

PLR
Yitong Liu
Fall 2018

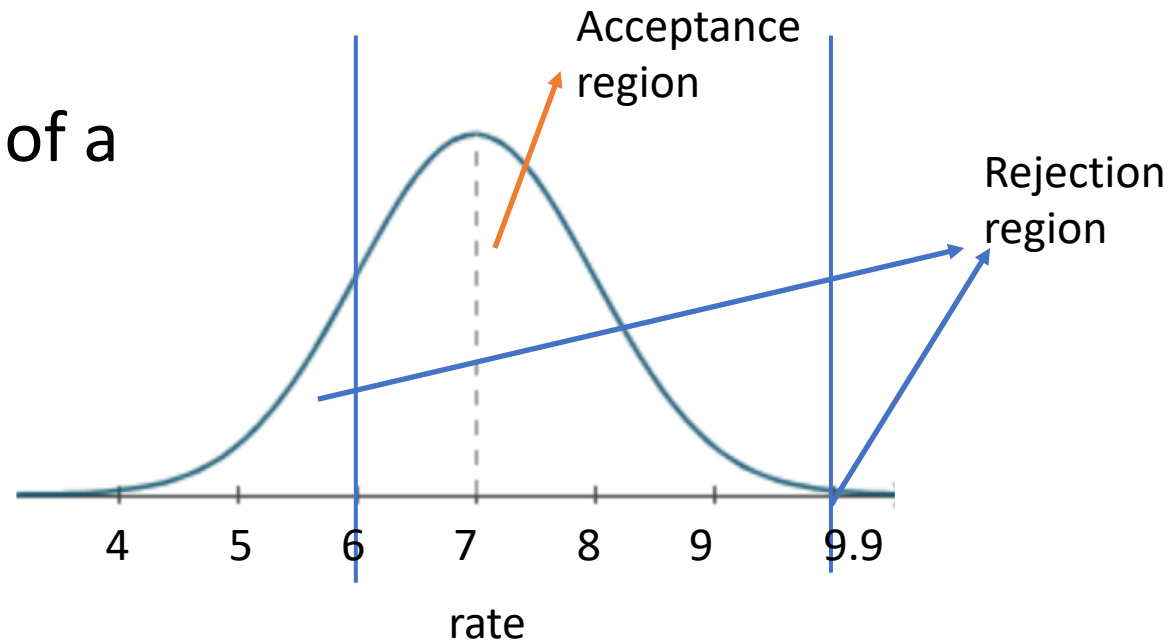
Hypothesis Test(procedure)

- 1. Choose null hypothesis(H_0).
- 2. Choose a test statistic -> to determine whether reject H_0 or not
- 3. PDF for the null hypothesis (probability vs. test statistic)
- 4. Determine the acceptance and rejection region
- 5. Compare the measurement(test statistic) of the data to the PDF and to see if it is within the acceptance region

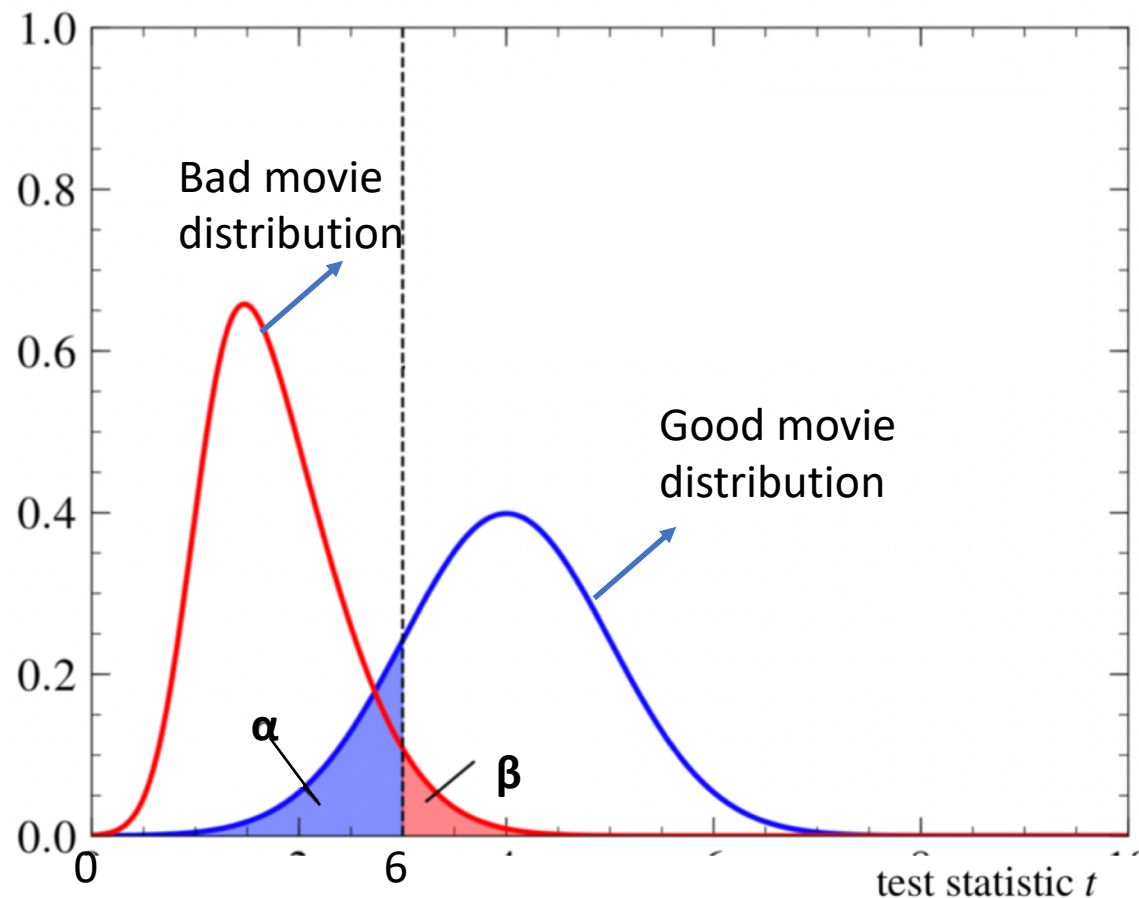
Hypothesis Test(example)

