Recent Progress in Parton Distributions and Implications for LHC Physics.

Robert Thorne

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University of Cambridge

Royal Society Research Fellow

Parton Uncertainties – Experiment – recently a lot of work. Number of approaches.

Hessian (Error Matrix) approach.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

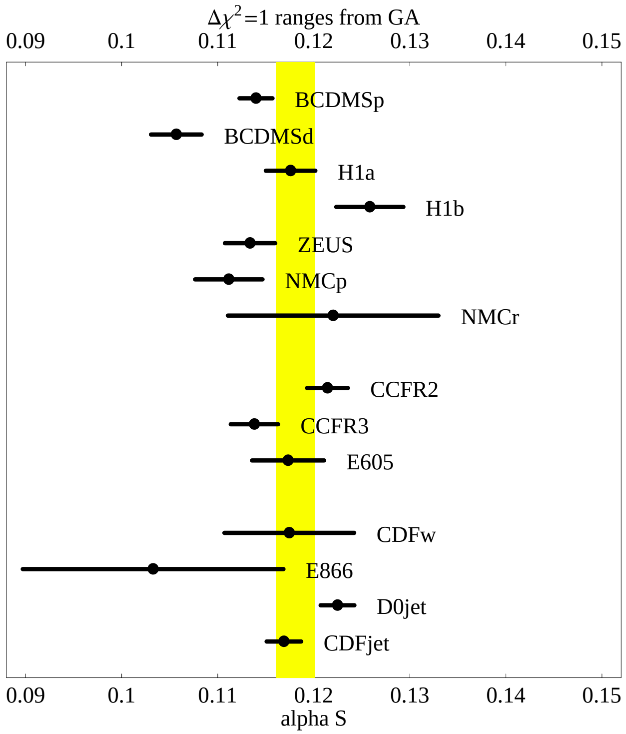
Simple method problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space – particularly with more parameters (more data). → numerical instability. Improved by CTEQ. Now used in slightly weaker form by MRST and ZEUS.

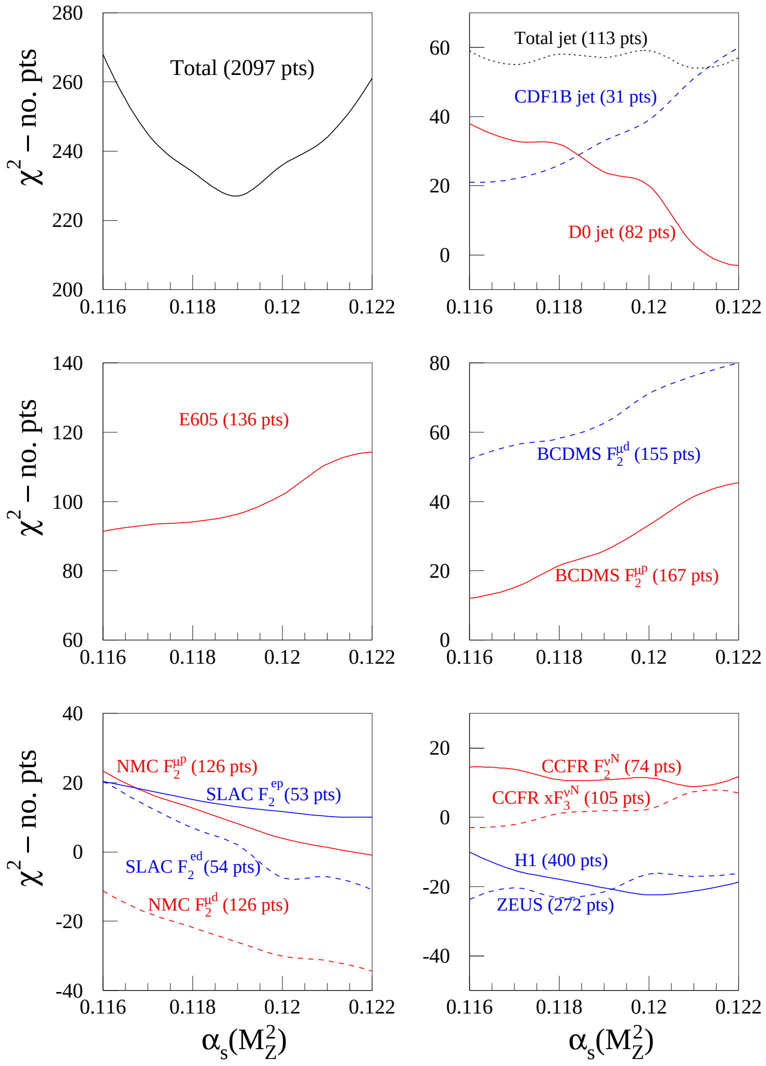
Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and concentrated on by MRST. Minimize

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a).$$

Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2.

