

PLR  
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Fall 2018

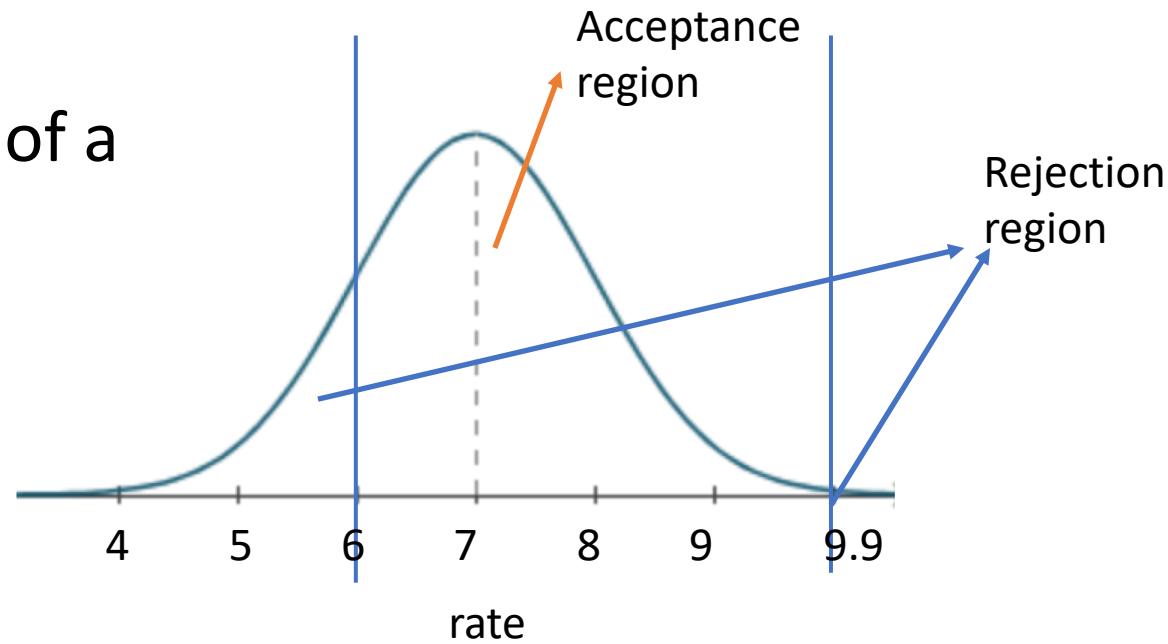
# Hypothesis Test(procedure)

- 1. Choose null hypothesis( $H_0$ ).
- 2. Choose a test statistic -> to determine whether reject  $H_0$  or not
- 3. PDF for the null hypothesis (probability vs. test statistic)
- 4. Determine the acceptance and rejection region
- 5. Compare the measurement(test statistic) of the data to the PDF and to see if it is within the acceptance region

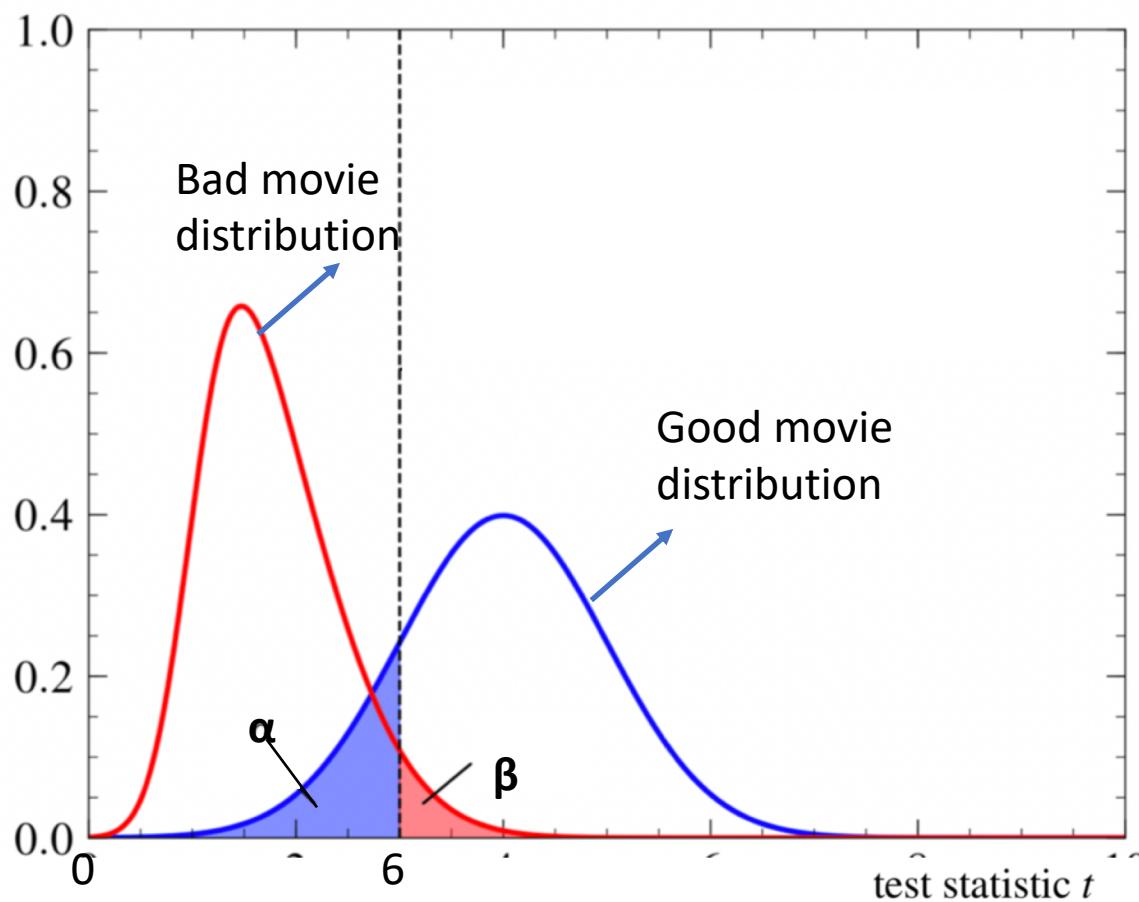
# Hypothesis Test(example)

