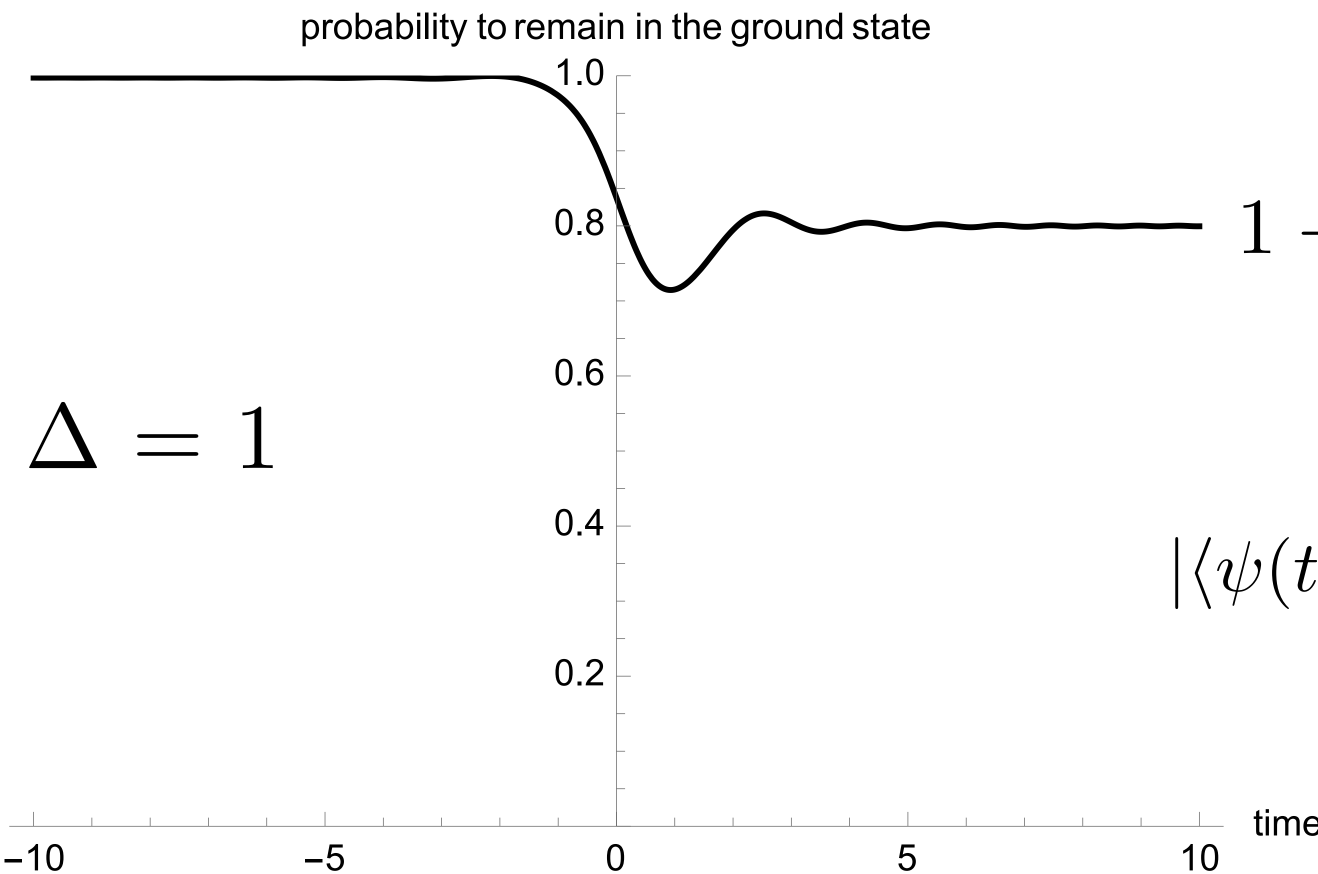
$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$H = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x \qquad H(t)\,|\psi(t)\rangle = i\hbar\,\frac{\partial|\psi(t)\rangle}{\partial t}$$

$$P = |\langle\psi(t)|g.s.(t)\rangle|^2$$

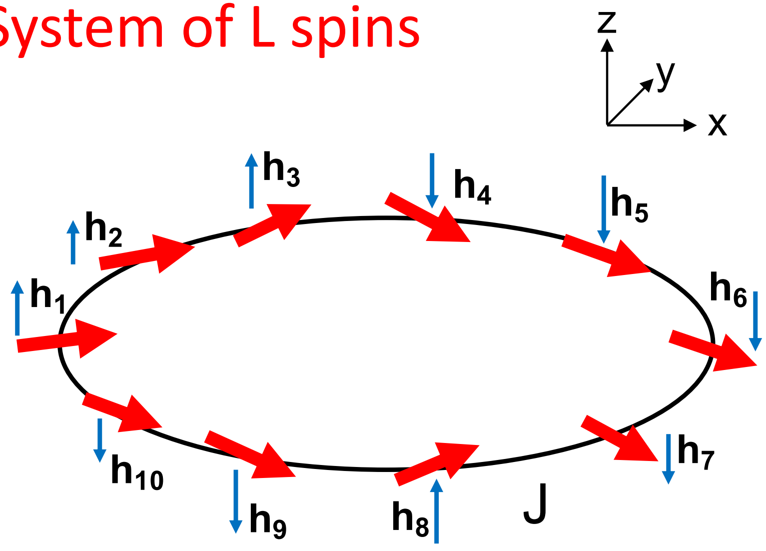


$$1 - e^{-\frac{\pi\Delta^2}{2v}}$$

$$|\langle\psi(t)|0\rangle|^2$$

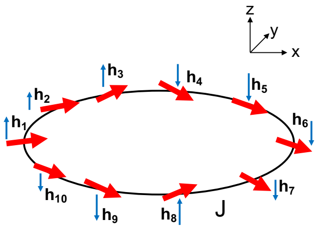
$$\mathrm{v} = 1,\ \Delta = 1$$

System of L spins



$$H_f = \sum_{i=1}^{L} J\, \sigma_z^i \sigma_z^{i+1} + \sum_{i=1}^{L} h_i\, \sigma_z^i$$

