# Tilted S2 Peak/Width

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## 1 Introduction

First, I am given $\frac{dn}{dz} = (\alpha * E) - (\beta * P) - (\gamma)$ with $\alpha = 137\frac{1}{kV}$, $\beta = 177\frac{1}{bar*cm}$, and $\gamma = 45.7\frac{1}{cm}$

## 2 Establishing Transformations and Coordinates

Define the x-y plane as that which the plates normally take. If we are transforming a plate, then any formation can be achieved by doing (in order for simplicity): a rotation about the z-axis, a tilt along some line orthogonal to the z-axis, then translations in the x,y, and z axes (the translations are likely severely limited by the geometry of the detector, but they are nonetheless plausible).

Short (and incomplete) Explanation:

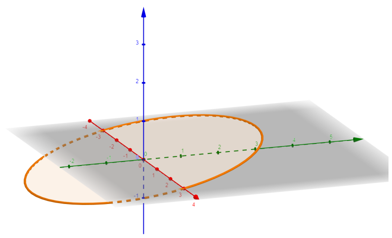
Take any transformation of the plate which keeps its original shape (ex. no scaling) Define a plane B that is orthogonal to the z-axis and intersects the center of the plate If the entire plate intersects plane B: Then the plate is parallel to its original position Thus it is only translated and rotated about the z-axis from its original position If the entire plate does not intersect B: Then it must intersect B at a straight line Because the plane intersects the center, this intersection line goes through the center of the plate, bisecting it Because the plate keeps its shape, the plate is symmetric along the line, with one half being above plane B and the other being equally below. Thus the transformation of the plate aside from the rotation/translation can be described as a single tilt along this line of intersection.

For now, I will not look at x-y translations due to edge effects getting even more complicated. I will later if time allows. Given that the plate is circular, I can ignore any rotations about the z-axis, as it does not change its interaction with the bottom plate if it is done first. Thus I will be looking at the tilt and the z-transformation.

I define the x-axis as the axis where the tilting is about, and the y axis as the axis orthogonal to both the x and z axes. Let $+y$ be where the tilt is above the x-y plane and $+x$ be $\pi/2$ clockwise from $+y$. Let the origin be at $x = 0, y = 0$ and, assuming that the plates are of 0 thickness, let the bottom plate be at

$z = 0$.. If a tilt is present in the detector, this will show a different distribution than what the detector will due to almost certainly different definitions of x and y, but it should just be a rotation of the actual distribution along the z-axis.

Figure 1: An example of what the x- and y-axes will look like (at $z = 0$ for easier viewing)



Let this angle of tilt about the x-axis be called $\theta$. Thus:

$\tan\theta = \frac{z-z_0-z_t}{y}, z = z_0 + z_t + y\tan\theta$

Where $z_0$ is the normal distance between the two plates and $z_t$ is any possible z translation of the plate.

Given some $\theta$ and $z_t$, I am operating with three variables: $z$, $x$ and $y$. $z$, however, can be expressed entirely in terms of y and some constants, so I am really only working with $x$ and $y$.

# 3 Finding Capacitance Between Normal Plates

Easily Enough:

$C = \epsilon\frac{A}{z} = \epsilon\frac{\pi r^2}{z_0}$

# 4 Finding Capacitance Between Plates

While the plates as a whole are nonparallel, at infinitesimally areas $da$ of the plates in the y-direction, I can treat the areas as parallel to that of the plate below them. In addition, this is true for every $dy$ along the x-axis, as they share the same $z$ for each $y$. Thus we can do this for the whole strip along the x-axis.

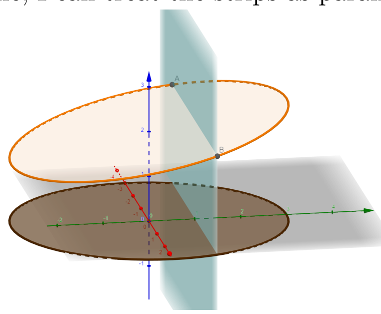
The area of each of these strips is $dA = x * dy$.