- Null hypothesis: The X is a good movie
- Test statistic: The rate from IMDb

Distribution of a  
good move



- Adding an alternative hypothesis:  $H_1$ : The movie X is a bad movie

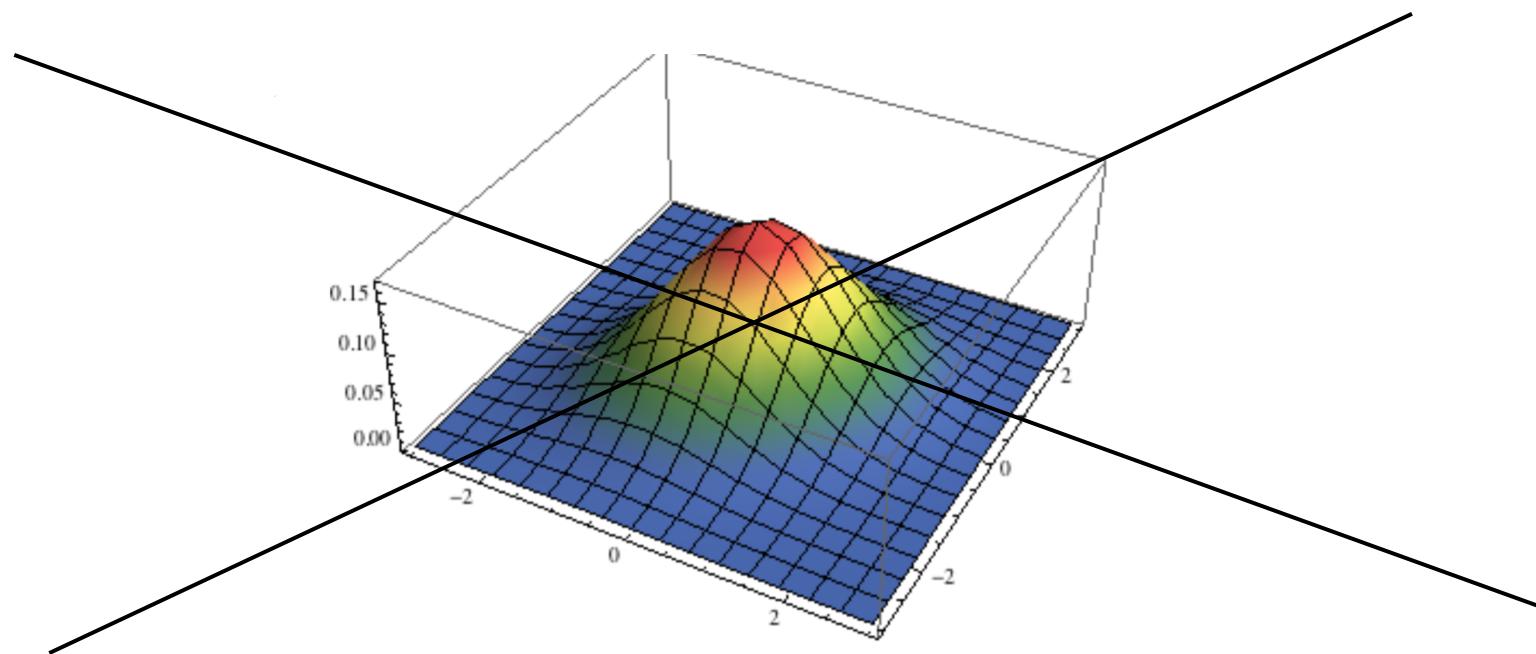


**$\alpha$ : Type 1 error: Reject  $H_0$ , even though it is true**

**$\beta$ : Type 2 error: Reject  $H_1$ , even though it is true**

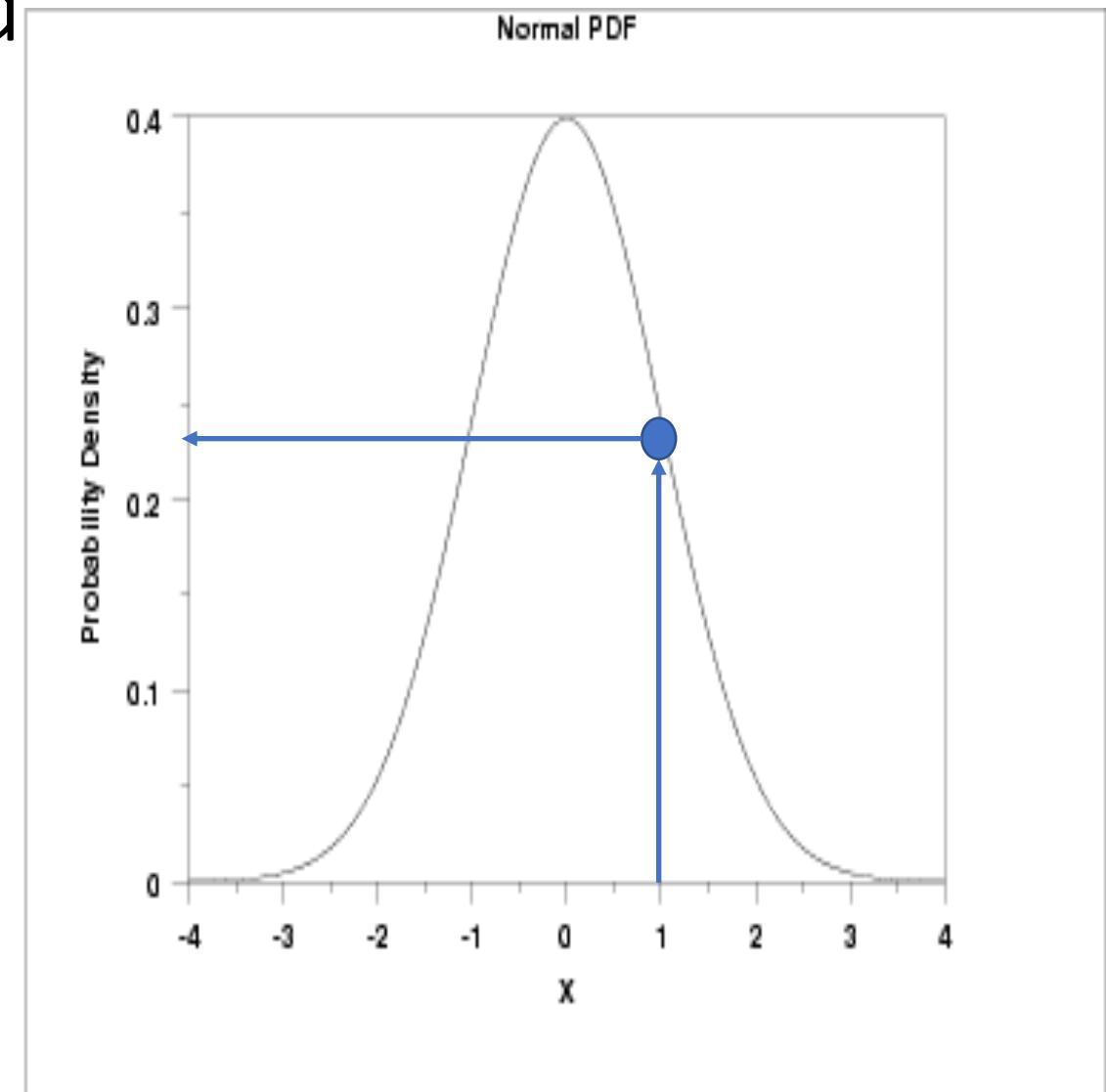
# Likelihood

- We want test statistic to be a function of multiple observables
  - Test statistic is a function of rate, box-office grosses.....
- Instead of creating N-D PDF, we use likelihood