- 2^L-by-2^L matrix
- Exponential increase in required resources
- Disordered systems not easy to handle in a personal computer. Many realizations should be performed.
- Difficult problems have exponentially small energy gaps O(exp(-L)). NP-hard, NP-complete problems can be mapped to Ising model (see: Ising formulations of many NP problems).
- Quantum adiabatic algorithm more efficient than classical optimization algorithms?



$$H_f = \sum_{i=1}^{L} J\,\sigma_z^i \sigma_z^{i+1} + \sum_{i=1}^{L} h_i\,\sigma_z^i$$

- $\{h_i\}$ selected randomly (uniform distribution) from $[-W,W]$. $W = 1$.
- 10^4 realizations performed.
- If L increases, much more memory needed.
- Memory required in a personal computer and CHTC computer are not the same. Memory spread out in CHTC, which can be nice.

L (system size)	Memory	Time	
		Without HTCondor	With HTCondor
1	Up to 256MB	a few days	a few hours
3	Up to 256MB	~ week	half day
8	Up to 256MB	~ a few weeks	less than a day
10	Around 1000MB	~ months	~ a day
12	More than 1000MB	~ half PhD!	a few days

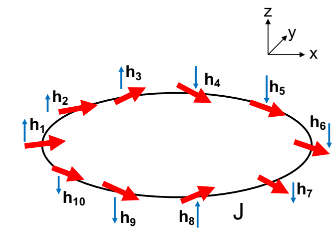
Quantum Adiabatic Algorithm

$$|\psi(0)\rangle$$

H(t = 0)

$$H_{initial}$$

$$H(t)\,|\psi(t)\rangle = i\hbar\,\frac{\partial|\psi(t)\rangle}{\partial t}$$



$$|\psi(t_a)\rangle$$

Hamiltonian changes slowly

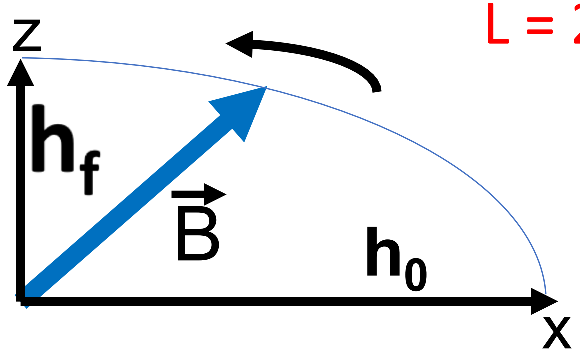
H(t = t_a)

$$H_{final}$$

$$H(t) = f_1(t)\,H_{initial} + f_2(t)\,H_{final}$$

$$P_1 = |\langle\psi(\tau=1)|g.s.(\tau=1)\rangle|^2$$

If there is a gap between ground level and rest of the Hamiltonian's spectrum, then the final state is the ground state of H_{final}.

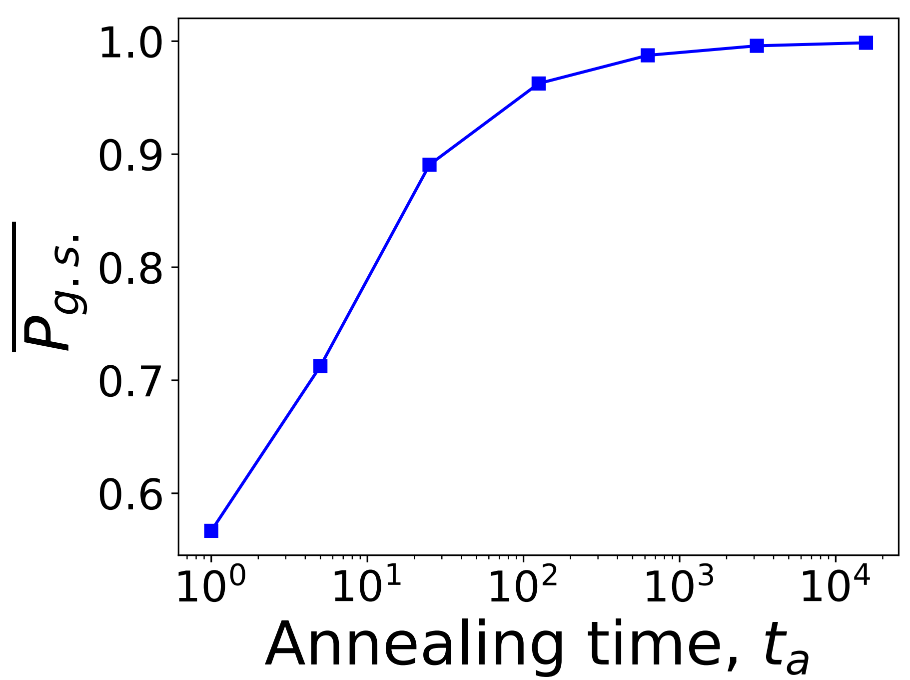


L = 2, without steering

$$H_i = h_0\, \sigma_x$$

$$H_f = h_f\, \sigma_z$$

$$H = h_0\, cos^2(\frac{\pi}{2}\frac{t}{t_a})\, \sigma_x + h_f\, sin^2(\frac{\pi}{2}\frac{t}{t_a})\, \sigma_z$$