We start with $C = \epsilon\frac{A}{z} = \epsilon\frac{A}{z_0+z_t+y\tan\theta}$

Thus $dC = \epsilon\frac{x*dy}{z_0+z_t+y\tan\theta}$ (We are treating the plates as parallel the z should be the same across this area).

For now, I will only be looking at the area directly under the top plate. This means that the area is of an oval, with major axis $a = r$ and minor axis $b = r\cos\theta$

Figure 2: Along this line, I can treat the strips as parallel to the bottom plate)



I know that there is a set radius $r$ of the plates, so I also know that $r^2 = x^2 + a^2 = x^2 + (y * \sec\theta)^2$. Therefore I can integrate across $y$, using the semicircle $x = \sqrt{r^2 - (y\sec\theta)^2}$and multiplying by 2.

$C = \int_A \epsilon \frac{dA}{z}$

$y = \pm r\cos\theta\sqrt{1 - \frac{x^2}{r^2}}, x = \pm r\sqrt{1 - \frac{y^2}{(r\cos\theta)^2}}$

Therefore $dC$ can be written more accurately:

$dC = 2\epsilon\frac{r\sqrt{1-\frac{y^2}{(r\cos\theta)^2}}}{z_0+z_t+y\tan\theta}dy$

First, look at case $\theta = 0, z_t = 0$

$dC = 2\epsilon\frac{\sqrt{r^2-y^2}}{z_0}dy$

$C = \frac{2\epsilon}{z_0}\int_{-r}^{r}\sqrt{r^2 - y^2}dy$

$C = \frac{2\epsilon}{z_0}\frac{1}{2}(y\sqrt{r^2 - y^2} + r^2 \arctan\frac{y}{\sqrt{r^2-y^2}})|_{-r}^{r}$

$C = \frac{\epsilon}{z_0}(0 + \frac{r^2\pi}{2} - 0 - \frac{-r^2\pi}{2})$

$C = \frac{\epsilon\pi r^2}{z_0}$

Now try with arbitrary $z_t$ and $\theta$

Thus $C = 2\epsilon r\int_{-r\cos\theta}^{r\cos\theta}\frac{\sqrt{1-\frac{y^2}{(r\cos\theta)^2}}}{z_0+z_t+y\tan\theta}dy$

Let $y = b\sin\phi, dy = r\cos\theta\cos\phi d\phi$

$C = 2\epsilon r^2\cos\theta\int_{-\pi/2}^{\pi/2}\cos\phi\frac{\sqrt{1-\sin^2\phi}}{z_0+z_t+r\sin\theta\sin\phi}d\phi$

# 5 Everything Below Here are Other Attempts That Don't Work

$$C = 2\epsilon r^2\cos\theta\frac{\cos\phi(-2\sqrt{z_0^2+2z_0z_t+z_t^2-r^2\sin^2\theta}\arctan\frac{(z_0+z_t)\tan\phi/2+r\sin\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-r^2\sin^2\theta}}+\phi(z_0+z_t)+r\sin\theta\cos\phi)}{r^2\sin^2\theta\sqrt{\cos^2\phi}}|_{-\pi/2}^{\pi/2}$$

$C = 2\epsilon r^2\cos\theta(\frac{-2\sqrt{z_0^2+2z_0z_t+z_t^2-r^2\sin^2\theta}\arctan(z_0+z_t)+r\sin\theta}{r^2\sin^2\theta} - \frac{-2\sqrt{z_0^2+2z_0z_t+z_t^2-r^2\sin^2\theta}\arctan -(z_0+z_t)+r\sin\theta}{r^2\sin^2\theta})$

Now look at $\theta = 0, z_t = 0$

$C = 2\epsilon r^2(\frac{-2\sqrt{z_0^2}\arctan z_0}{r^2\sin^2 0} - \frac{-2\sqrt{z_0^2}\arctan -z_0}{r^2\sin^2 0})$