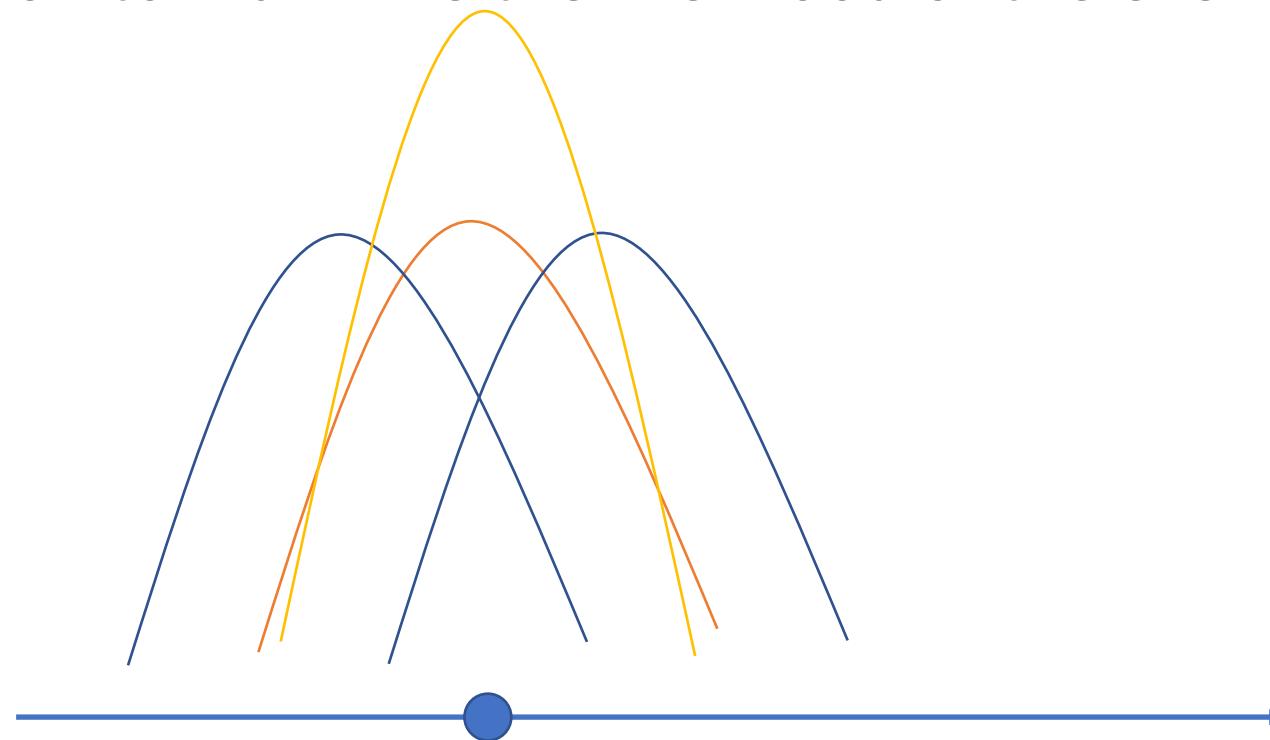
# Probability vs. Likelihood

- Probability: given the fixed distribution(PDF), the probability that an event can occur
  - Fixed PDF/model parameter
  - Unfixed observed data
- Likelihood: Given the event is true, the probability that this will happen
  - Unfixed parameters(shape or location of the distribution)
  - Fixed observed data



# Maximize the likelihood

- So for likelihood, we can change the location and the shape of the distribution to maximize the likelihood of the event.



# PLR

Parameter of Interest  
(cross section)

$$\lambda(\mu) = \frac{L(\mu, \hat{\theta})}{L(\hat{\mu}, \hat{\theta})}.$$

H0      H1

Nuisance parameters

- $L(\hat{u}, \theta) = P(\hat{u}, x) * \text{Gaus}(\hat{u}, \theta)$ ,  $\theta$  is the nuisance parameter and float to maximize  $L$ 
  - $\hat{u} = f(\theta)$

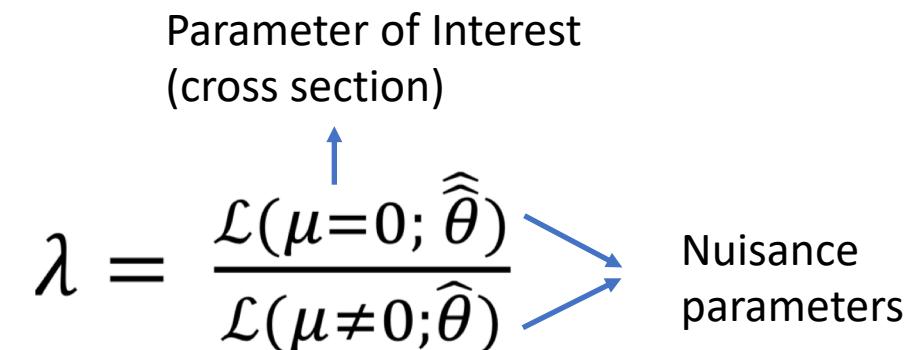
# PLR for WIMPs

- Test Statistic:

$$\lambda = \frac{\mathcal{L}(\mu=0; \hat{\bar{\theta}})}{\mathcal{L}(\mu \neq 0; \hat{\bar{\theta}})}$$

Parameter of Interest  
(cross section)

Nuisance  
parameters



- Discover WIMP

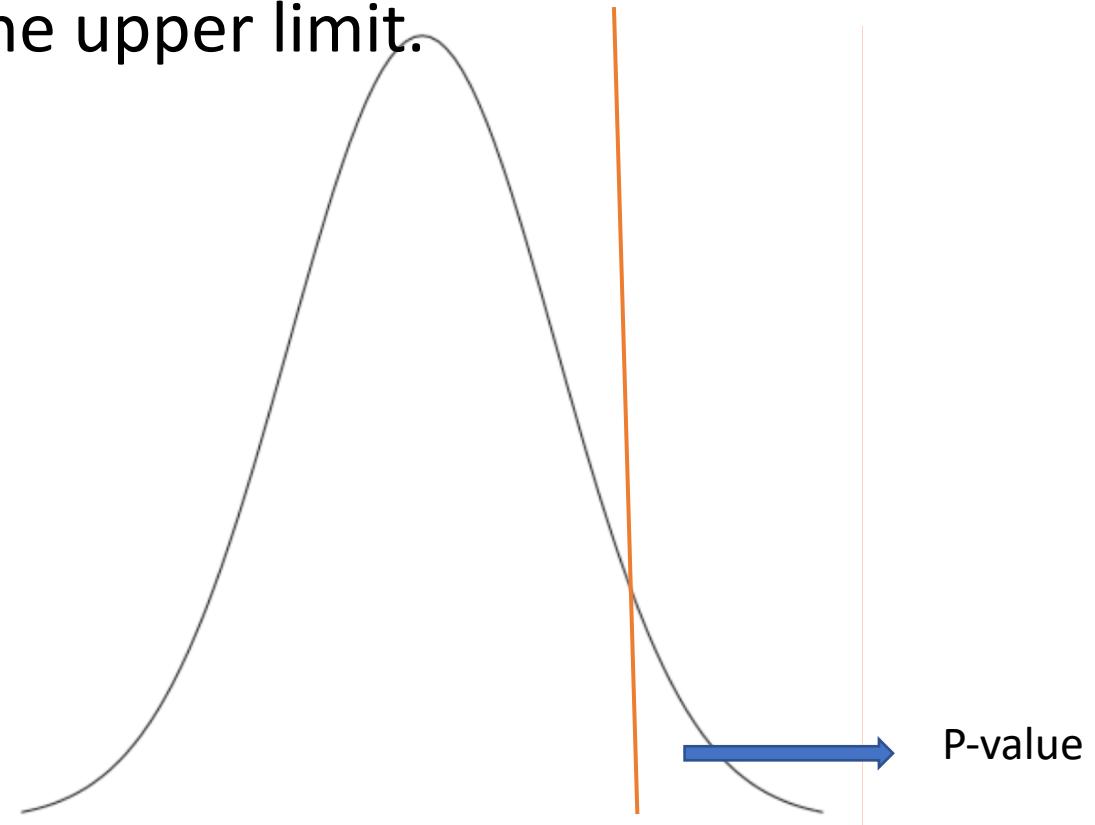
- H0: There are no WIMPs,  $u=0$  (Background only)
- H1: There are WIMPs,  $u \neq 0$