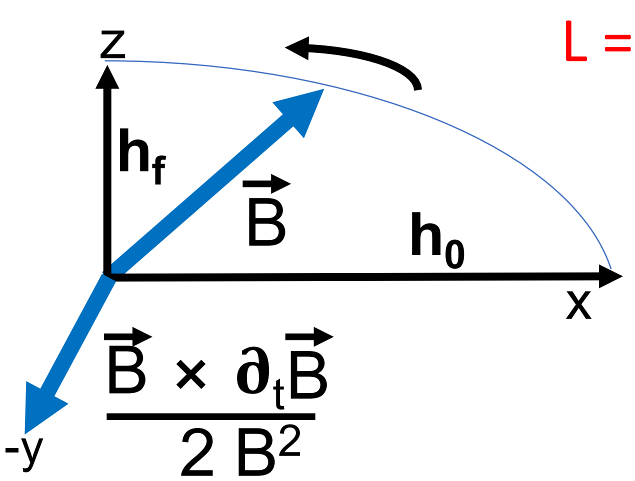
Shortcut?

Transitionless tracking algorithm* gives the correction Hamiltonian.
To drive the ground state, we can use it as:

$$H_B(t) = i\hbar \sum_{m=2}^{2^L} \frac{|m\rangle \langle m| \partial_t H_0 |1\rangle \langle 1|}{E_1 - E_m} + (\text{h.c.})$$

$H_0 + H_B$ drives ground state of H_0 without transitions.

** Berry, M. V., Journal of Physics A (2009)*

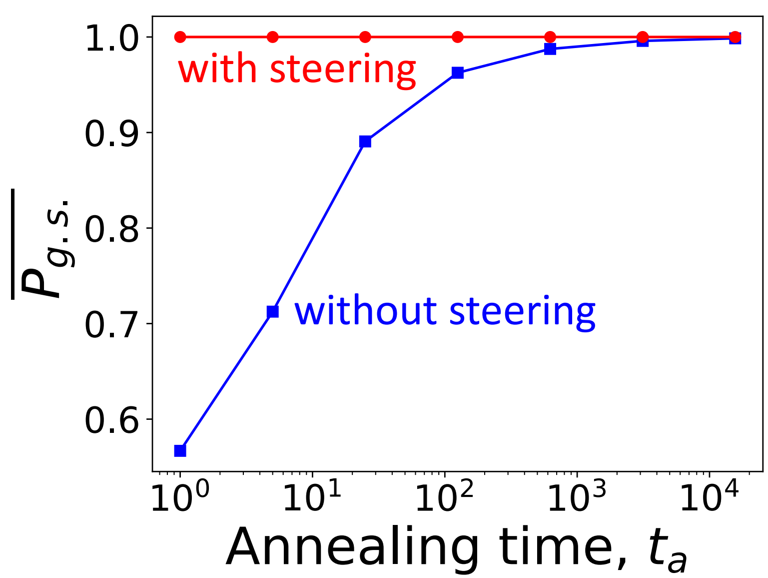


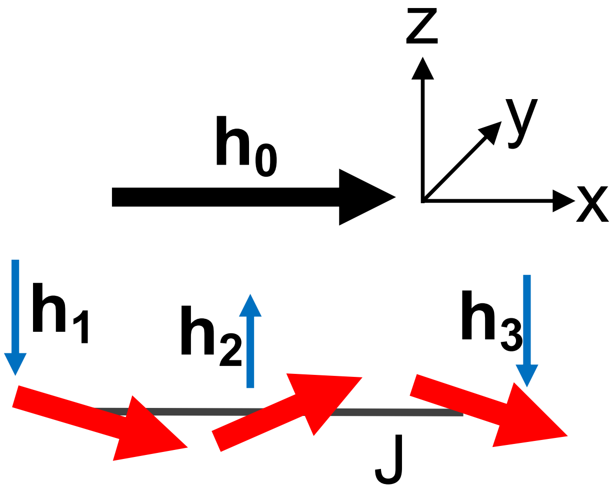
L = 2, with steering

$$H_i = h_0\,\sigma_x$$

$$H_f = h_f\,\sigma_z$$

$$H_{steering} = \frac{-h_0 h_f \pi \sin\left(\pi\,\frac{t}{t_a}\right)}{4\,t_a[h_0^2\cos^4(\frac{\pi}{2}\,\frac{t}{t_a}) + h_f^2\sin^4(\frac{\pi}{2}\,\frac{t}{t_a})]}\,\sigma_y$$



