Averaging of DIS Cross Section Data

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Abstract. A method to combine measurements of the structure functions performed by several experiments in a common kinematic domain is presented. This method generalises the standard averaging procedure by taking into account point-to-point correlations which are introduced by the systematic uncertainties of the measurements. The method is applied to the neutral and charged current deep inelastic scattering cross section data published by the H1 and ZEUS collaborations. The averaging improves accuracy owing to the cross calibration of the H1 and ZEUS measurements.

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Modern QCD fit procedures (Alekhin [1], CTEQ [2], MRST [3], H1 [4], ZEUS [5]) use data from a number of individual experiments directly to extract the parton distribution functions (PDF). All modern programs use both the central values of measured cross section data as well as information about the correlations among the experimental data points.

This direct extraction procedure has some drawbacks. Firstly the number of input datasets is large consisting of many individual publications. The data points are correlated through common systematic uncertainties, within and also across the publications. Handling of the experimental data without additional expert knowledge often becomes very difficult. In addition, the treatment of the correlations produced by the systematic errors is not unique [6]. In the Lagrange Multiplier method [7] each systematic error is treated as a parameter and thus fitted to QCD. Error propogation is then used to estimate resulting uncertainties on PDFs. In the so-called “offset” method (see e.g. [5]) the datasets are shifted in turn by each systematic error before fitting. The resulting fits are used to form an envelope function to estimate the PDF uncertainty. Each method has its own advantages and shortcomings, and it is difficult to select the standard one. Finally, some global QCD analyses use non-statistical criteria to estimate the PDF uncertainties ($\Delta\chi^2 >> 1$). This is driven by the apparent discrepancy between different experiments which is often difficult to quantify. Without a model independent consistency check of the data it might be the only safe procedure.

These drawbacks can be significantly reduced by averaging of the input structure function data in a model independent way before performing a QCD analysis of that data. One combined dataset of deep inelastic scattering (DIS) cross section measurements is much easier to handle compared to a scattered set of individual experimental measurements, while retaining the full correlations between data points. The averaging method proposed here is unique and removes the drawback of the offset method, which fixes the size of the systematic uncertainties. In the averaging procedure the correlated systematic uncertainties are floated coherently allowing in some cases reduction of the uncertainty. In addition, study of a global χ^2/dof of the average and distribution of the pulls allows a model independent consistency check between the experiments. In case of discrepancy between the input datasets, localised enlargement of the uncertainties for the average can be performed.

A standard way to represent a cross section measurement of a single experiment is given in the case of the F_2 structure function by:

$$\chi^2_{exp}\left(\left\{F_2^{i,true}\right\},\{\alpha_j\}\right) = \sum_i \frac{\left[F_2^{i,true} - \left(F_2^i + \sum_j \frac{\partial F_2^i}{\partial \alpha_j}\alpha_j\right)\right]^2}{\sigma_i^2} + \sum_j \frac{\alpha_j^2}{\sigma_{\alpha_j}^2}. \qquad (1)$$

Here F_2^i (σ_i^2) are the measured central values (statistical and uncorrelated systematic uncertainties) of the F_2 structure function[1], α_j are the correlated systematic uncertainty sources and $\partial F_2^i/\partial \alpha_j$ are the sensitivities of the measurements to these systematic sources. Eq. 1 corresponds to the correlated probability distribution functions for the structure function $F_2^{i,true}$ and for the systematic uncertainties α_j.

The χ^2 function Eq. 1 by construction has a minimum $\chi^2 = 0$ for $F_2^{i,true} = F_2^i$ and $\alpha_j = 0$. One can show that the total uncertainty for $F_2^{i,true}$ determined from the formal minimisation of Eq. 1 is equal to the sum in quadrature of the statistical and systematic uncertainties. The reduced covariance matrix $cov(F_2^{i,true}, F_2^{j,true})$ quantifies the correlation between experimental points.

In the analysis of data from more than one experiment, the χ^2_{tot} function is taken as a sum of the χ^2 functions Eq. 1 for each experiment. The QCD fit is then performed in terms of parton density functions which are used to calculate predictions for $F_2^{i,true}$.

Before performing the QCD fit, the χ^2_{tot} function can be minimised with respect to $F_2^{i,true}$ and α_j. If none of correlated sources is present, this minimisation is equivalent to taking an average of the structure function measurements. If the systematic sources are included, the minimisation corresponds to a generalisation of the averaging procedure which contains correlations among the measurements.

Being a sum of positive definite quadratic functions, χ^2_{tot} is also a positive definite quadratic and thus has a unique minimum which can be found as a solution of a system of linear equations. Although this system of the equations has a large dimension it has a simple structure allowing fast and precise solution.

A dedicated program has been developed to perform this averaging of the DIS cross section data (`http://www.desy.de/~glazov/f2av.tar.gz`). This program can calculate the simultaneous averages for neutral current (NC) and charged current (CC) electron- and positron-proton scattering cross section data including correlated systematic sources. The output of the program includes the central values and uncorrelated uncertainties of the average cross section data. The correlated systematic uncertainties can be represented in terms of (i) covariance matrix, (ii) dependence of the average cross section on the original systematic sources together with the correlation matrix for the systematic sources, (iii) and finally the correlation matrix of the systematic sources

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[1] The structure function is measured for different Q^2 (four momentum transfer squared) and Bjorken-x values which are omitted here for simplicity.