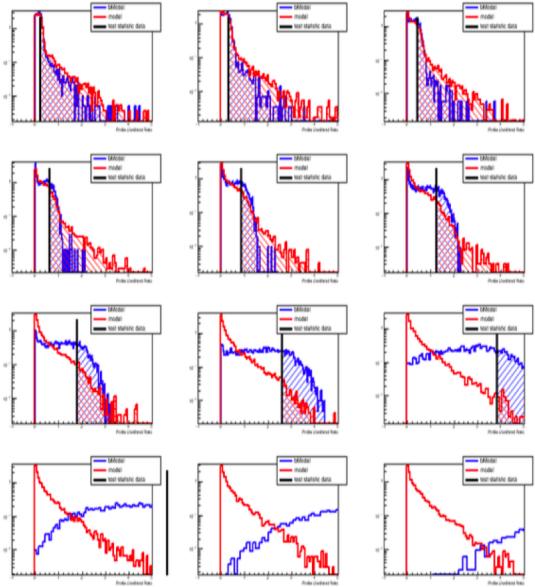
- WIMP limit set

- H0: WIMPs exist with specific  $u = u_{\text{test}}$  (signal+background)
- H1: WIMPs exist with  $u \neq u_{\text{test}}$

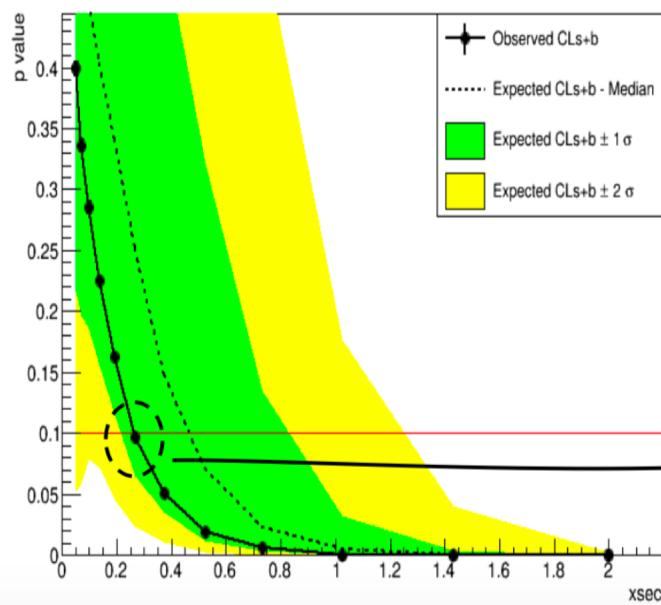
# Setting the limit

- For WIMP with each mass, varying  $u_{\text{test}}$  and when the  $u_{\text{test}}$  leads to a  $p\text{-value}=0.1$ , the  $u_{\text{test}}$  is the upper limit.
- Then varying the WIMP mass