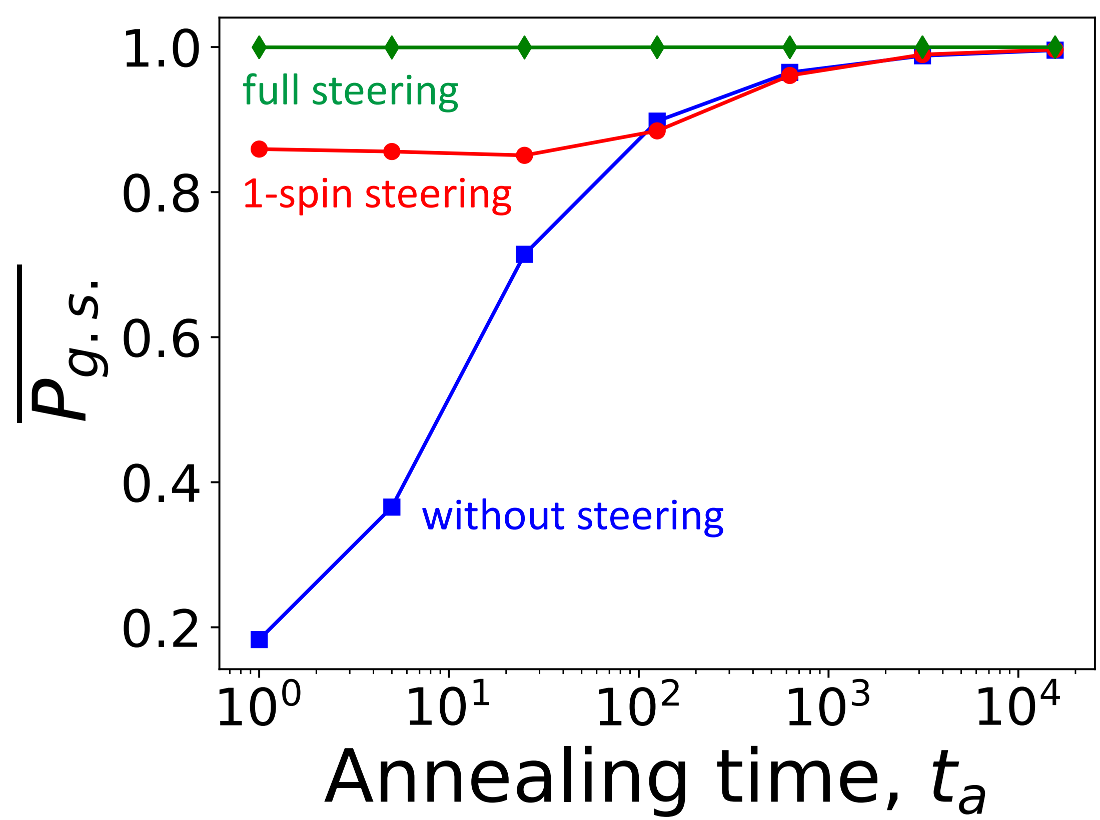


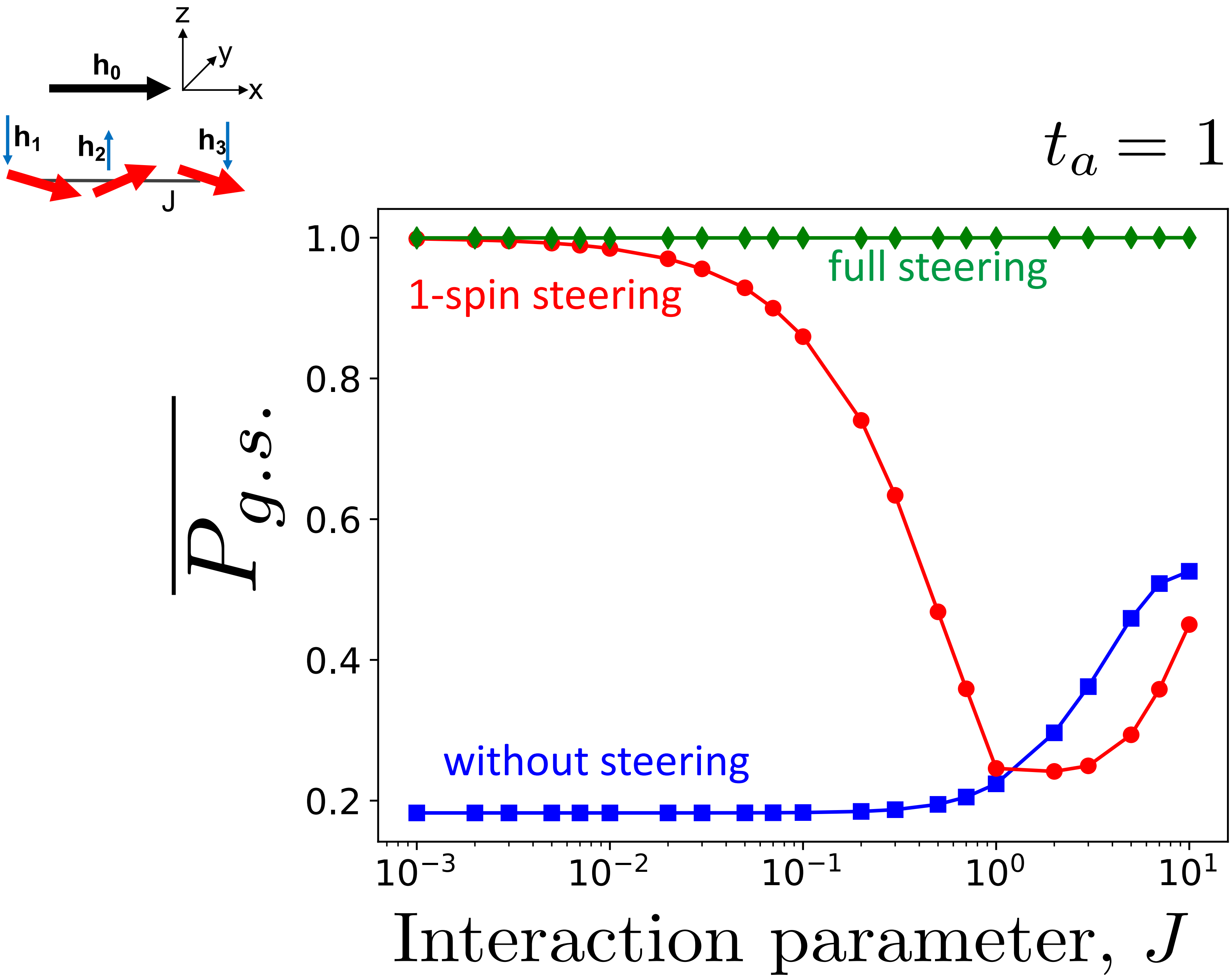
L = 3

$$H_i = h_0 \sum_{i=1}^{L} \sigma_x^i$$

$$H_f = J\,\sigma_2^z\,(\sigma_1^z + \sigma_3^z) + \sum_{i=1}^{3} h_i \sigma_i^z$$

