Vacuum Stability & Higgs Mass Bound

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Outline

- What is an effective potential?
- What is vacuum stability?
- How does vacuum stability give a lower bound on the Higgs mass?

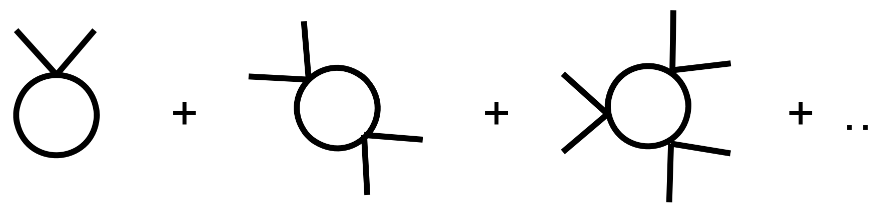
What is an effective potential?

- The effective potential is the tree level potential from the Lagrangian written in terms of the classical field (no uncertainty) plus corrections due to quantum fluctuations (all loop diagrams)
- The loop corrections depend on the value of the classical field (sometimes called the “background” field) and the particles created in the loops have contributions to their masses (“shifted” masses) due to the classical field value

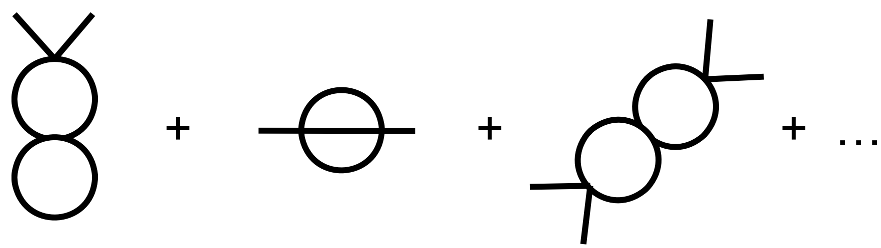
- Schematically, the effective potential can be organized into a loop expansion:

$$V_{eff} = V^{(tree)} + V^{(1\ loop)} + V^{(2\ loop)} + \cdots$$

- E.g., φ^4 theory
- One loop potential



- Two loop potential



- (In practice, it is almost never calculated this way.)

- In the standard model (Landau gauge), we have the tree level Higgs potential:

$$V_H^{(tree)} = m^2 H^\dagger H + \frac{\lambda}{6}\left(H^\dagger H\right)^2, \qquad (m^2 < 0)$$

- The Higgs is an SU(2) gauge doublet of complex scalar fields:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}\phi_1 + i\phi_2 \\ h + i\phi_4\end{bmatrix}$$

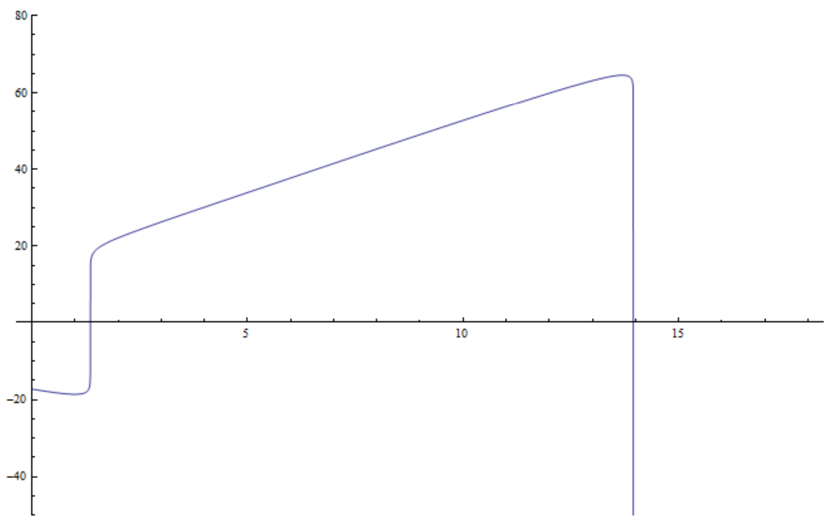
- The (MS-BAR renormalized) one loop contributions to the potential are given by:

$$V^{(1\,loop)} = \sum_{\substack{i=W,Z,t,\\ h,\phi}} \frac{n_i}{64\pi^2} M_i^4(h_c)\left[\ln\frac{M_i^2(h_c)}{\mu^2} - c_i\right]$$

What is vacuum stability?