$C = 2\epsilon\int_{-r}^{r}\frac{x*dy}{z_0+z_1+a\sin\theta} = 2\epsilon\int_{-r}^{r}\frac{\sqrt{r^2-y\sec\theta^2}*dy}{z_0+z_1+y\tan\theta}$
Let $z = r\sin\phi, da = dr\sin\phi$

Let $a = r\cos\phi, da = dr\cos\phi$ $C = 4\epsilon\int_{-\pi/2}^{\pi/2}\frac{r\sin\phi\cos\phi}{z_0+z_t+r\sin\theta\cos\phi}d\phi$ $C = 4\epsilon(\frac{(z_0+z_t)\log(z_0+z_t+r\sin\theta\cos\phi)-r\sin\theta\cos\phi}{r^2\sin^2\theta})$

$C = 4\epsilon(\frac{(z_0+z_t)\log(z_0+z_t)}{r^2\sin^2\theta})|_{-\pi/2}^{\pi/2}$

$C = \frac{2\epsilon}{\sin^2\theta}(-\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}\log(\sqrt{r^2-a^2}\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}+z_0a+z_ta+\sin\theta r$ $\sqrt{-z_0{}^2-2z_0z_t-z_t{}^2+\sin^2\theta r^2}\log(z_0+z_t+\sin\theta a)+(z_0+z_t)\arctan\frac{a}{\sqrt{r^2-a^2}}+$ $c\sqrt{r^2-a^2})|_{-r}^{r}$ $C = \frac{2\epsilon}{\sin^2\theta}(-\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}\log(z_0r+z_tr+\sin\theta r^2)+$ $\sqrt{-z_0{}^2-2z_0z_t-z_t{}^2+\sin^2\theta r^2}\log(z_0+z_t+\sin\theta r)+(z_0+z_t)\frac{\pi}{2}+\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}\log(-z_0r+-z$ $\sqrt{-z_0{}^2-2z_0z_t-z_t{}^2}\log(z_0+z_t-\sin\theta r)+(z_0+z_t)\frac{\pi}{2}) = \frac{1}{\sin^2\theta}(-\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}\log(z_0r+z_tr+$ $\sqrt{-z_0{}^2-2z_0z_t-z_t{}^2+\sin^2\theta r^2}\log(z_0+z_t+\sin\theta r)+(z_0+z_t)\pi+\sqrt{-z_0{}^2-2z_0z_t-z_t^2+\sin^2\theta r^2}\log(-z_0r+-z$ $\sqrt{-z_0{}^2-2z_0z_t-z_t{}^2}\log(z_0+z_t-\sin\theta r))$ Test Case: $z_t = 0, \theta = 0$ $C = \frac{1}{\sin^2 0}(-\sqrt{-z_0^2})\log(z_0r)+$ $\sqrt{z_0r}\log z_0+z_0\pi+\sqrt{-z_0^2}\log -z_0$

$C = \frac{1}{\sin^2\theta}(-z_0\log)$

$C = \epsilon\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}\cos\theta}^{\sqrt{r^2-x^2}\cos\theta}\frac{dy*dx*\sec\theta}{z_0+z_1+y\tan\theta} = \epsilon\sec\theta\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}\cos\theta}^{\sqrt{r^2-x^2}\cos\theta}\frac{dy*dx}{z_0+z_1+y\tan\theta}$

$C = \epsilon\sec\theta\int_{-r}^{r}2(\cot\theta\log(z_0+z_t+\sqrt{r^2-x^2}\sin\theta))dx = 2\epsilon\csc\theta\int_{-r}^{r}\log(z_0+z_t+\sqrt{r^2-x^2}\sin\theta)$