$J = 0.1$



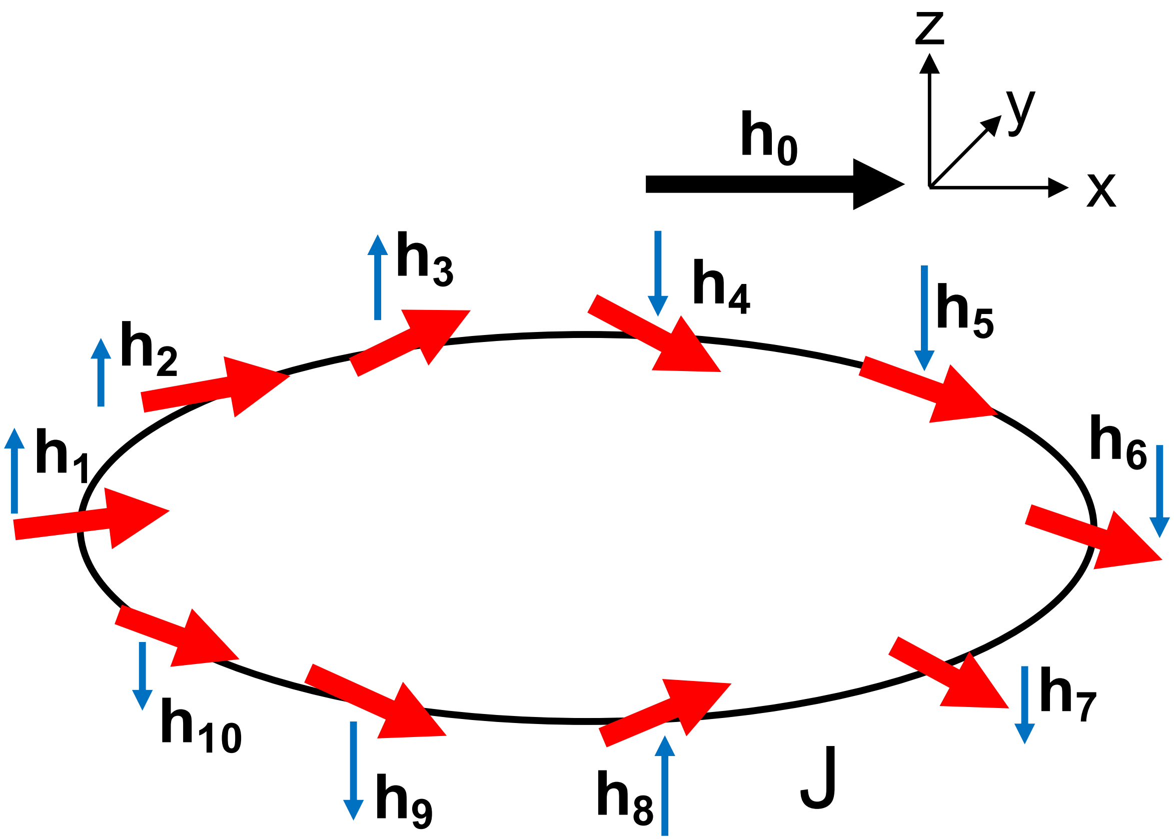


z
y
x
h_0
h_1
h_2
h_3
J
$t_a = 1$
1.0
0.8
0.6
0.4
0.2
$\overline{P_{g.s.}}$
full steering
1-spin steering
without steering
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Interaction parameter, J