- Null hypothesis: The X is a good movie
- Test statistic: The rate from IMDb

Distribution of a
good move



- Adding an alternative hypothesis: H_1 : The movie X is a bad movie

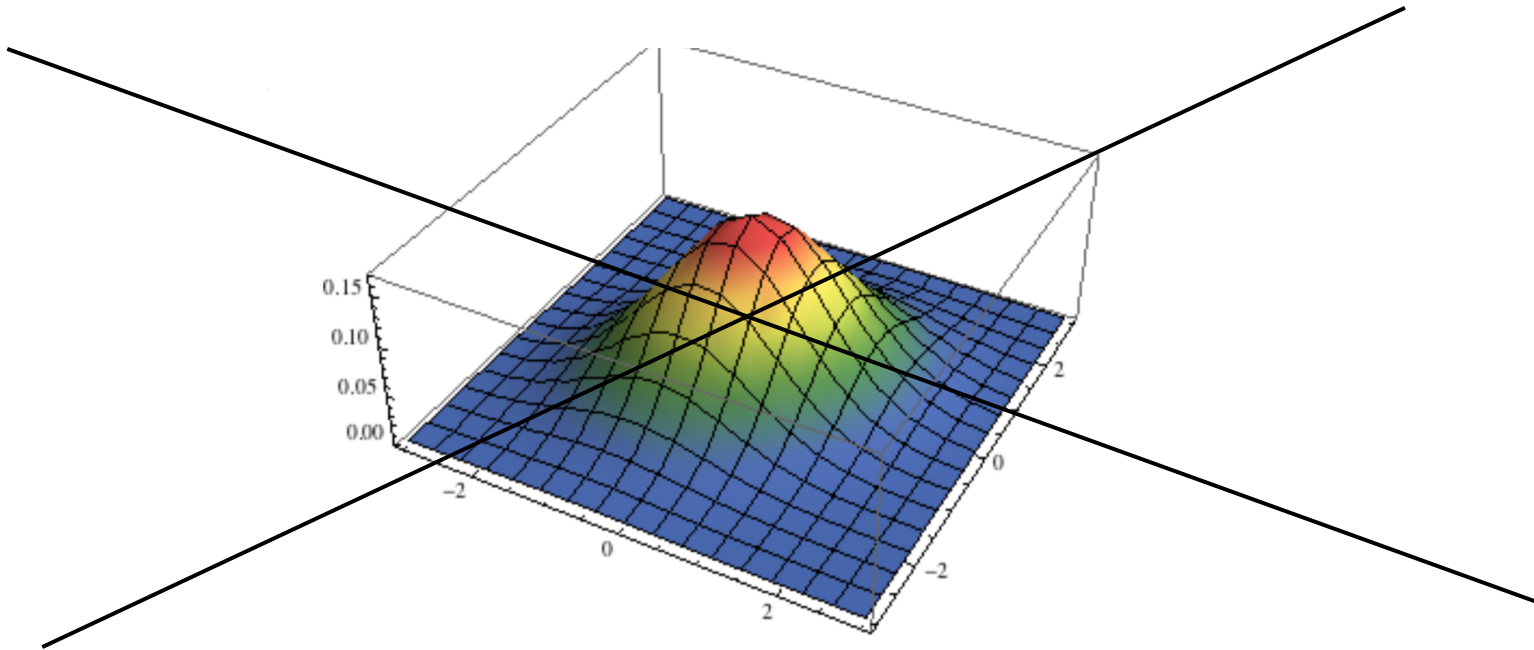


α : Type 1 error: Reject H_0 , even though it is true

β : Type 2 error: Reject H_1 , even though it is true

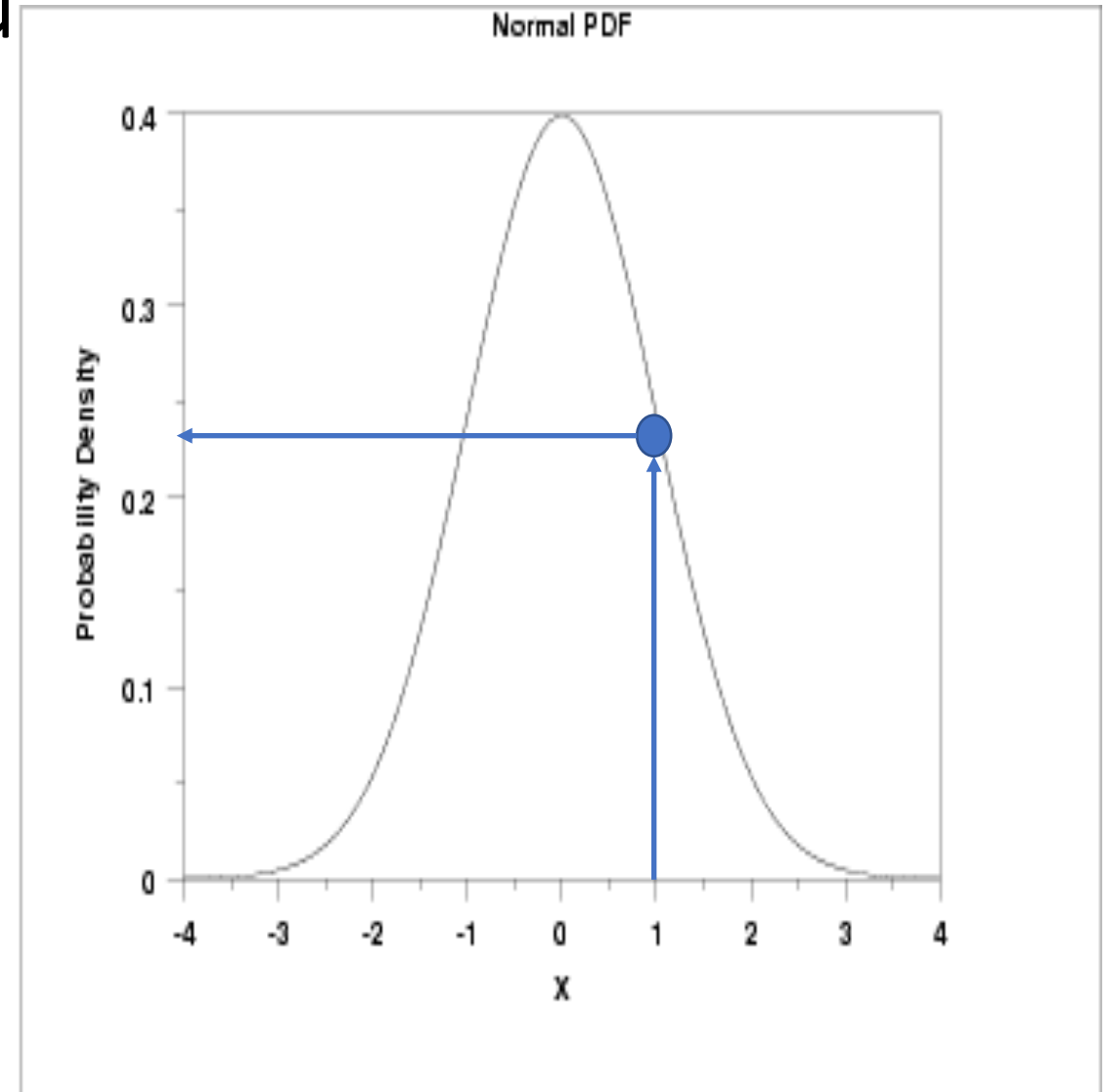
Likelihood

- We want test statistic to be a function of multiple observables
 - Test statistic is a function of rate, box-office grosses.....
- Instead of creating N-D PDF, we use likelihood