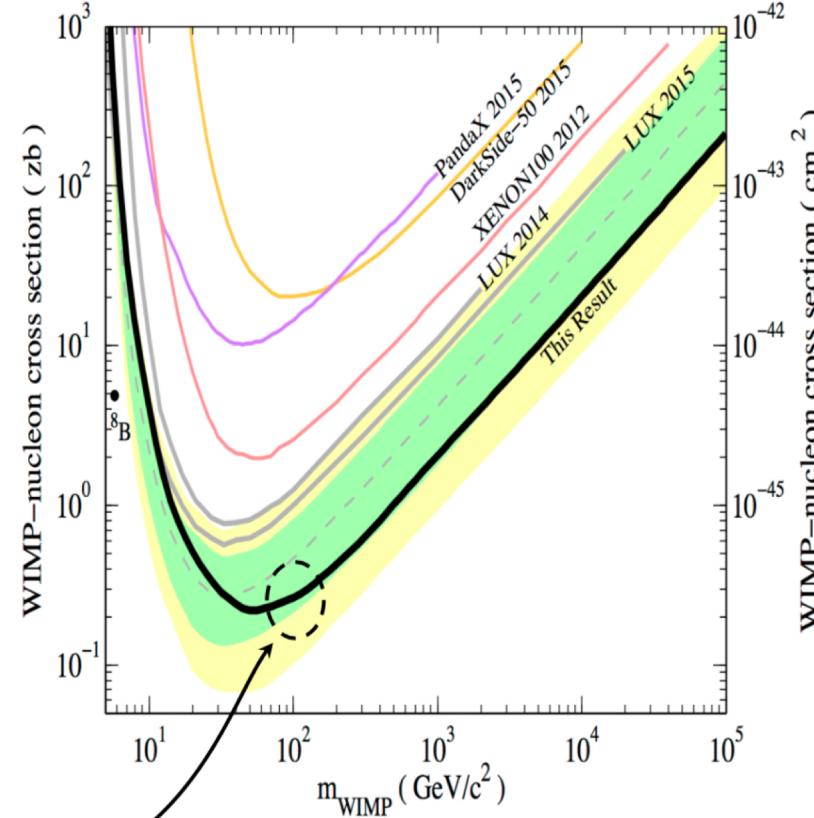




HypoTestInverter Result For xsec



# Limit Setting for One Mass Illustration



Slide from Shaun

# TO DO

- Structure of the code

└ laidbax 

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└ laidbax.egg-info

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└ notebooks 

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└ old\_notebooks 

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└ HISTORY.rst

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└ laidbax\_logo.png

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└ LICENSE

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└ MANIFEST.in

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└ README.rst

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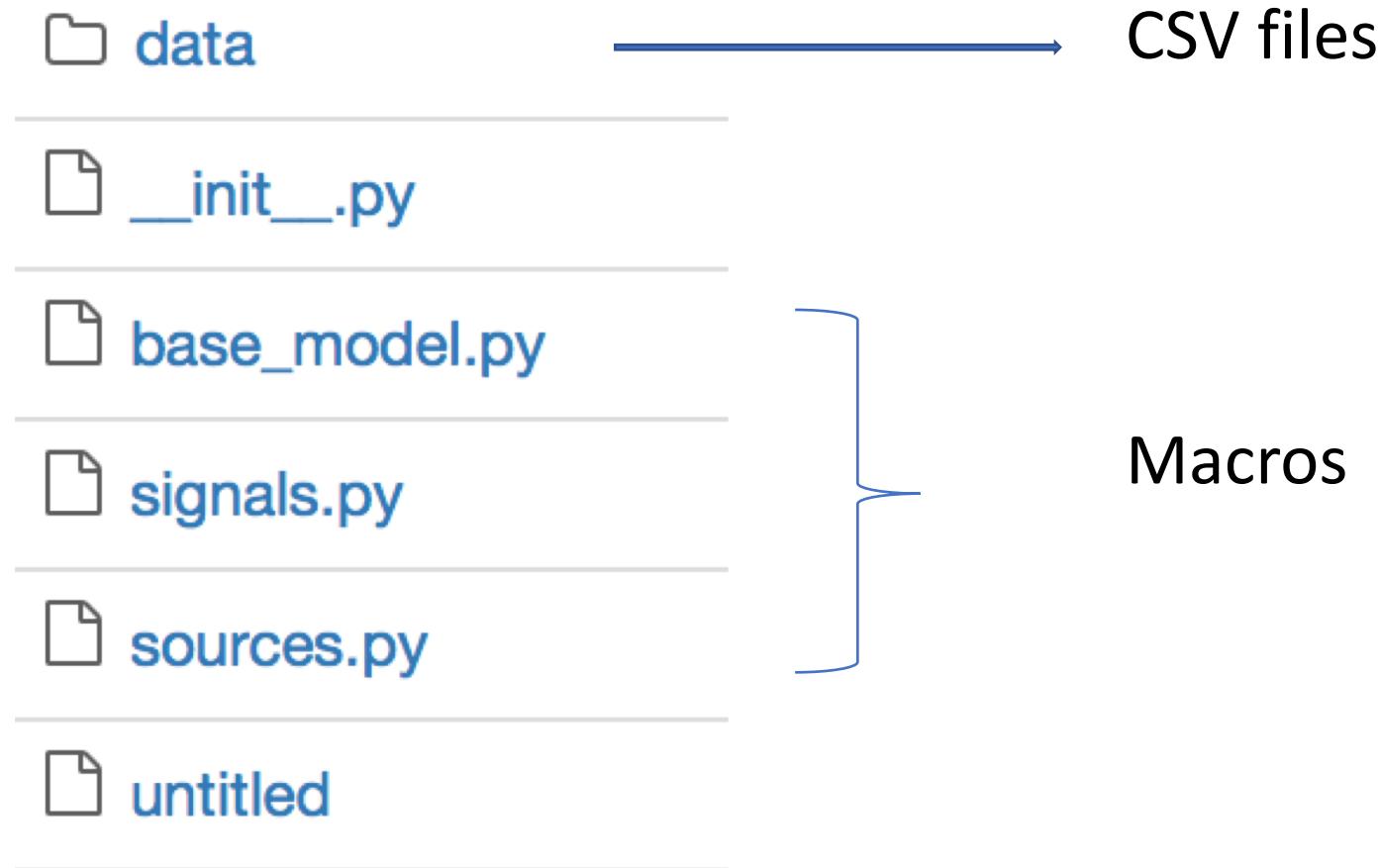
└ requirements.txt

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└ setup.py

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# Laidbax



# Notebooks

 [pdf\\_cache](#)

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 [Basic checks.ipynb](#)

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 [SR0 limit.ipynb](#)

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 [Tutorial.ipynb](#)

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 [WIMP limits.ipynb](#)

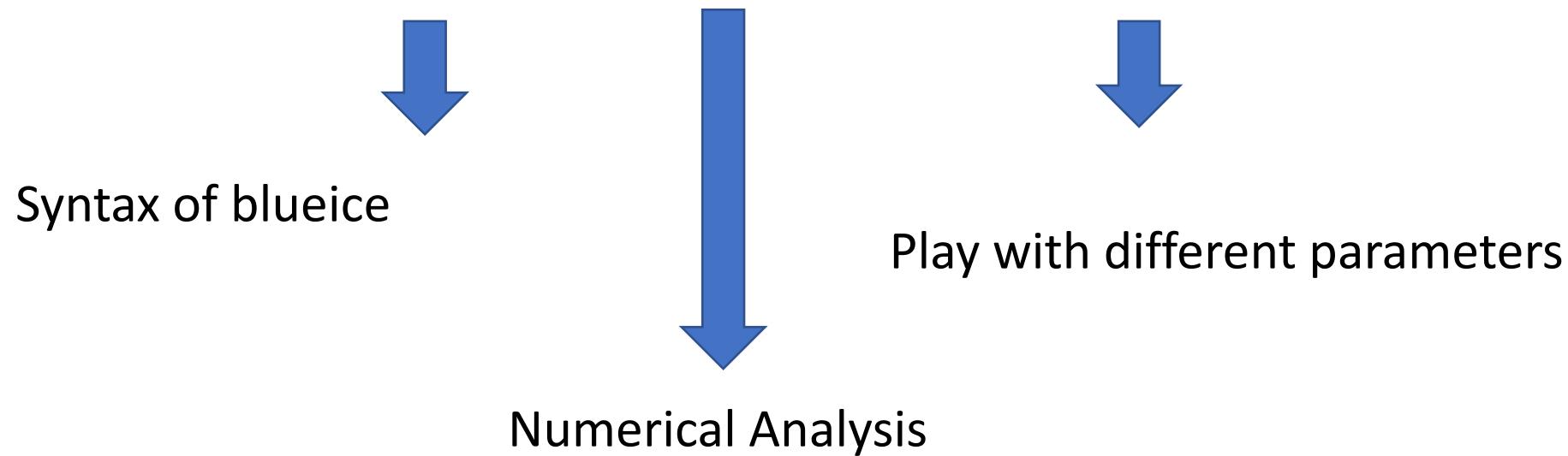
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 [compute\\_limits.py](#)

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# This week

Figure out what is going on in the code, WIMP limit



# General idea of the code

Use blueice to create 1) likelihood function and 2) its interval to find the roots by using Brent's method and the roots will be plotted in the limit graph as the cross-section.

For likelihood function: 1) adds two rate parameters: WIMP recoil, electron recoil 2) adds 1 shape parameter: WIMP mass  
(why is the WIMP recoil rate parameter = 0 to make a discovery?)