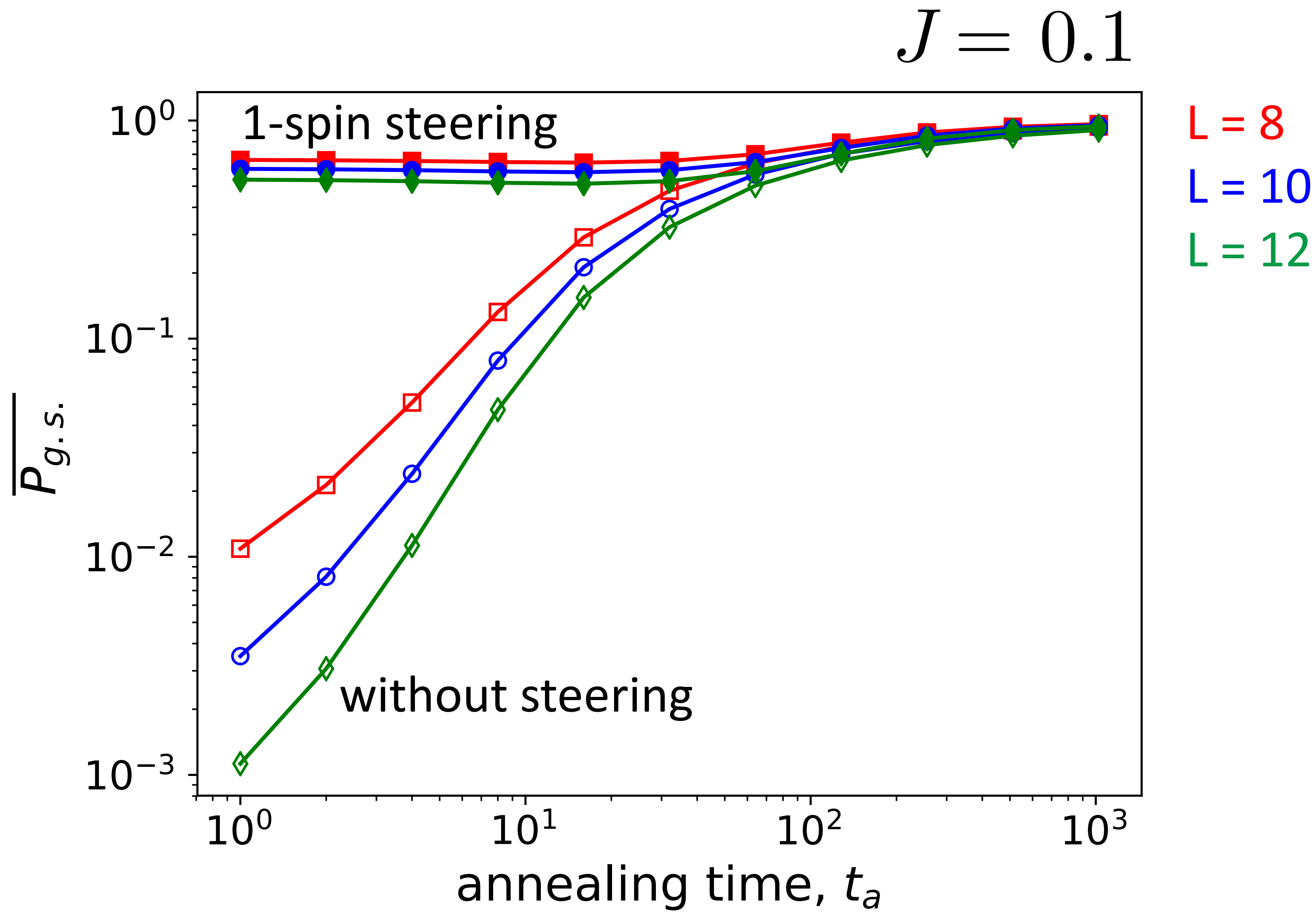
L ≥ 8

$$H_i = h_0 \sum_{i=1}^{L} \sigma_x^i$$

$$H_f = \sum_{i=1}^{L} J\, \sigma_z^i \sigma_z^{i+1} + \sum_{i=1}^{L} h_i\, \sigma_z^i$$

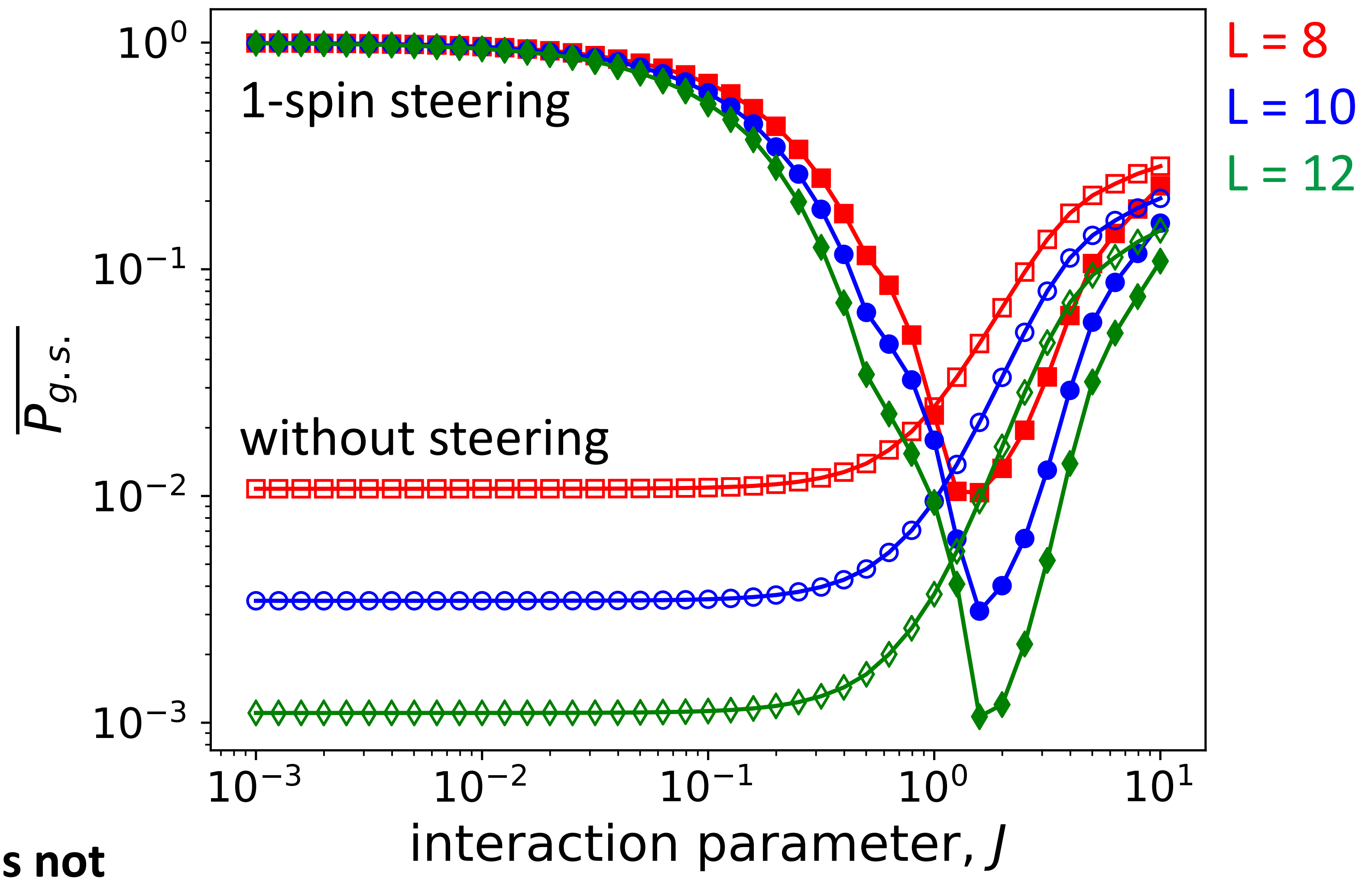


100x speed-up!