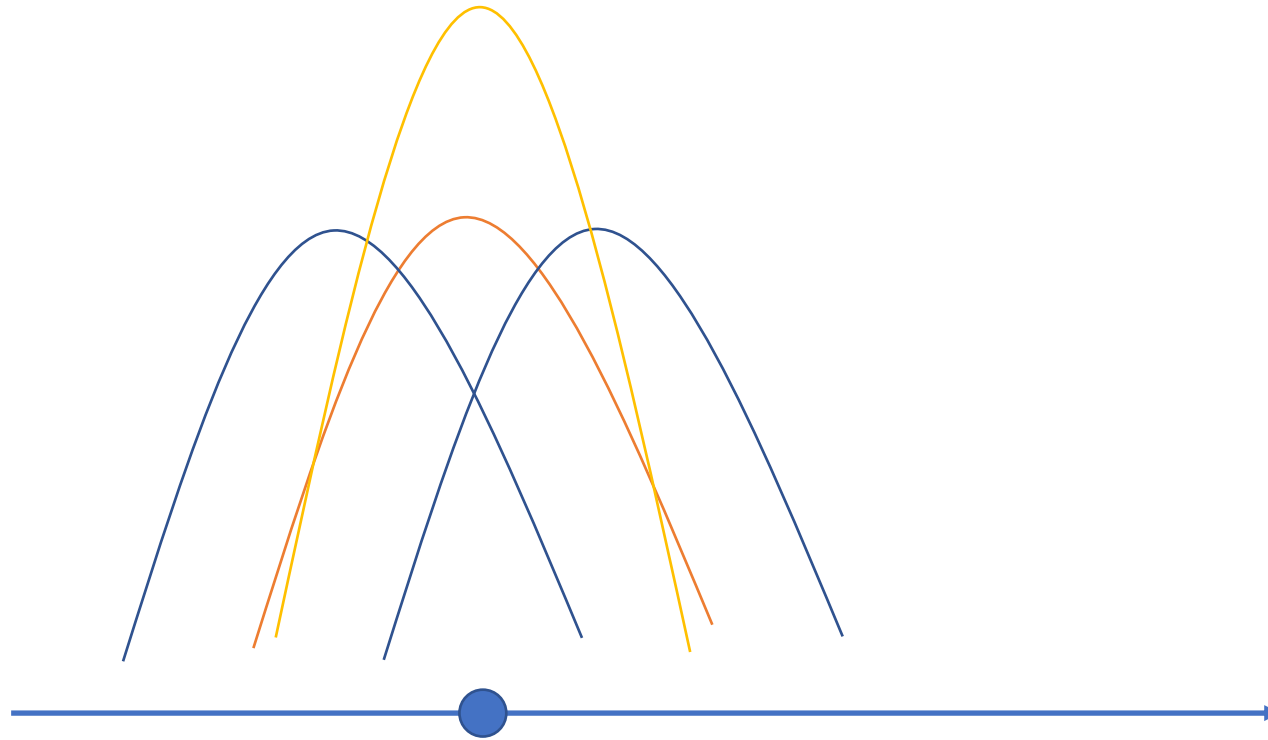
Probability vs. Likelihood

- Probability: given the fixed distribution(PDF), the probability that an event can occur
 - Fixed PDF/model parameter
 - Unfixed observed data
- Likelihood: Given the event is true, the probability that this will happen
 - Unfixed parameters(shape or location of the distribution)
 - Fixed observed data

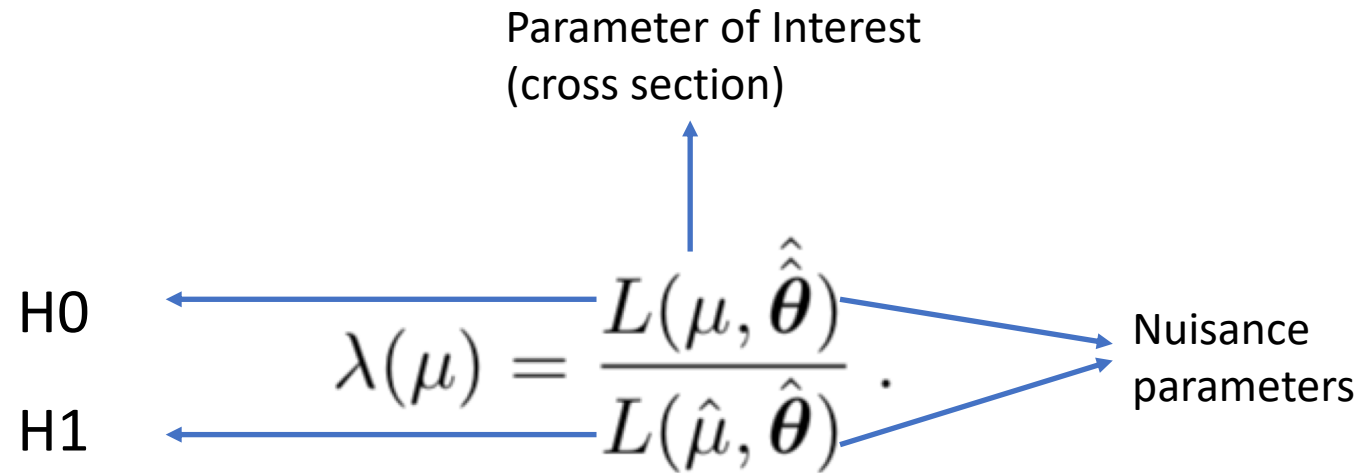


Maximize the likelihood

- So for likelihood, we can change the location and the shape of the distribution to maximize the likelihood of the event.



