

# Heavy flavours: theory summary

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**Abstract.** I summarize the theory talks given in the Heavy Flavours Working Group. In particular, I discuss heavy-flavour parton distribution functions, threshold resummation for heavy-quark production, progress in fragmentation functions, quarkonium production, heavy-meson hadroproduction.

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## INTRODUCTION

Heavy-flavour physics is currently one of the main fields of investigation, in both theoretical and experimental particle physics. In this talk, I summarize the main theoretical issues that were presented in the Heavy Flavour session of DIS 2005. Among the topics discussed, we had updates on heavy-flavour parton distributions, large- $x$  resummation for heavy quark production in DIS, heavy-quark fragmentation functions to next-to-next-to-leading order, progress in heavy-meson and quarkonium production.

## HEAVY-QUARK PARTON DISTRIBUTIONS

Relevant work was carried out on the subject of heavy-quark parton distribution functions. Progress was reported by R. Thorne, from the MRST collaboration, concerning the formulation of the Variable Flavour Number Scheme (VFNS) for heavy quarks at NNLO.

In fact, there are problems in defining VFNS for heavy quarks. When switching naively from a given order in the coupling constant  $\alpha_S$  with  $n_f$  flavours, to the same order, but with  $n_f + 1$  flavours, one would get a discontinuity when  $Q^2$  is equal to the heavy quark mass. At NNLO, for example, the contribution to a heavy-quark structure function  $F_2^H(x, Q^2)$  with  $n_f$  flavours is  $\sim \alpha_S^3 C_2^{\text{FF},3} \otimes f^{n_f}$ , where  $C_2^{\text{FF},3}$  is the fixed-flavour (FF) coefficient function and  $f$  the parton distribution function. The corresponding contribution, in the region of  $n_f + 1$  flavours, is instead  $\sim \alpha_S^2 C_2^{\text{VF},2} \otimes f^{n_f+1}$ , where  $C_2^{\text{VF},2}$  is the variable-flavour (VF) coefficient function. As a result,  $F_2^H(x, Q^2)$  would be discontinuous through the heavy-quark mass threshold  $Q^2 = m_H^2$ .

The Thorne–Roberts (TR) prescription handles this discontinuity by freezing higher-order terms when crossing the value  $Q^2 = m_H^2$ . At LO, for instance, the TR prescription reads, in terms of gluon ( $g$ ) and heavy quark ( $h, \bar{h}$ ) densities:

$$\begin{aligned} \frac{\alpha_S(Q^2)}{4\pi} C_{2,Hg}^{\text{FF},1} \left( \frac{Q^2}{m_H^2} \right) \otimes g^{n_f}(Q^2) &\rightarrow \frac{\alpha_S(M^2)}{4\pi} C_{2,Hg}^{\text{FF},1}(1) \otimes g^{n_f}(m_H^2) \\ &+ C_{2,HH}^{\text{VF},0} \left( \frac{Q^2}{m_H^2} \right) \otimes (h + \bar{h})(Q^2). \end{aligned} \quad (1)$$

In Eq. (1), we note the ‘frozen’ term  $\sim \alpha_S(M^2) C_{2,Hg}^{\text{FF},1}(1)$ . In order to apply this prescription to NNLO, we would need the FF  $\mathcal{O}(\alpha_S^3)$  heavy-quark coefficient functions for  $Q^2 \leq m_H^2$ , which have not been computed yet. However, a reliable approximation of such functions can be obtained, gaining information from the available calculations which resum threshold logarithms and leading  $\ln(1/x)$  terms, and from the known NNLO coefficient functions and space-like splitting functions.

Using this method, parton densities are still discontinuous at  $Q^2 = m_H^2$ , but structure functions are now continuous. Results were presented for the charm-quark structure function  $F_2^c(x, Q^2)$ : the NNLO VFNS prediction fits rather well the ZEUS and H1 data, while the NLO result is always below the data.

F. Olness, from the CTEQ collaboration, also presented recent work on heavy-quark parton distributions, implemented in the CTEQ6HQ set. Mainly, the new set resums logarithms of the heavy-quark mass  $\ln(m_H^2/\mu_F^2)$  in the full kinematic range, by reabsorbing them in the heavy-quark distribution function. This yields a small, but visible difference with respect to the previous set CTEQ6M. The predictions obtained using the CTEQ6HQ parton distributions fit the HERA data on  $F_2^c(x, Q^2)$  quite well.