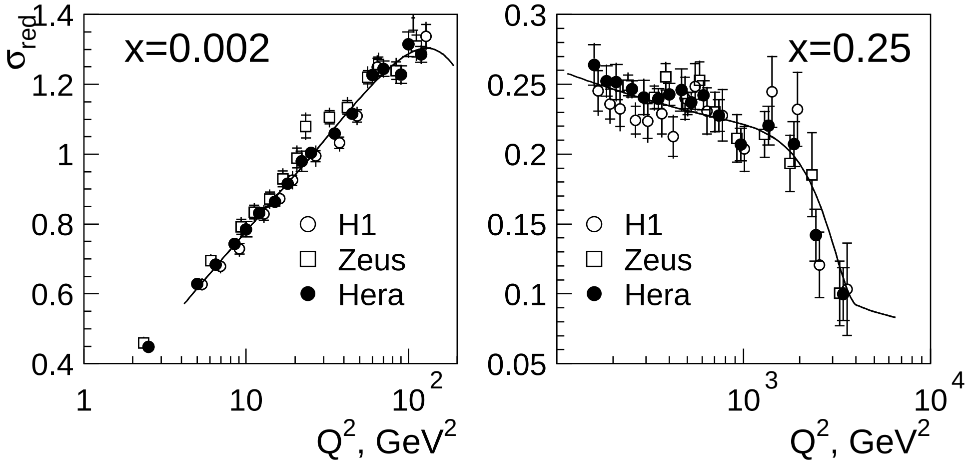

FIGURE 1. Q^2 dependence of the NC reduced cross section for $x = 0.002$ and $x = 0.25$ bins. H1 data is shown as open circles, ZEUS data is shown as open squares and the average of H1 and ZEUS data is shown as filled circles. The line represents the expectation from the H1 PDF 2000 QCD fit.

can be diagonalised, in this case the form of χ^2 for the average data is identical to Eq. 1 but the original systematic sources are not preserved.

The first application of the averaging program has been a determination of the average of the published H1 and ZEUS data [4,10-18]. Nine individual NC and CC cross section measurements are included from H1 and seven are included from ZEUS. Several sources of systematic uncertainties are correlated between datasets, the correlations among H1 and ZEUS datasets are taken from [4] and [8], respectively. No correlations are assumed between H1 and ZEUS systematic uncertainties apart from a common 0.5% luminosity measurement uncertainty. The total number of data points is 1153 (552 unique points) and the number of correlated systematic sources, including normalisation uncertainties, is 43.

The averaging can take place only if most of the data from the experiments are quoted at the same Q^2 and x values. Therefore, before the averaging the data points are interpolated to a common Q^2, x grid. This interpolation is based on the H1 PDF 2000 QCD fit [4]. The interpolation of data points in principle introduces a model dependency. For H1 and ZEUS structure function data both experiments employ rather similar Q^2, x grids. About 20% of the input points are interpolated, for most of the cases the correction factors are small (few percent) and stable if different QCD fit parametrizations [2, 3] are used.

The cross section data have also been corrected to a fixed center of mass energy squared $S = 101570$ GeV^2. This has introduced a small correction for the data taken at $S = 90530$ GeV^2. The correction is based on H1-2000 PDFs, it is only significant for high inelasticity $y > 0.6$ and does not exceed 6%.

The HERA data sets agree very well: χ^2/dof for the average is 521/601. The distribution of pulls does not show any significant tensions across the kinematic plane. Some systematic trends can be observed at low $Q^2 < 50$ GeV^2, where ZEUS NC data lie systematically higher than the H1 data, although this difference is within the normalisation uncertainty. An example of the resulting average DIS cross section is shown in Fig. 1, where the data points are displaced in Q^2 for clarity.

A remarkable side feature of the averaging is a significant reduction of the correlated systematic uncertainties. For example the uncertainty on the scattered electron energy measurement in the H1 backward calorimeter is reduced by a factor of three. The reduction of the correlated systematic uncertainties thus leads to a significant reduction of the total errors, especially for low $Q^2 < 100$ GeV^2, where systematic uncertainties limit the measurement accuracy. For this domain the total errors are often reduced by a factor two compared to the total errors of the individual H1 and ZEUS measurements.

The reduction of the correlated systematic uncertainties is achieved since the dependence of the measured cross section on the systematic sources is significantly different between H1 and ZEUS experiments. This difference is due mostly to the difference in the kinematic reconstruction methods used by the two collaborations, and to a lesser extent to the individual features of the H1 and ZEUS detectors. For example, the cross section dependence on the scattered electron energy scale has a very particular behaviour for H1 data which relies on kinematic reconstruction using only the scattered electron in one region of phase space. ZEUS uses the double angle reconstruction method where the pattern of this dependence is completely different leading to a measurement constraint.

In summary, a generalised averaging procedure to include point-to-point correlations caused by the systematic uncertainties has been developed. This averaging procedure has been applied to H1 and ZEUS DIS cross section data. The data show good consistency. The averaging of H1 and ZEUS data leads to a significant reduction of the correlated systematic uncertainties and thus a large improvement in precision for low Q^2 measurements. The goal of the averaging procedure is to obtain HERA DIS cross section set which takes into account all correlations among the experiments.

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