PLR



- $L(\hat{u}, \theta) = P(\hat{u}, x) * \text{Gaus}(\hat{u}, \theta)$, θ is the nuisance parameter and float to maximize L
 - $\hat{u} = f(\theta)$

PLR for WIMPs

- Test Statistic:

Parameter of Interest
(cross section)

$$\lambda = \frac{\mathcal{L}(\mu=0; \hat{\hat{\theta}})}{\mathcal{L}(\mu \neq 0; \hat{\theta})}$$

Nuisance parameters

- Discover WIMP

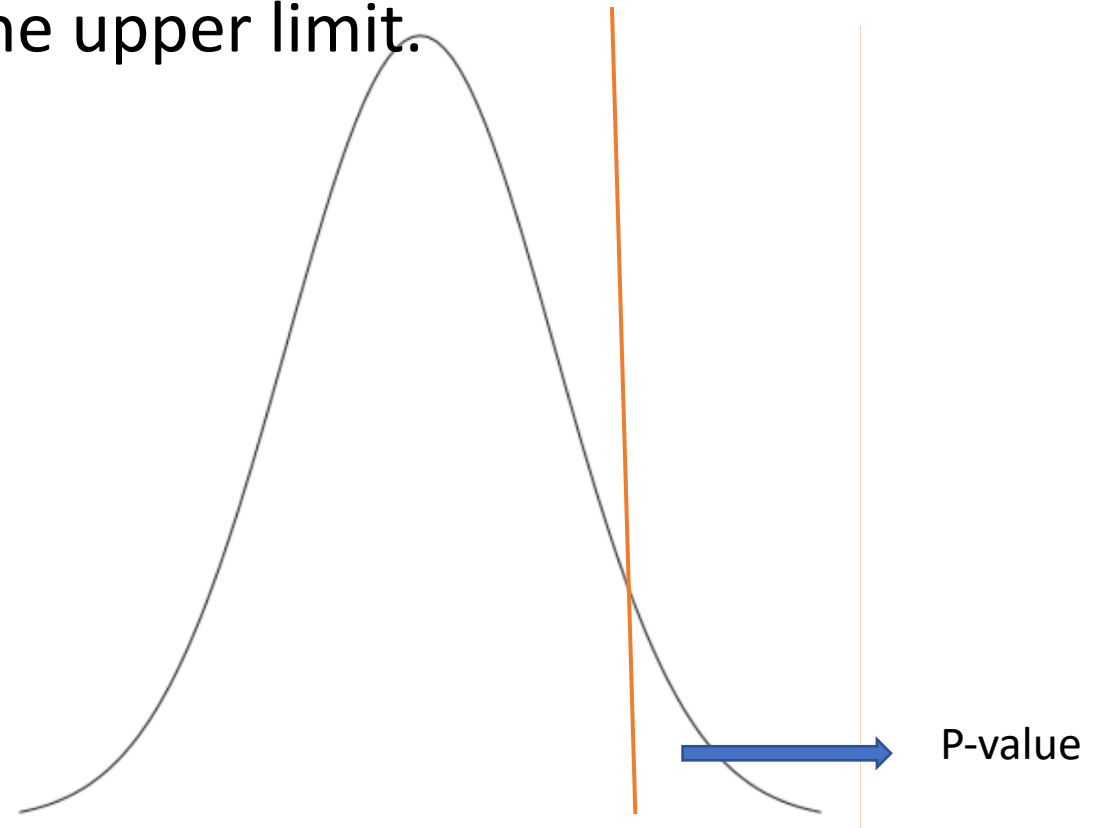
- H0: There are no WIMPs, $u=0$ (Background only)
- H1: There are WIMPs, $u \neq 0$

- WIMP limit set

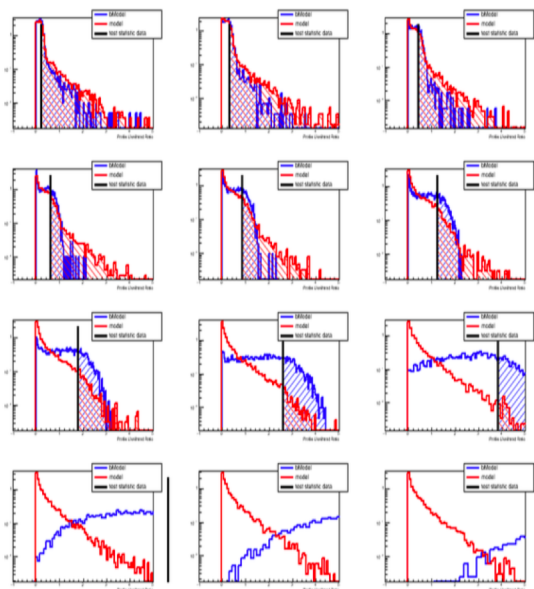
- H0: WIMPs exist with specific $u = u_{\text{test}}$ (signal+background)
- H1: WIMPs exist with $u \neq u_{\text{test}}$

Setting the limit

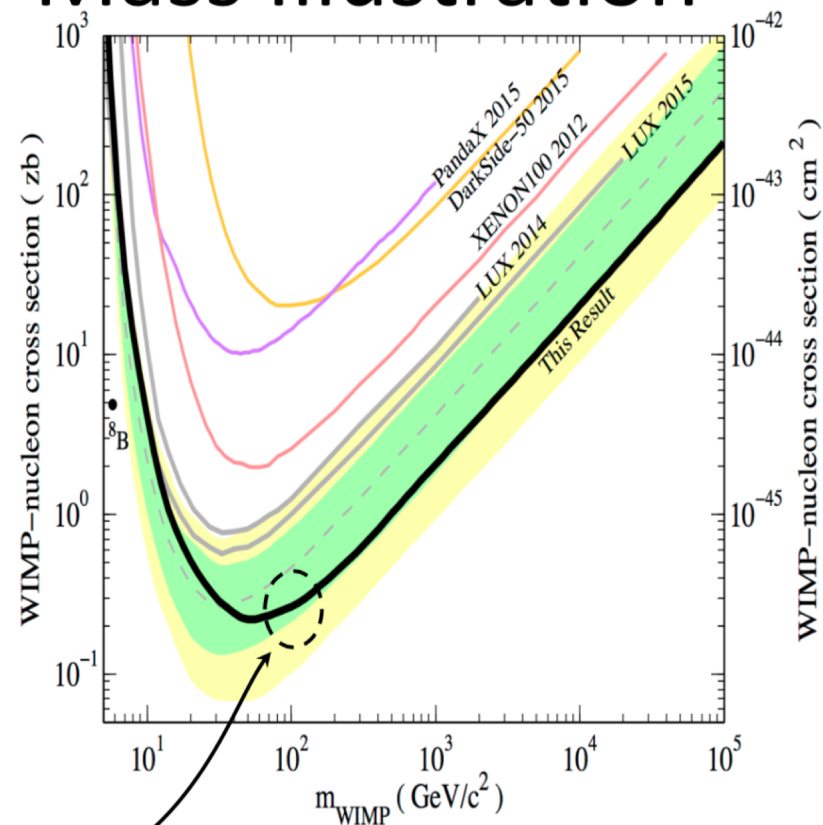
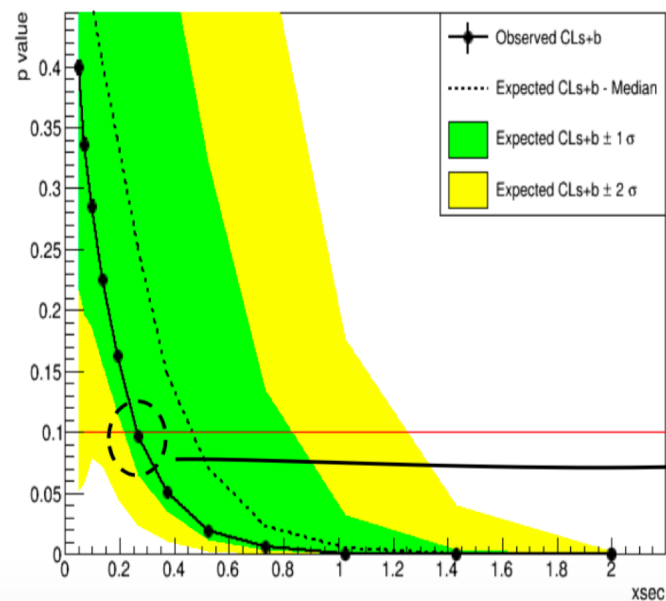
- For WIMP with each mass, varying u_{test} and when the u_{test} leads to a $p\text{-value}=0.1$, the u_{test} is the upper limit.
- Then varying the WIMP mass