Progress was reported on the determination of the strange-quark density, which, in the previous CTEQ sets, was tied to the  $\bar{u}$  and  $\bar{d}$  distributions via the relation  $s = \bar{s} = \kappa(\bar{u} + \bar{d})/2$ . This assumption might lead to an underestimate of the uncertainty on the  $s$ -quark density. Rather, the  $s$  quark can be treated as an additional set, described by a more general parametrization, which can be fitted to data. Preliminary results with an independent  $s$ -quark density give reasonable  $\chi^2$  when comparing with DIS and Tevatron data.

Moreover, the talk by F. Olness also discussed soft-resummation for  $b$  production in DIS, which might also be included in future fits of CTEQ parton distribution functions.

## THRESHOLD RESUMMATION FOR HEAVY-QUARK PRODUCTION IN DIS

Soft-gluon resummation for heavy-quark production in DIS was discussed in the talk by A. Mitov. In fact, the DIS  $\overline{\text{MS}}$  coefficient functions present, at  $\mathcal{O}(\alpha_S)$ , terms  $\sim 1/(1-x)_+$  and  $\sim [\ln(1-x)/(1-x)]_+$ , which become large for  $x \rightarrow 1$  and need to be resummed (threshold resummation). Such contributions correspond to collinear- or soft-gluon radiation.

It was pointed out that the large- $x$  behaviour of the coefficient functions crucially depend, via the ratio  $m/Q$ , on the mass of the final-state quark. In fact, a light quark emits both soft- and collinear-divergent radiation, while gluon radiation off a heavy quark can

only be soft-enhanced. As a result, the two regimes  $m/Q \simeq 1$  and  $m/Q \ll 1$  must be treated separately when performing large- $x$  resummation.

The threshold resummation reported by A. Mitov was performed in the next-to-leading logarithmic approximation (NLL), which corresponds to keeping in the Sudakov exponent terms  $\sim \alpha_S^n \ln^{n+1} N$  (LL) and  $\sim \alpha_S^n \ln^n N$  (NLL), where  $N$  is the Mellin moment variable. In fact, soft resummation is analytically performed in  $N$ -space, and the results are then inverted to  $x$ -space. Predictions were presented for the charm-quark structure function  $F_2^c(x, Q^2)$ , for charged-current interactions, in the environment of the HERA and NuTeV experiments. The results showed that threshold resummation is relevant, especially at small  $Q^2$ . Moreover, resumming large- $x$  terms in the  $\overline{\text{MS}}$  coefficient function yields a milder dependence on both factorization and renormalization scales, which corresponds to a reduction of the theoretical uncertainty.

## NNLO PERTURBATIVE FRAGMENTATION FUNCTION

A. Mitov also reported on progress in heavy-quark fragmentation functions. The energy distribution of a heavy quark presents, at fixed order, terms  $\sim \alpha_S^n \ln^k(Q^2/m^2)$  ( $k \leq n$ ), where  $Q$  is the hard scale of the process, that are large for  $m \ll Q$ , which is often the case. Such logarithms can be resummed using the approach of perturbative fragmentation functions, which expresses the energy spectrum of a heavy quark as the convolution of a coefficient function, describing the emission of a massless parton, and a perturbative fragmentation function  $D(m, \mu_F)$ , associated with the fragmentation of a massless parton (a light quark or a gluon) into a massive quark. The dependence of  $D(m, \mu_F)$  on the factorization scale  $\mu_F$  is determined by solving the Dokshitzer–Gribov–Altarelli–Parisi (DGLAP) evolution equations, once an initial condition at a scale  $\mu_{0F}$  is given. Neglecting powers  $(m/Q)^p$ , the initial condition of the perturbative fragmentation function was proved to be process independent and calculated several years ago to NLO. This talk discussed the recent calculation of NNLO contributions, i.e. up to  $\mathcal{O}(\alpha_S^2)$ .

Denoting by  $Q$  and  $q$  heavy and light quarks respectively, NNLO corrections to the initial condition of the perturbative fragmentation function come from the elementary processes:  $Q \rightarrow Qgg$ ,  $Q \rightarrow Qq\bar{q}$ ,  $Q \rightarrow Q\bar{Q}$ ,  $\bar{Q} \rightarrow Q\bar{Q}\bar{Q}$ ,  $q(\bar{q}) \rightarrow \bar{Q}q(\bar{q})$  and  $g \rightarrow \bar{Q}g$ , which have now been fully computed.

