Extra Dimensions

Daniel Lecoanet

December 2, 2008

1 Introduction

- Extra dimensions are invoked in string theory, so we might want to know how they act
- Extra dimensions can also help solve other problems with the SM, i.e., hierarchy problem and dark matter

Outline:

- Construct toy model:
  - Derive and solve EOM from action
  - Average over extra dimension
- What we gain:
  - Hierarchy problem
  - Modification of Newton's Law
  - KK mode could be dark matter

2 Toy Model

Imagine that the standard model lives in (3+1) dimensions, but there is an extra circular dimension that gravitons be in. Say the radius of the extra dimension is R. The full action for a massive scalar with mass $\hat{m}$ is

$$S = \int d^4x dy \frac{1}{2}\partial_M \phi \partial^M \phi - \hat{m}^2\phi^2 - \hat{\lambda}\phi^4, \tag{1}$$

where the extra dimension is given by the y coordinate, M indexes $0, 1, 2, 3, 5$, and I've put in a ϕ^4 interaction term with coupling $\hat{\lambda}$. We can rewrite this as

$$S = \int d^4x dy \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\partial_y \phi \partial_y \phi - \hat{m}^2\phi^2 - \hat{\lambda}\phi^4. \tag{2}$$

Notice that we have the following mass dimensions:

- $[S] = 0$
- $[d^5x] = -5$
- $[\partial] = 1$
- $[\phi] = 3/2$
- $[\hat{m}] = 1$
- $[\hat{\lambda}] = -1$

Now we can vary the Lagrangian to get the equation of motion for (free-field) ϕ:

$$(\Box - \partial_y^2 + m^2)\phi = 0 \tag{3}$$

This is separable, so we can write

$$\phi(x^\mu, y) = \sum_n \phi^{(n)}(x^\mu) f_n(y) \tag{4}$$

which satisfies

$$\frac{(\Box + \hat{m}^2)\phi^{(n)}(x)}{\phi^{(n)}(x)} = -M_n^2 = \frac{\partial_y^2 f_n(y)}{f_n(y)}. \tag{5}$$

Thus,

$$f_n(y) = N_n \sin(M_n y) + N_n' \cos(M_n y). \tag{6}$$

Remember that these modes are living on a circle, so we have the boundary condition that $f_n(y) = f_n(y + 2\pi R)$, which implies that the allowed M_n are discrete:

$$M_n = \frac{n}{R}. \tag{7}$$

Thus, we will take

$$f_n(y) = N_n \sin(\frac{ny}{R}) + N_n \cos(\frac{ny}{R}). \tag{8}$$

This infinite series of particles $\phi^{(n)}(x)$ are called a Kaluza-Klein tower.

We must pick a normalization that will yield the correct mass dimension. A standard choice is

$$N_n = \frac{1}{\sqrt{\pi R}} \tag{9}$$

when n is not 0, and

$$N_0 = \frac{1}{\sqrt{2\pi R}} \tag{10}$$

for $n = 0$. Now we can average over y to find an effective field theory. This effective field theory will be valid when probing scales larger than R. One finds that

$$S_{4D} = \int d^4x \sum_n \frac{1}{2}\partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - \left(\frac{n^2}{R^2} + m^2\right) \phi^{(n)2} - \sum_{m,n,p,q} \lambda_{m,n,p,q} \phi^{(n)} \phi^{(m)} \phi^{(p)} \phi^{(q)}. \tag{11}$$

Notice that each of the KK modes has mass squared $n^2/R^2 + m_0^2$, where m_0 is the mass of the lowest excitation. The coupling $\lambda_{m,n,p,q}$ is proportional to $1/R$, and has certain selection rules on n, m, p, q essentially dictating conservation of momentum.

3 Why we did this

3.1 Hierarchy Problem

This toy model can help us see how we can get around the hierarchy problem using extra dimensions. The coupling strength gets altered (by a factor proportional to $1/R$) in the effective field theory, i.e., what we observe today. If we average over the extra dimensions in a $4+\delta$ dimensional general relativity action, we find that the strength of gravity changes according to

$$M_{pl}^2 \sim M_\delta^{\delta-2} R^{\delta-4}, \tag{12}$$

where M_{pl} is the Planck mass in 4D, and M_δ is the strength of gravity in all $4+\delta$ dimensions. Thus, a large Planck mass could be due to the geometric factor, and not that M_δ is large.

3.2 Modification to Newton

Newton's law is also modified by the scaling above. Instead of having

$$V(r) \sim \frac{1}{M_{pl}^2 r} \tag{13}$$

we would have

$$V(r) \sim \frac{1}{M_\delta^{\delta-2} R^{\delta-4} r} \tag{14}$$

when our effective field theory is valid, i.e., when $r >> R$, but when probing at smaller length scales, we find

$$V(r) \sim \frac{1}{M_\delta^{\delta-2} r^{\delta-3}}. \tag{15}$$

Experiments see no deviation from Newton's law to length scales around 0.1 mm, so R must be smaller than this. This puts constraints on how large large extra dimensions could be.

3.3 Dark Matter

If we take the point of view that the standard model lives in a higher dimensional space, then all the particles we see could be the lowest excition of the KK tower (this only makes sense if $1/R$ is large). However, as mentioned above, the ϕ^4 theory imposes conservation laws on the types of excitations of the KK tower which can interact with each other. Thus, dark matter could be higher excitations of the KK tower. These higher excitations could not interact with normal matter alone, and they would be heavy if $1/R$ is large.

4 References

W.M. Yao et al. J. Phys. G 33 2006
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