Limit Setting for One Mass Illustration



HypoTestInverter Result For xsec



Slide from Shaun

TO DO

- Structure of the code

 `laidbax` 

 `laidbax.egg-info`

 `notebooks` 

 `old_notebooks` 

 `HISTORY.rst`

 `laidbax_logo.png`

 `LICENSE`

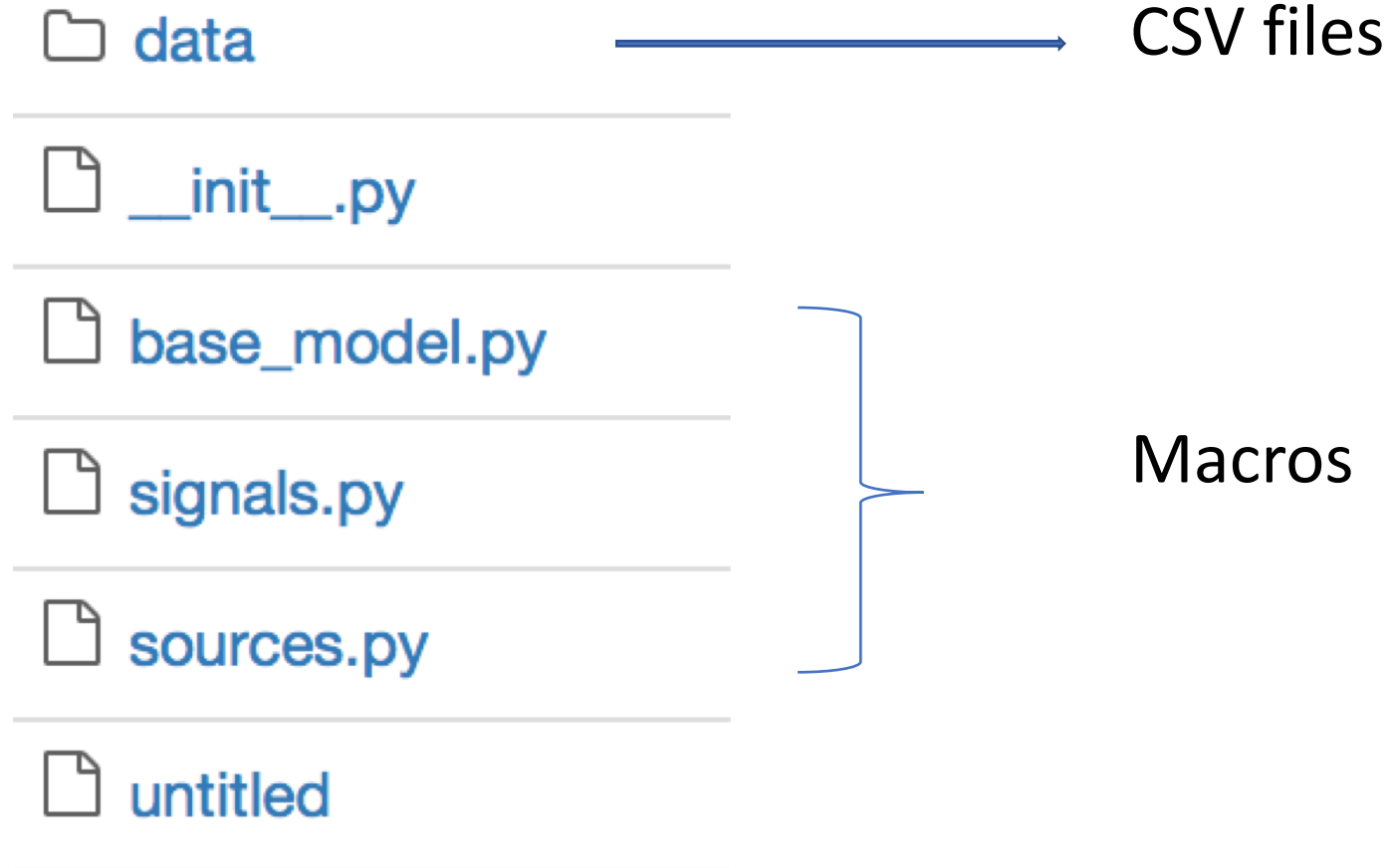
 `MANIFEST.in`

 `README.rst`

 `requirements.txt`

 `setup.py`

Laidbax



Notebooks

 pdf_cache

 Basic checks.ipynb

 SR0 limit.ipynb

 Tutorial.ipynb

 WIMP limits.ipynb

 compute_limits.py

This week

Figure out what is going on in the code, WIMP limit



Syntax of blueice



Numerical Analysis



Play with different parameters

General idea of the code

Use blueice to create 1) likelihood function and 2) its interval to find the roots by using Brent's method and the roots will be plotted in the limit graph as the cross-section.

For likelihood function: 1) adds two rate parameters: WIMP recoil, electron recoil 2) adds 1 shape parameter: WIMP mass

(why is the WIMP recoil rate parameter = 0 to make a discovery?)