Solving the DGLAP equations, for an evolution from  $\mu_{0F}$  to  $\mu_F$ , allows us to resum terms  $\sim \ln(\mu_F^2/\mu_{0F}^2)$ , i.e.  $\sim \ln(Q^2/m^2)$  if we choose  $\mu_{0F} \simeq m$  and  $\mu_F \simeq Q$  (collinear resummation). In particular, the leading logarithms are  $\alpha_S^n \ln^n(Q^2/m^2)$ , the NLLs  $\alpha_S^n \ln^{n-1}(Q^2/m^2)$ , the NNLLs  $\alpha_S^n \ln^{n-2}(Q^2/m^2)$ . In principle, the calculation of the initial condition of the perturbative fragmentation function to NNLO would allow one to study the spectrum of heavy quarks in the NNLO approximation, with NNLL collinear resummation. However, for this level of accuracy to be achieved, one would also need NNLO Altarelli–Parisi time-like splitting functions, which are currently known to NLO. The computation of NNLO corrections to such functions is in progress.

## HADROPRODUCTION OF HEAVY MESONS IN A MASSIVE VFNS

We had a presentation from I. Schienbein on hadroproduction of heavy mesons in a Massive Variable Flavour Number Scheme (MVFNS). Considering, for example,  $D$ -meson production at the Tevatron ( $p\bar{p} \rightarrow DX$ ), the MVFNS subtracts and resums logarithms of the heavy-quark mass  $\ln(\mu_F^2/m^2)$ . Unlike the perturbative fragmentation function approach discussed above, it keeps powers of  $m/Q$  in the hard-scattering cross section. This prescription is equivalent to  $\overline{\text{MS}}$  mass factorization in a scheme where the heavy-quark mass regularizes the collinear divergences. It was numerically checked that the short-distance coefficient function corresponds to the  $\overline{\text{MS}}$  one in the limit where the heavy-quark mass tends to zero. As a result, predictions can be obtained still using  $\overline{\text{MS}}$  parton distributions and fragmentation functions, but convoluted with a massive hard-scattering cross section.

The considered partonic subprocesses that were calculated are  $gg \rightarrow c\bar{c}$  and  $q\bar{q} \rightarrow c\bar{c}$ , at LO;  $qq \rightarrow c\bar{c}g$ ,  $q\bar{q} \rightarrow c\bar{c}g$  and  $gq \rightarrow c\bar{c}q$ , at NLO. If the heavy (charm) quark is in the initial state, it is treated in the massless approximation.

Using the CTEQ6M parton distribution function set and the Binnewies–Kniehl–Krämer (BKK) NLO fragmentation functions, fitted to the OPAL data, predictions were given on the transverse momentum of  $D^0$ ,  $D^{*+}$ ,  $D^+$  and  $D_s^+$  mesons at the Tevatron. It was considered just prompt charm production, while  $D$  production from  $B$ -meson decays was not taken into account. Within the error range, agreement was found with the CDF data, although the ratio of the central values of theoretical predictions and data is about 1.5–1.8, which may warrant further investigation. A prediction was finally given for the transverse momentum distribution of  $c$ -flavoured baryons  $\Lambda_c^+$ .

This approach will be in the near future extended to  $B$ -meson production at the Tevatron, and both  $D$  and  $B$  mesons in Deep Inelastic Scattering.

## HEAVY-QUARKONIUM PRODUCTION

The talk by J.–P. Lansberg discussed the production of heavy quarkonium ( $Q\bar{Q}$ ) in a new model, which consists of an extension of the Colour Singlet Model (CSM) and the Colour Octet Model (COM). The naive CSM factorizes heavy-quarkonium production in a hard and a soft part. In the hard process,  $Q$  and  $\bar{Q}$  are assumed to be on-shell, in a colour-singlet state, with zero relative momentum, in a  $^3S_1$  angular-momentum state (for  $J/\psi$ ,  $\psi'$  and  $\Upsilon$ ). As far as the soft part is concerned, the amplitude for the quark-binding probability is a wave function, solution of the Schrödinger equation. This model is, however, unable to reproduce the Tevatron Run I data from CDF on  $J/\psi$  and  $\psi'$  direct production.

The COM proposes instead that quarkonium states are produced by the fragmentation of a gluon, which is transversely polarized, so that, according to Non-Relativistic QCD (NRQCD), the quarkonium is to have itself transverse polarization. Nevertheless, this prediction disagrees with CDF measurements of unpolarized or slightly longitudinally polarized quarkonium states.

The new model goes beyond the static and on-shell approximations of the CSM.  $^3S_1$

quarkonium ( $\mathcal{Q} = Q\bar{Q}$ ) is produced via gluon fusion  $gg \rightarrow \mathcal{Q}g$ , but it includes new contributions with respect to the usual CSM (see J.-P. Lansberg's presentation in these proceedings for details).