$C = \epsilon\csc^2\theta(-\sqrt{z_0^2+2z_0z_t+z_t^2-\sin^2\theta r^2}\arctan\frac{x(z_0+z_t)}{\sqrt{r^2-x^2}\sqrt{z_0^2+2z_0z_t+z_t^2-\sin^2\theta r^2}}+\sqrt{z_0^2+z_t^2+2z_0z_t-\sin^2\theta r^2}$ $\sin\theta x)|_-^r r$ $C = \epsilon\csc\theta(-\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta r^2}\arctan\infty+\sqrt{z_0^2+z_t^2+2z_0z_t-\tan^2\theta r^2}\arctan\frac{\tan\theta}{\sqrt{z_0^2+2z_0z_t+z}}$ $\tan\theta r\log(z_0+z_t)+(z_0+z_t)\arctan\infty-\tan\theta r-(-\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta r^2}\arctan -\infty+$ $\sqrt{z_0^2+z_t^2+2z_0z_t-\tan^2\theta r^2}\arctan\frac{-\tan\theta r}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta r^2}}-\tan\theta r\log(z_0+z_t)+$ $(z_0+z_t)\arctan -\infty+\tan\theta r))$

$C = \epsilon\csc\theta\cot\theta(-\pi\sqrt{z_0^2+z_t^2+2z_0z_t-\tan^2\theta r^2}+2\sqrt{z_0^2+z_t^2+2z_0z_t-\tan^2\theta r^2}\arctan\frac{\tan\theta r}{\sqrt{z_0^2+z_t^2+2z_0z_t-\tan^2\theta r^2}}$ $2\tan\theta r\log z_0+z_t+\pi(z_0+z_t)-2\tan\theta r)$

Let $x = r\cos t$, $dx = -r\sin t dt$, where $t$ is the angle formed with $x$ and $a$ as shorter legs

$C = 4\epsilon\csc\theta\int_{-\pi/2}^{\pi/2}-r\sin t\log(z_0+z_t+\sin t\tan\theta)dt = -4r\epsilon\csc\theta\int_{-\pi/2}^{\pi/2}\sin t\log(z_0+z_t+\sin t\tan\theta)dt$

$C = 4\epsilon\csc\theta\cot\theta(\frac{1}{\tan\theta}(-2\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}\arctan\frac{(z_0+z_t)\tan\frac{t}{2}+\tan\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}}-$ $\tan\theta\cos t(\log(z_0+z_t+\tan\theta\sin t)-1)+t(z_0+z_t)))|_{-\pi/2}^{\pi/2}$

$C = 4\epsilon\csc\theta\cot\theta(\frac{1}{\tan\theta}(-2\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}\arctan\frac{(z_0+z_t)+\tan\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}})-$ $4\epsilon\csc\theta\cot\theta(\frac{1}{\tan\theta}(-2\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}\arctan\frac{-(z_0+z_t)+\tan\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}})$

$C = -16\epsilon\csc\theta\cot\theta\frac{1}{\tan\theta}\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}(\arctan\frac{(z_0+z_t)+\tan\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}}-$ $\arctan\frac{-(z_0+z_t)+\tan\theta}{\sqrt{z_0^2+2z_0z_t+z_t^2-\tan^2\theta}})$

Test Case: $\theta = 0, z_t = 0$

$C = \epsilon\csc\theta\cot\theta(-\pi\sqrt{z_0^2}+2\sqrt{z_0^2}\arctan 0$

$C = \sqrt{2}\epsilon\pi r^2 \cot a$ (cotangent goes to infinity as r approaches zero but the r squared going to zero makes the whole term go to zero)

# 6 Finding Electric Field

$V = Ed = Q/C$

I know the voltage of the plates from data, so I know $Q = CV$

$E = \frac{Q}{Cz} = \frac{Q}{z\sqrt{2}\epsilon\pi r^2 \cot a}$

$E = \frac{Q}{\sqrt{2}\epsilon\pi r^2 \cot a}$

# 7 Finding Relative s2 Peak

Assume for simplicity that m times more photons produced results in n times greater s2 peak.

$\frac{dn}{dz} = (\alpha * E) - (\beta * P) - (\gamma)$

$\frac{dn}{dz} = (\alpha * \frac{Q}{\sqrt{2}\epsilon\pi r^2 \cot a}) - (\beta * P) - (\gamma)$

$n = \int (\alpha * \frac{Q}{\sqrt{2}\epsilon\pi r^2 \cot a}) - (\beta * P) - (\gamma) dz$