Problem

- Set cross section = $1e^{-45}$, wimp interaction = SD, got an error.

```
~/Desktop/blueice/blueice/inference.py in one_parameter_interval(lf, target, bound, confidence_level, kind, bestfit_routine, t_ppf, **kwargs)
    376         return brentq(t, bound, global_best, args=[1 - confidence_level])
    377     elif kind == 'upper':
--> 378         return brentq(t, global_best, bound, args=[confidence_level])
    379
    380

/anaconda3/lib/python3.7/site-packages/scipy/optimize/zeros.py in brentq(f, a, b, args, xtol, rtol, maxiter, full_output, disp)
    508     if rtol < _rtol:
    509         raise ValueError("rtol too small (%g < %g)" % (rtol, _rtol))
--> 510     r = _zeros._brentq(f,a,b,xtol,rtol,maxiter,args,full_output,disp)
    511     return results_c(full_output, r)
    512
```

ValueError: f(a) and f(b) must have different signs

Interpretation of the syntax

- `one_parameter_interval(lf, target, bound, confidence_level, kind, bestfit_routine, t_ppf, **kwargs)`
- Set a `confidence_level(100%)` interval of `kind(a range of WIMP mass)` on the parameter `target(WIMP rate multiplier)` of likelihood function

`lf` = likelihood function

`target` = WIMP rate multiplier

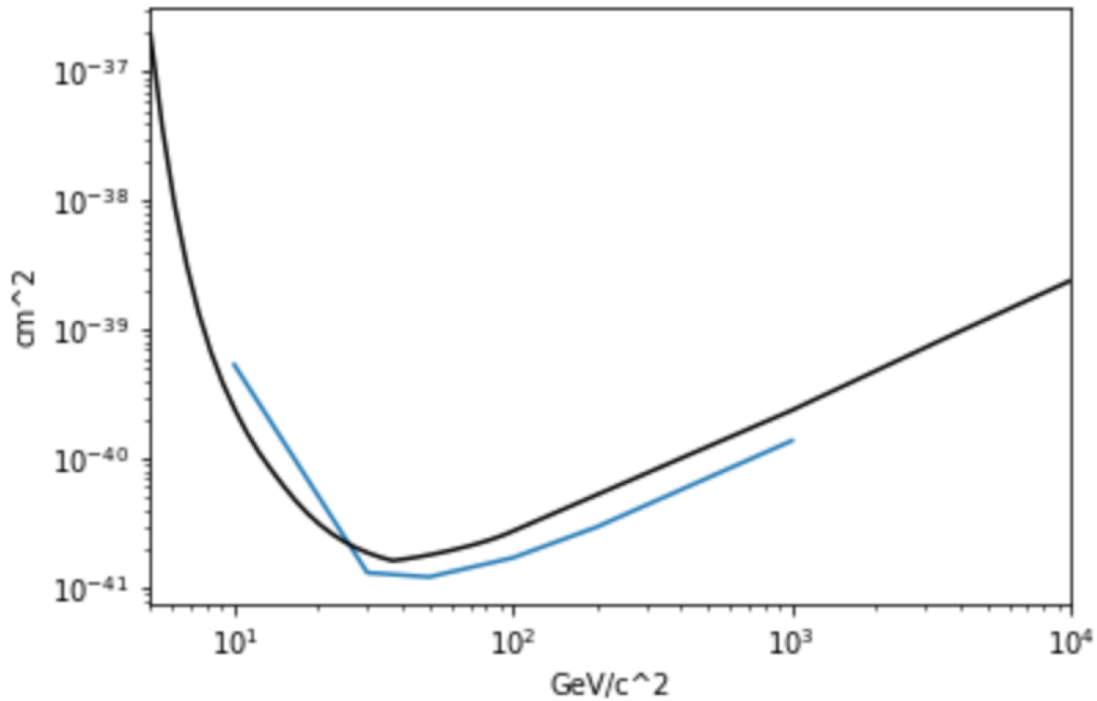
`bound` = 10

`kind` = WIMP mass

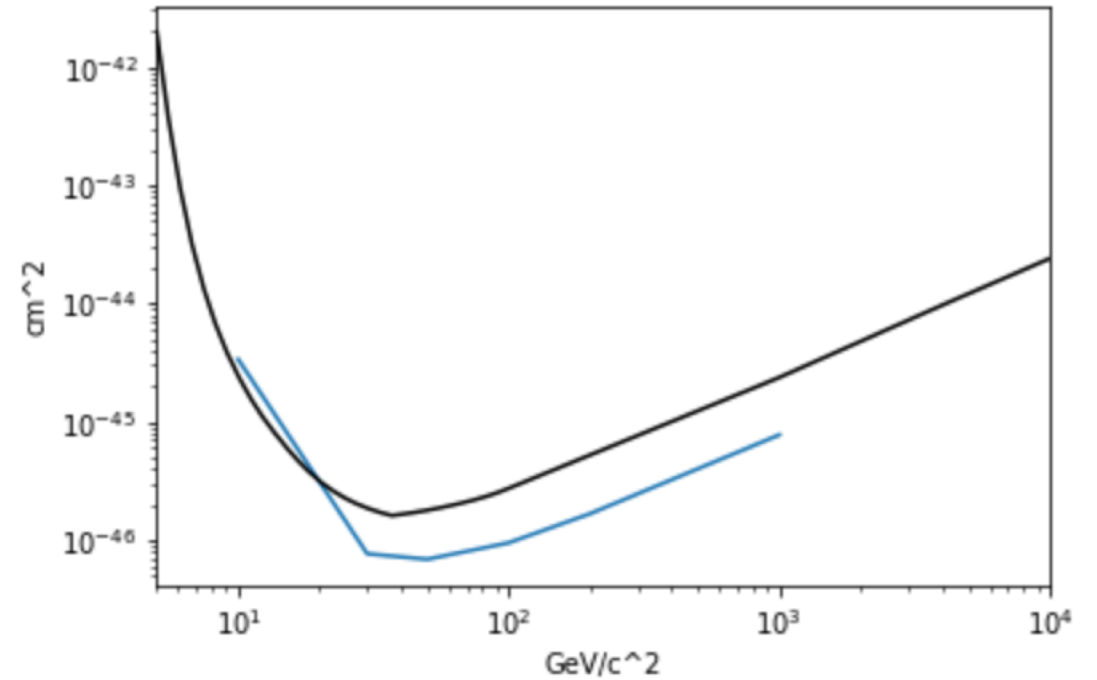
What I think.....

- Using Brent's method to find roots of the function and it requires $f(a)$ and $f(b)$ have different signs.
- Brent's Method: iteration of (bisection method + secant method)(I could make a slide on this method, if anyone is interested.....)
- When set CS to $1e^{-45}$, the likelihood is too small, and the function curve is around zero-axis.
- Solution: increase the likelihood by decreasing the WIMP energy in the macro ---> No Effect
- Does it worth to dig more into it?