To describe the soft, non-perturbative part, two phenomenological vertex functions are chosen:  $\psi(\vec{p}_{\text{rel}}) \sim \exp[-\vec{p}_{\text{rel}}^2/\Lambda^2]$  and  $\psi(\vec{p}_{\text{rel}}) \sim (1 + \vec{p}_{\text{rel}}^2/\Lambda^2)^{-2}$ , where  $\vec{p}_{\text{rel}}$  is the relative  $Q\bar{Q}$  momentum and  $\Lambda$  is a free size parameter. The results yielded by this model are in good agreement with RICH data on  $J/\psi$ , and Tevatron data on  $J/\psi$ ,  $\psi'$  and  $\Upsilon(1S)$  production. Fragmentation contributions are then taken from the COM, which gives transverse polarization, and included in the model. The agreement with polarization measurements at the Tevatron is shown to be now pretty good.

## NLO CHARMONIUM PRODUCTION IN $\gamma\gamma$ COLLISIONS

The talk given by B. Kniehl discussed NLO charmonium production in photon–photon collisions. The computation of NLO corrections to such processes is a relevant improvement, since it reduces renormalization and factorization scale dependence and allows a test of NRQCD.

New results were presented for processes  $\gamma\gamma \rightarrow J/\psi X$ , with direct photons and prompt  $J/\psi$ 's, within the framework of NRQCD. In the considered processes,  $X$  can be a purely hadronic state, or a hadronic state with a prompt photon. Phenomenological results were presented for  $e^+e^-$  colliders, with characteristics similar to the TESLA Linear Collider project, and a centre-of-mass energy  $\sqrt{s} = 500$  GeV. Cuts on transverse momentum and rapidity of final-state photons were set to  $p_T^\gamma > 3$  GeV and  $|y^\gamma| < 2.79$ .

It was shown that, unlike the LO, the NLO prediction yields very little dependence on the phase-space slicing parameters adopted to cancel soft and collinear singularities. Results were presented for the  $J/\psi$  rapidity and transverse momentum spectrum at LO and NLO, along with the NLO  $K$ -factor. NLO corrections exhibit a remarkable impact on both shape and normalization of the distributions which were shown.

In fact, for  $\gamma\gamma \rightarrow J/\psi X$ , the  $K$  factor is large because of the subprocesses  $\gamma\gamma \rightarrow c\bar{c}[^3S_1^{(8)}]g$  and  $\gamma\gamma \rightarrow c\bar{c}[^3S_1^{(8)}]q\bar{q}$ , which are mediated by the gluon splitting  $g \rightarrow c\bar{c}$ , where the  $c\bar{c}$  pair is in a  $^3S_1^{(8)}$  state.

The presented analysis will be extended to electron–proton photoproduction and hadroproduction, which will yield predictions that could be compared with data from HERA II, the Tevatron Run II and, ultimately, the LHC.

## CONCLUSIONS

The heavy-flavour session of DIS 2005 had a number of interesting theory talks.

The reported progress in heavy-quark parton distribution functions will be a key ingredient for precision studies of heavy-flavour physics at present and future high-energy colliders, such as the LHC.

Large- $x$  resummation in the coefficient function for heavy-quark production in DIS will allow us to extract resummed parton densities as well. The NNLO calculation of

the initial condition of heavy-quark perturbative fragmentation functions could allow the promotion of the perturbative fragmentation function approach to NNLO/NNLL accuracy if time-like Altarelli–Parisi splitting functions were to be known at NNLO. Given the process independence of the perturbative fragmentation function, we shall be able to apply such results to any process whose coefficient functions are known to NNLO.

An alternative approach to address heavy-quark (hadron) production is the MVFNS, which was used to predict charm-flavoured hadron production at the Tevatron. Some discrepancies between theory and CDF data in the central values of the  $D$  transverse-momentum distribution may require further investigation. It may also be worthwhile comparing MVFNS and perturbative fragmentation function approaches.

Quarkonium production studies were also discussed. A new model was proposed, which goes beyond the static approximation of the CSM, uses the COM for the fragmentation, and is able to reproduce Tevatron and RICH data on  $J/\psi$ ,  $\psi'$  and  $\Upsilon$  production.

NLO charmonium production in two-photon collisions was also discussed. The results, shown for  $J/\psi$  production at an  $e^+e^-$  collider with  $\sqrt{s} = 500$  GeV, exhibit a relevant effect of the inclusion of NLO corrections.

In summary, all the given talks show active work and progress on heavy-flavour phenomenology, which will allow the performance of increasingly more accurate measurements in Deep Inelastic Scattering experiments, as well as at any present and future hadron collider facility.