# Problem

- Set cross section =  $1e^{-45}$ , wimp interaction = SD, got an error.

```
~/Desktop/blueice/blueice/inference.py in one_parameter_interval(lf, target, bound, confidence_level, kind, bestfit_r
outine, t_ppf, **kwargs)
  376     return brentq(t, bound, global_best, args=[1 - confidence_level])
  377     elif kind == 'upper':
--> 378     return brentq(t, global_best, bound, args=[confidence_level])
  379
  380

/anaconda3/lib/python3.7/site-packages/scipy/optimize/zeros.py in brentq(f, a, b, args, xtol, rtol, maxiter, full_out
put, disp)
  508     if rtol < _rtol:
  509         raise ValueError("rtol too small (%g < %g)" % (rtol, _rtol))
--> 510     r = _zeros._brentq(f,a,b,xtol,rtol,maxiter,args,full_output,disp)
  511     return results_c(full_output, r)
  512

ValueError: f(a) and f(b) must have different signs
```

# Interpretation of the syntax

- `one_parameter_interval(lf, target, bound, confidence_level, kind, bestfit_routine, t_ppf, **kwargs)`
- Set a `confidence_level`(100%) interval of `kind`(a range of WIMP mass) on the parameter `target`(WIMP rate multiplier) of likelihood function

`lf` = likelihood function

`target` = WIMP rate multiplier

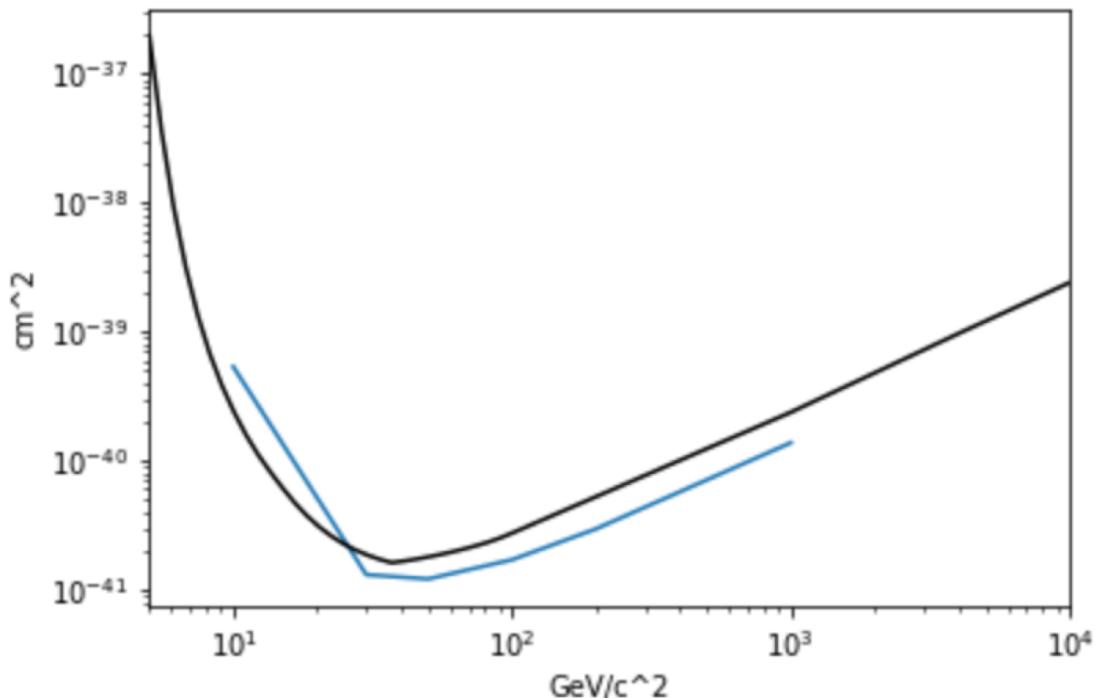
`bound` = 10

`kind` = WIMP mass

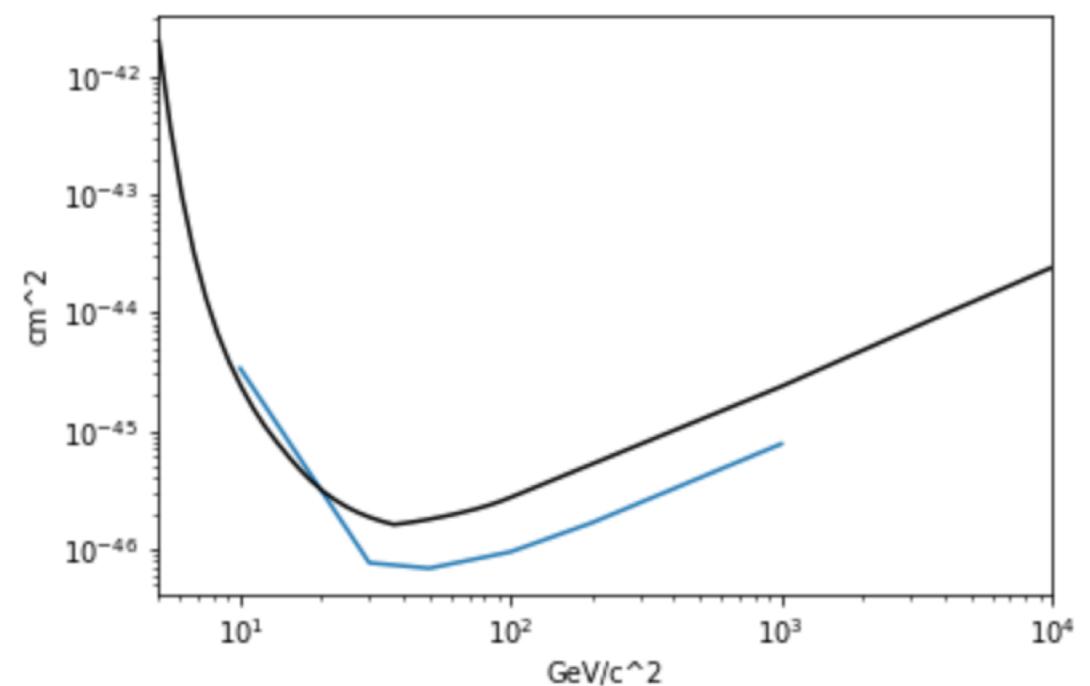
# What I think.....

- Using Brent's method to find roots of the function and it requires  $f(a)$  and  $f(b)$  have different signs.
- Brent's Method: iteration of (bisection method + secant method)(I could make a slide on this method, if anyone is interested.....)
- When set CS to  $1e^{-45}$ , the likelihood is too small, and the function curve is around zero-axis.
- Solution: increase the likelihood by decreasing the WIMP energy in the macro ---> No Effect
- Does it worth to dig more into it?