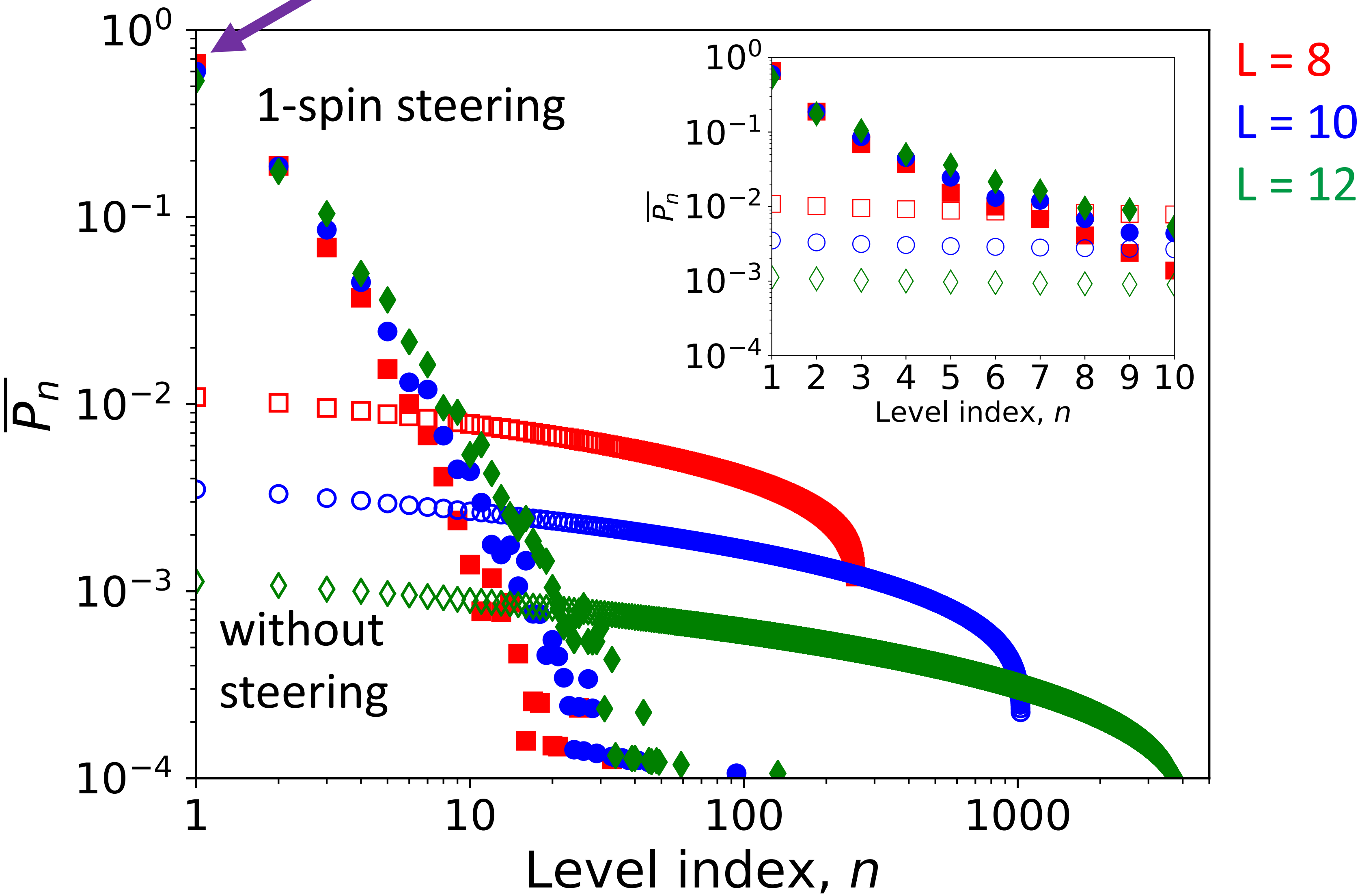
SIGNIFICANT IMPROVEMENT FOR $J \lesssim 0.4$

$t_a = 1$



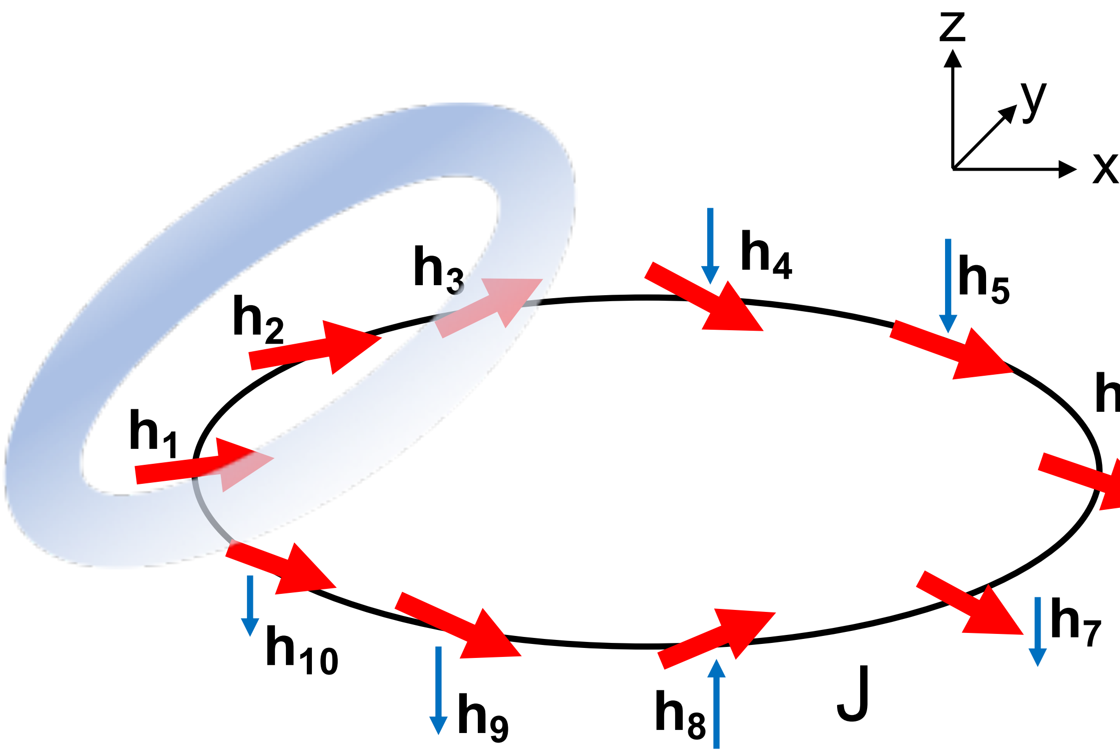
$\overline{P_{g.s.}}$ **is not sensitive to L in the strong disorder limit!**