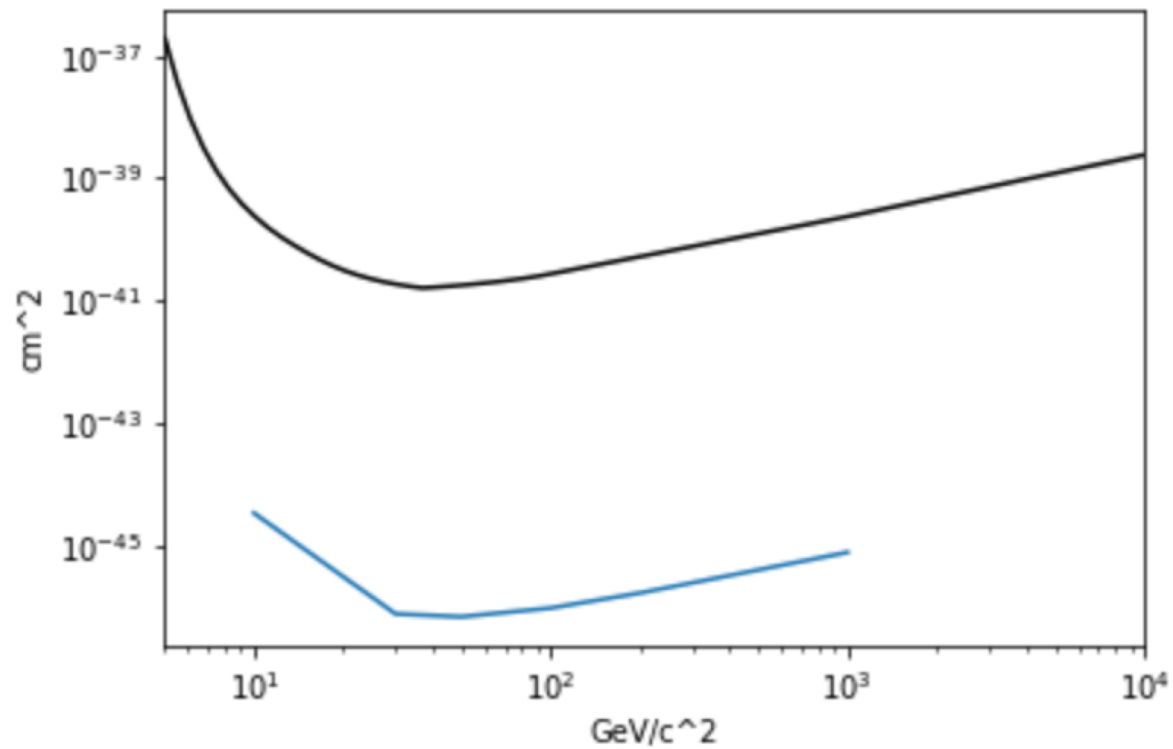
Cross-section: $1e^{-40}$
WIMP Interaction:
SD_n_central (spin dependent)



Cross section: $1e^{-45}$
WIMP Interaction: SI
(spin independent)



Cross-section: $1e^{-40}$
WIMP Interaction: SI

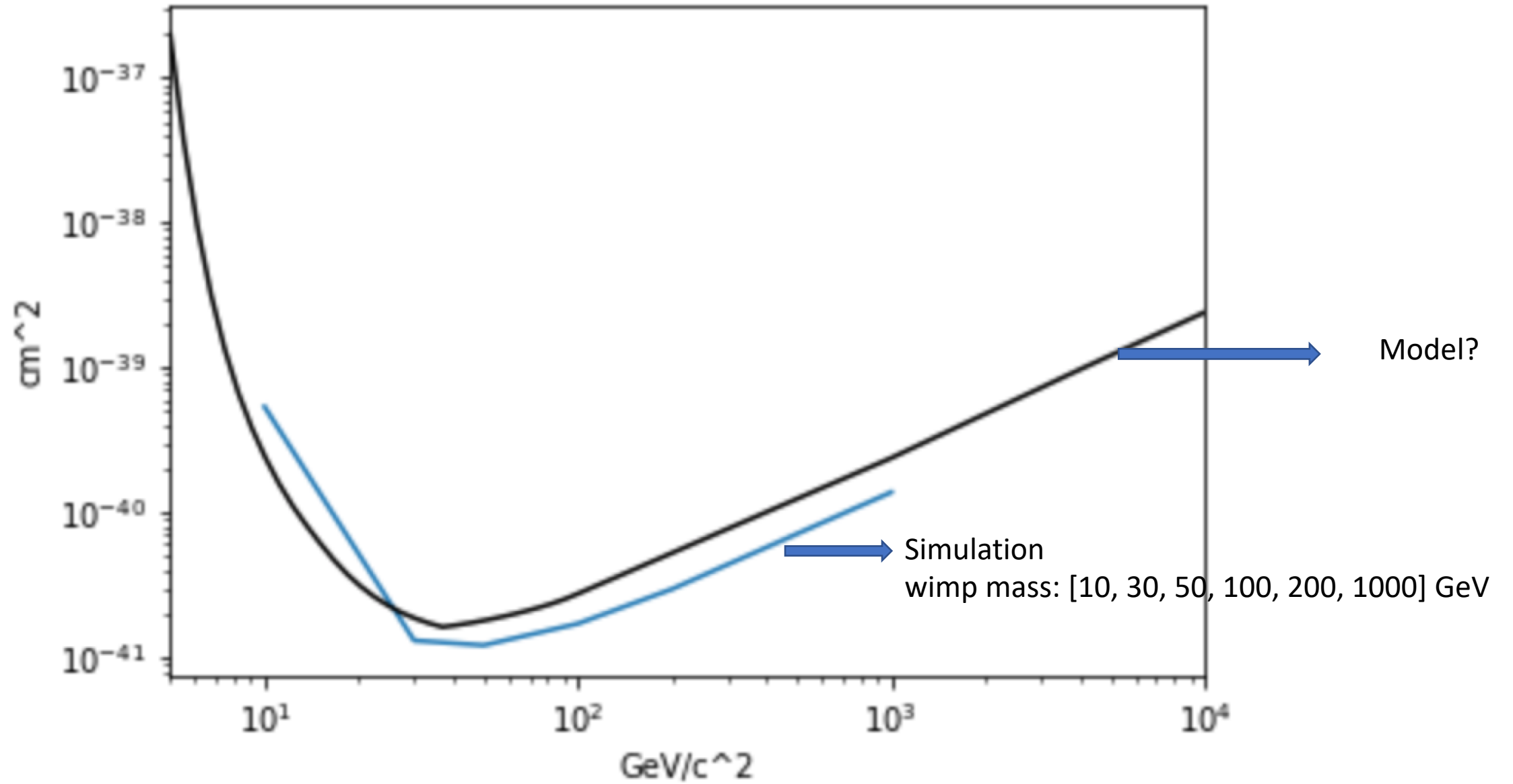


Next Week

- Analyze the simulation Macro
- Any remaining/new problems from this meeting

Macro(Xenon1T)

- Set Sources: energy distribution: $er_background + nr_background(cnns \text{ and radiogenic neutrons}) + 50\text{Gev } 10^{-45} \text{ WIMP}(nr)$
- Set parameters



Info for the black line

5 GeV	10 GeV	10^2 GeV	10^3 GeV	10^4 GeV
$1.98e^{-37}$ cm ²	$2.37e^{-40}$ cm ²	$2.75e^{-41}$ cm ²	$2.38e^{-40}$ cm ²	$2.41e^{-39}$ cm ²

Base model(Xenon 1T)

- WIMP Parameters:

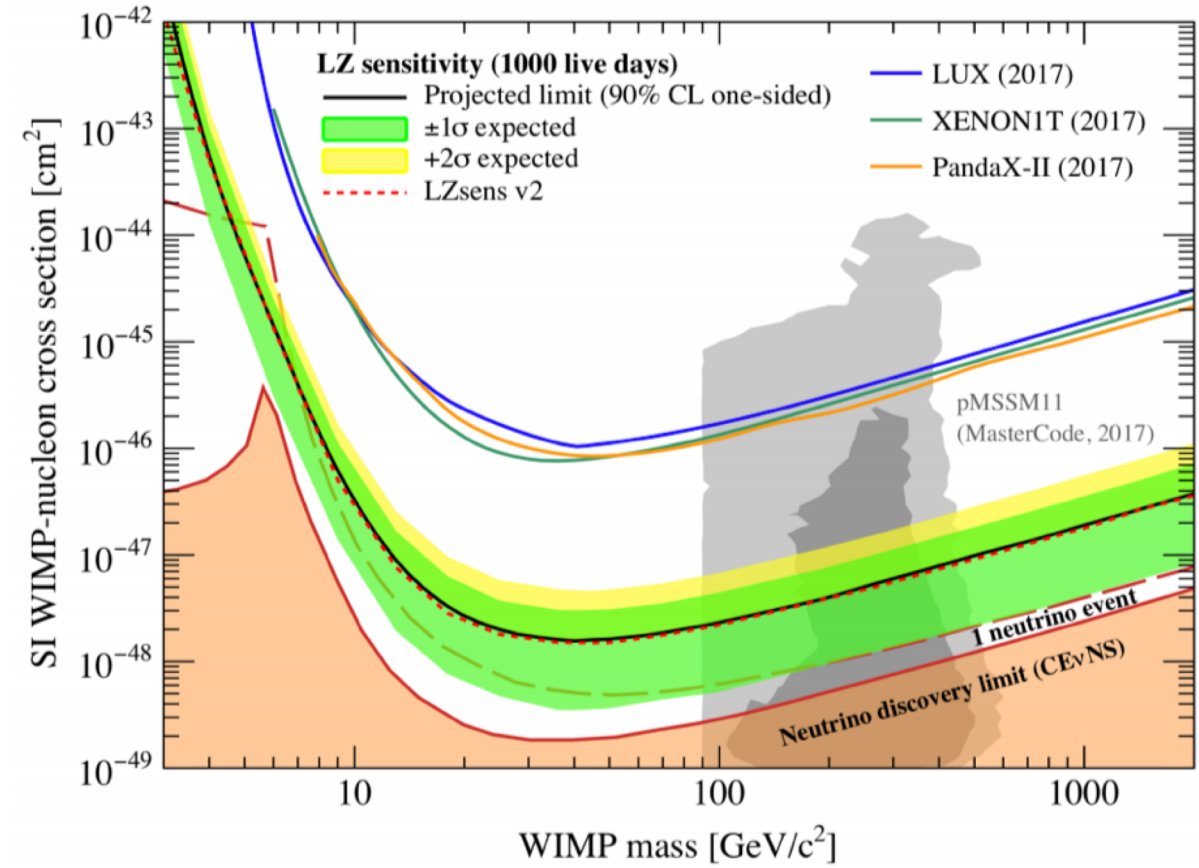
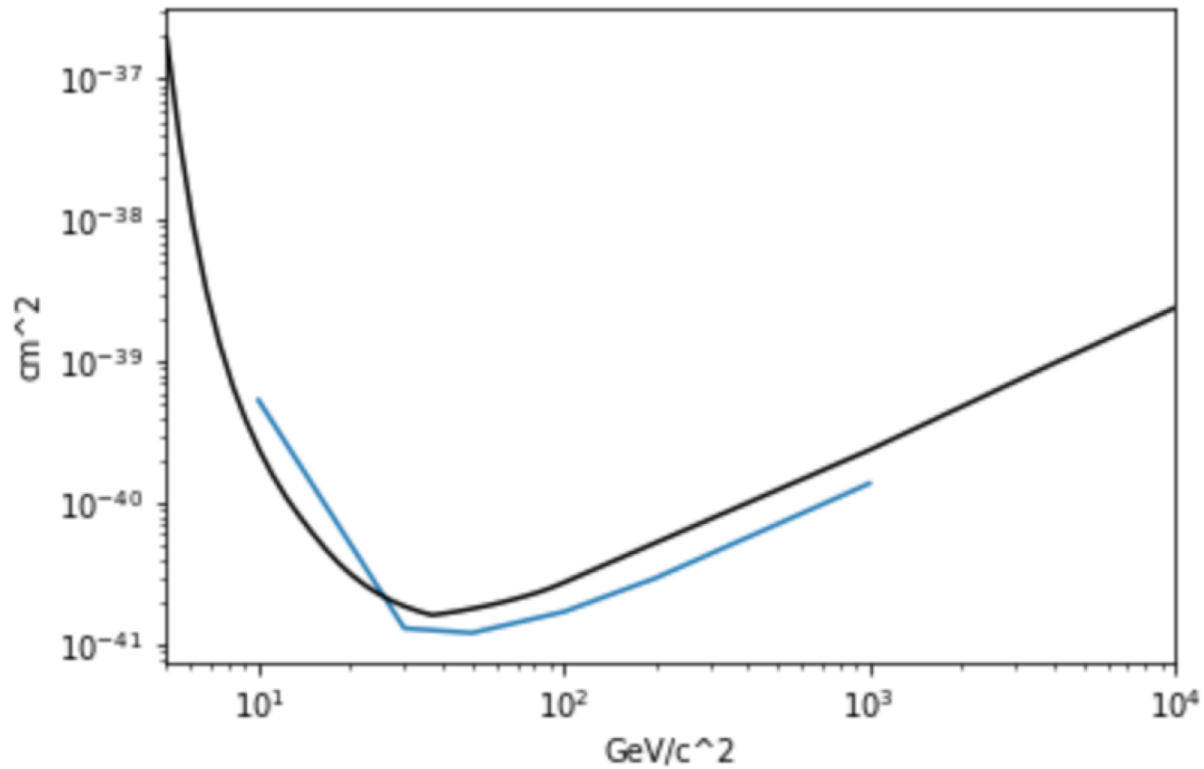
Detection	Sigma_nucleon	Wimp mass	NR cutoff	ER cutoff	WIMP interaction
Elastic NR	$1e^{-45} \text{ cm}^2$ → $10 e^{-40} \text{ cm}^2$	50 Gev/c ²	1 keV	0.18keV	SI → SD_n_central

- Detector parameters:

Fiducial mass	Electron lifetime	Drift velocity
1042 kg	452 us	1.44 m/ms

Pdf_sampling_multiplier:1->0.1

Wimp limits curve



Questions?

- What other parameters do I need to know/change?