Cross-section:  $1e^{-40}$   
WIMP Interaction:  
SD\_n\_central (spin dependent)

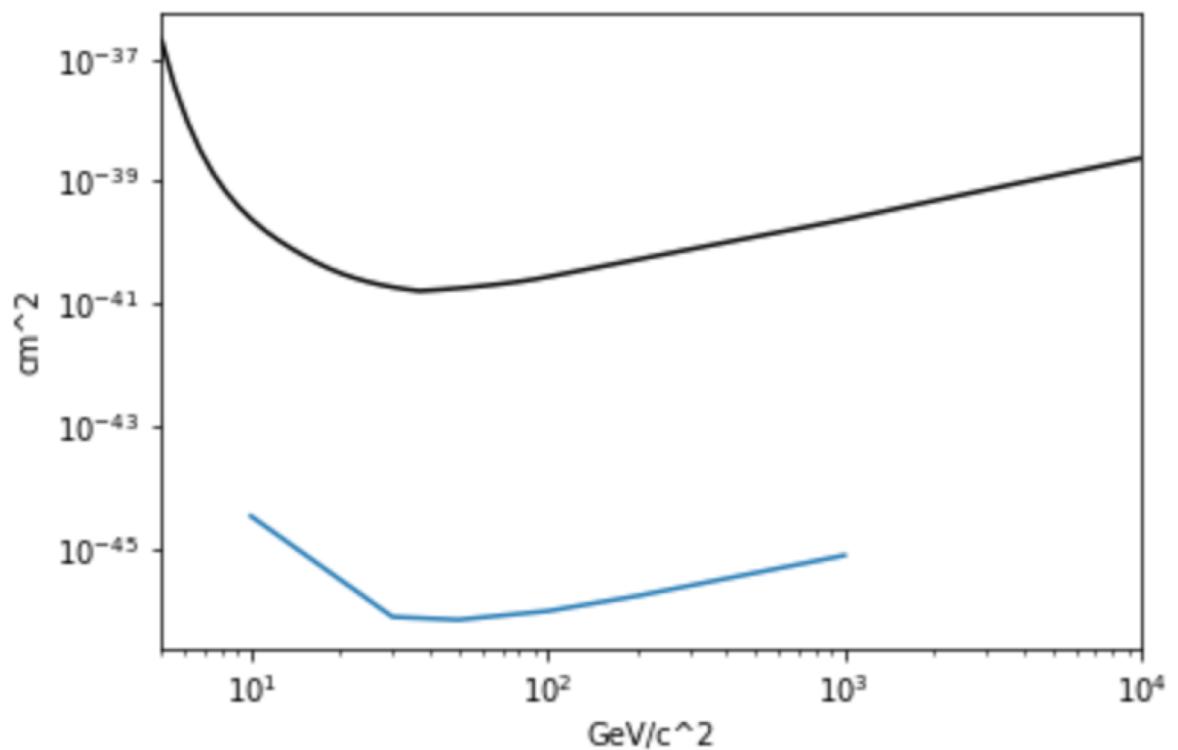


Cross section:  $1e^{-45}$   
WIMP Interaction: SI  
(spin independent)



Cross-section:  $1e^{-40}$

WIMP Interaction: SI

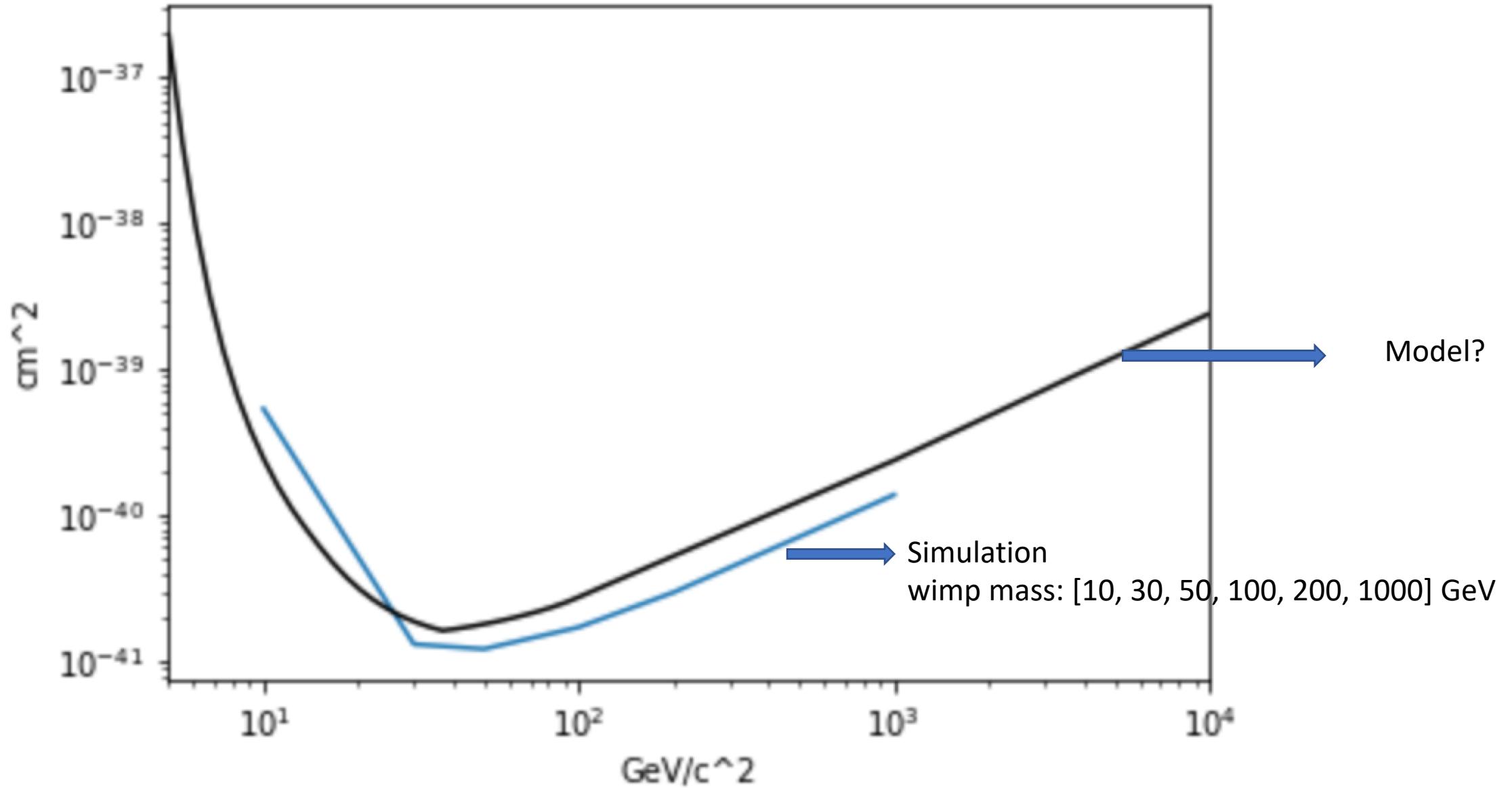


# Next Week

- Analyze the simulation Macro
- Any remaining/new problems from this meeting

# Macro(Xenon1T)

- Set Sources: energy distribution: `er_background + nr_background`(  
cnns and radiogenic neutrons) + 50Gev  $10^{-45}$  WIMP(nr)
- Set parameters



# Info for the black line

5 GeV	10 GeV	$10^2$ GeV	$10^3$ GeV	$10^4$ GeV
$1.98e^{-37}$ cm <sup>2</sup>	$2.37e^{-40}$ cm <sup>2</sup>	$2.75e^{-41}$ cm <sup>2</sup>	$2.38e^{-40}$ cm <sup>2</sup>	$2.41e^{-39}$ cm <sup>2</sup>