$t_a = 1, J = 0.1$

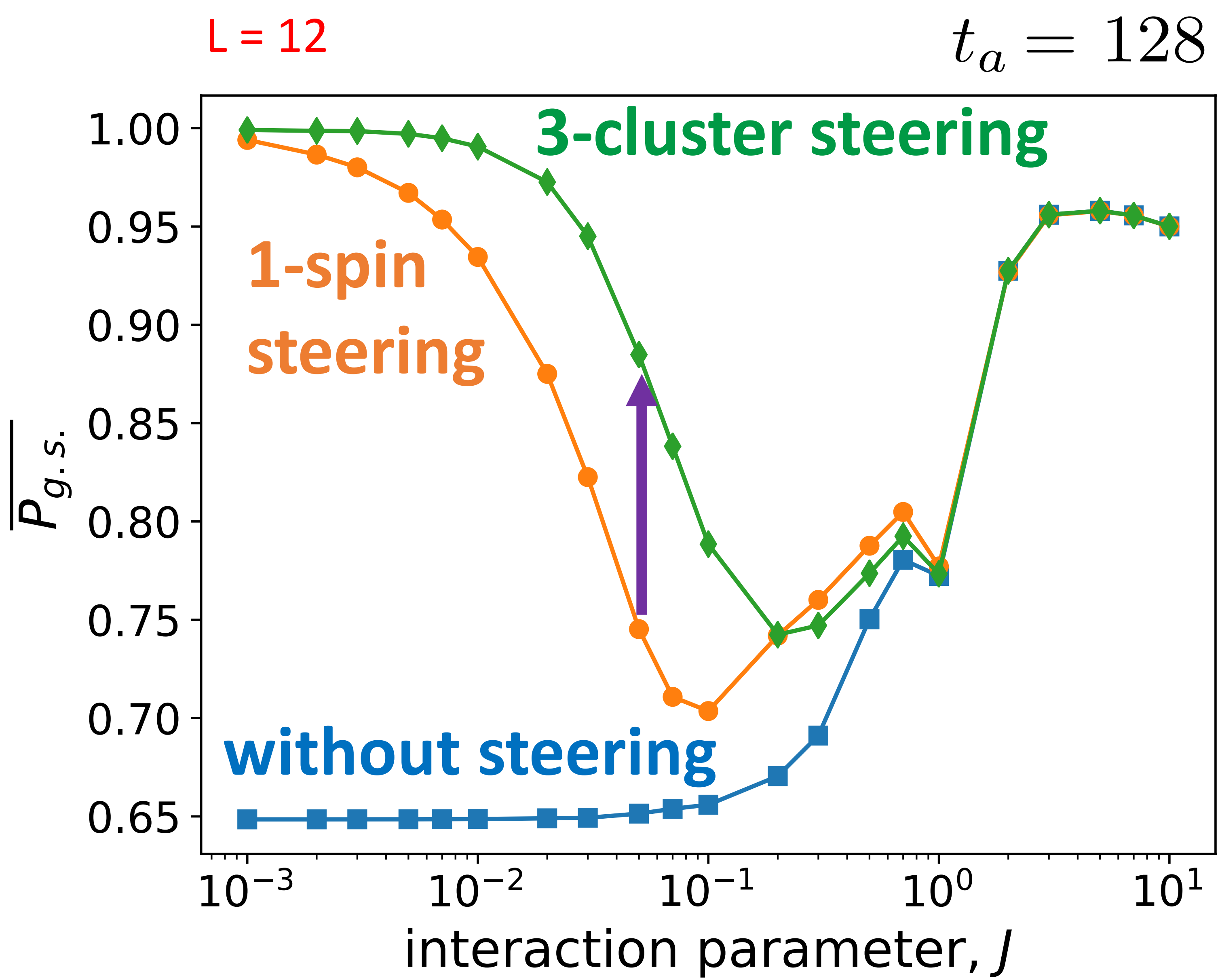


When steering is applied, the total probability of the first ~ 10 states is large. The rest is negligible.

Cluster steering



$\{h_i\}$ is shifted so that h_2 is the min absolute magnetic field.



L = 12
$t_a = 128$
3-cluster steering
1-spin steering
without steering
$\overline{P_{g.s.}}$
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
interaction parameter, J

Conclusions

- transitionless tracking algorithm, a shortcut to adiabaticity method, used in disordered systems
- in the limit of strong random field, 1-spin steering is successful even with short driving time
- cluster approach provides even more efficiency
- simulations performed using the computing resources of CHTC

Thanks to HT CENTER FOR HIGH THROUGHPUT COMPUTING