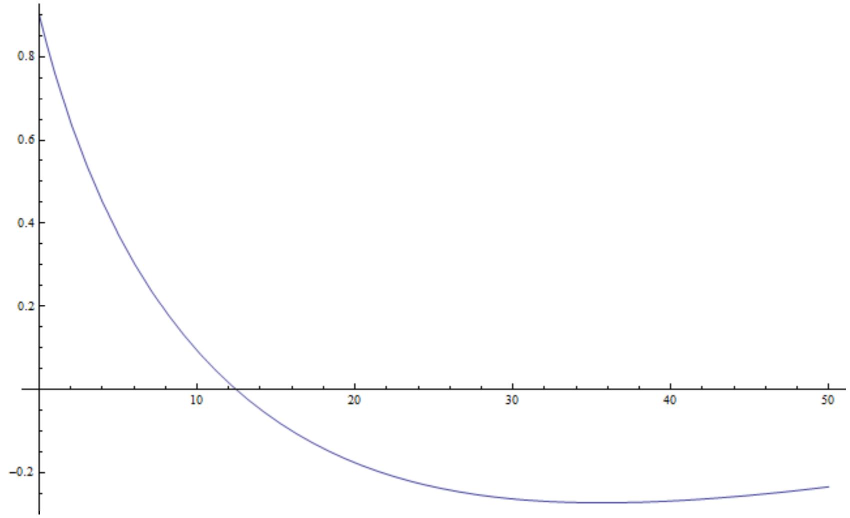
- The standard model Higgs potential must have a minimum at 246 GeV because this is the measured Higgs VEV that gives particles their masses
- Vacuum stability is the statement that the standard model minimum is the global minimum of the potential, at least up to some cutoff scale (Λ) where new physics kicks in

- It is possible for the standard model potential to be unstable:



- At some scale the potential reaches a maximum and begins to decrease until it reaches a second, very large negative minimum

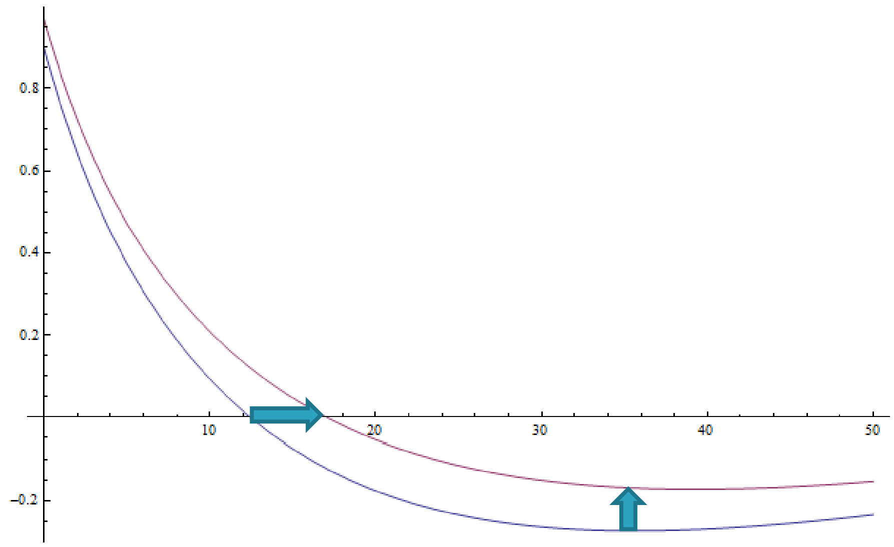
- This is a result of the renormalization scale dependence of the Higgs coupling λ beginning at one loop:



- When λ runs negative at a large scale due to the top quark Yukawa coupling, this will cause the potential to reach a maximum and begin decreasing

How does vacuum stability give a bound on the Higgs mass?

- Increasing the cutoff scale Λ requires that the scale where the potential turns over also increase
- This means that the scale at which λ runs negative must increase because it is λ, the quartic term in the potential, which supports the potential at high scales
- So λ has to be shifted upward:



- At the minimum (246 GeV) of the potential, there are two conditions:

$$\left.\frac{\partial V_{eff}}{\partial h}\right|_{h=v} = 0$$

$$\left.\frac{\partial^2 V_{eff}}{\partial h^2}\right|_{h=v} = m_H^2$$

- From these conditions, there is the result that

$$m_H^2 \propto \lambda v^2$$

- As Λ increases and λ is shifted upward, the value of λ at the minimum and the Higgs mass both increase

- A lower bound on the cutoff scale results in a lower bound on the Higgs mass
- A couple scenarios:

Λ	Higgs mass
~ 1 TeV	~ 55 GeV
~ 10^{19} GeV	~ 134 GeV

What is the scalar singlet?

- We can augment the scalar sector of the standard model with an extra real scalar field that is a gauge singlet and couples to the Higgs:

$$V_{H,S}^{(tree)} = m^2 H^\dagger H + \frac{\lambda}{6}\left(H^\dagger H\right)^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4 + a_1 H^\dagger H S + a_2 H^\dagger H S^2$$

- The scalar singlet can play a role in dark matter (if it has zero VEV) or the electroweak phase transition (for non-zero VEV)
- In both cases, it mixes with the Higgs, resulting in some implications for Higgs phenomenology at the LHC