# Base model(Xenon 1T)

- WIMP Parameters:

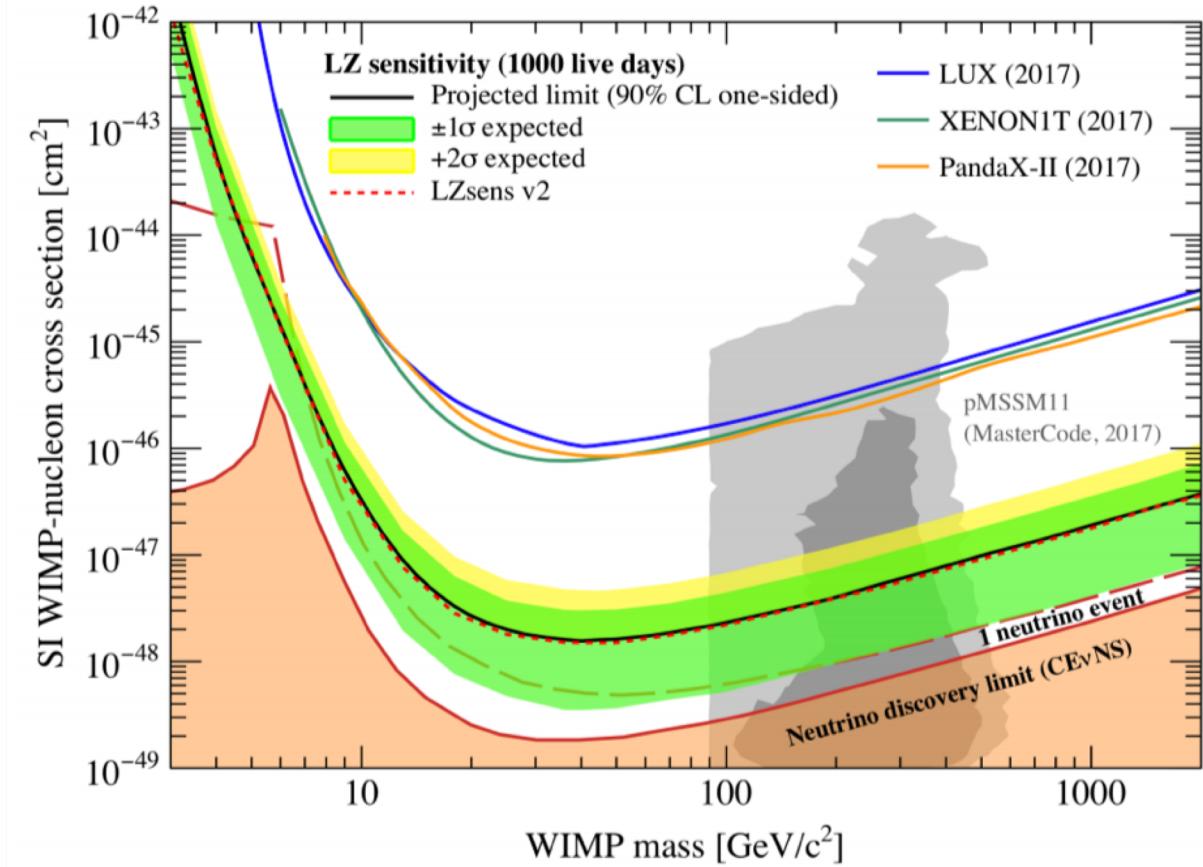
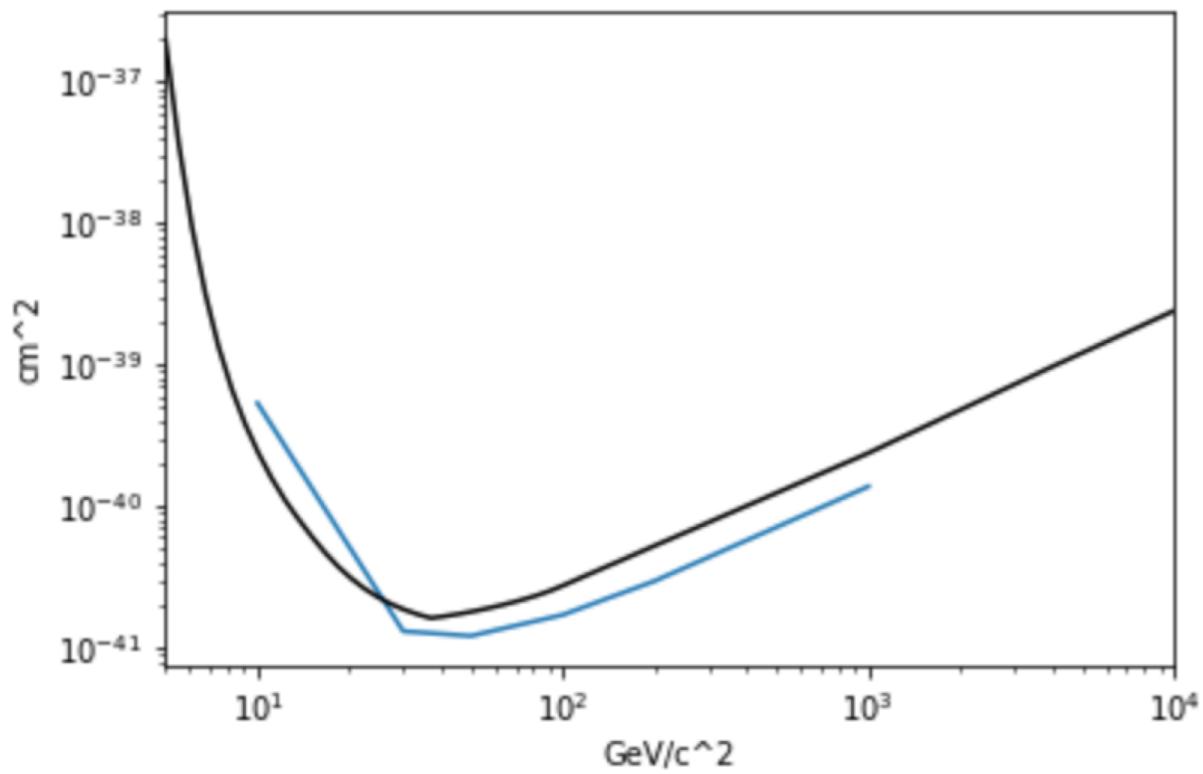
Detection	Sigma_nucleon	Wimp mass	NR cutoff	ER cutoff	WIMP interaction
Elastic NR	$1e^{-45} \text{ cm}^2$ $\rightarrow 10 e^{-40} \text{ cm}^2$	50 Gev/c <sup>2</sup>	1 keV	0.18keV	SI $\rightarrow SD\_n\_central$

- Detector parameters:

Fiducial mass	Electron lifetime	Drift velocity
1042 kg	452 us	1.44 m/ms

Pdf\_sampling\_multiplier:1->0.1

# Wimp limits curve



# Questions?

- What other parameters do I need to know/change?