In full **global** fit art in choosing “correct” $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.





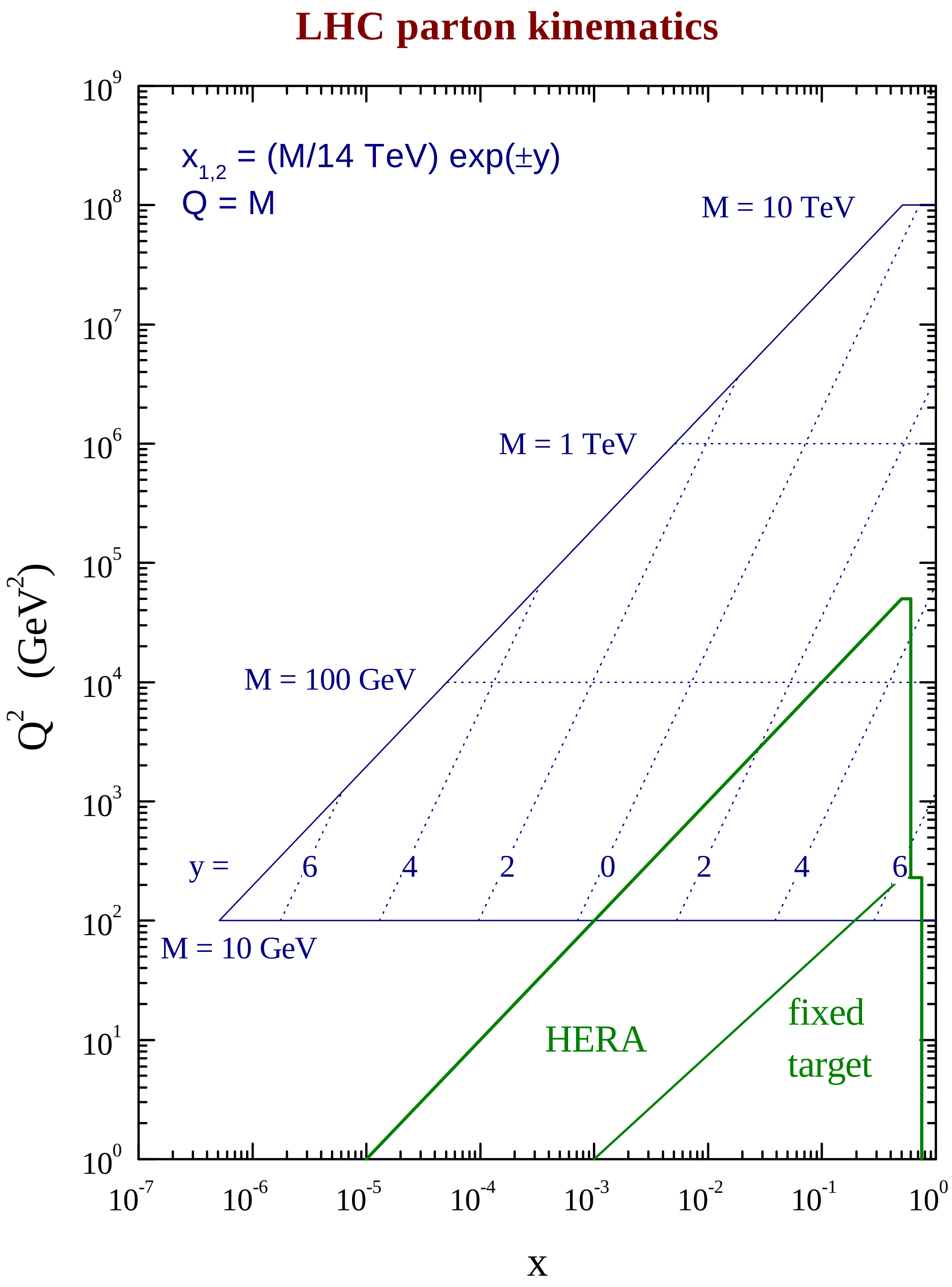
Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ for 90% confidence limit, i.e. ~ 40 for $1-\sigma$ error. MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

LHC **Physics**

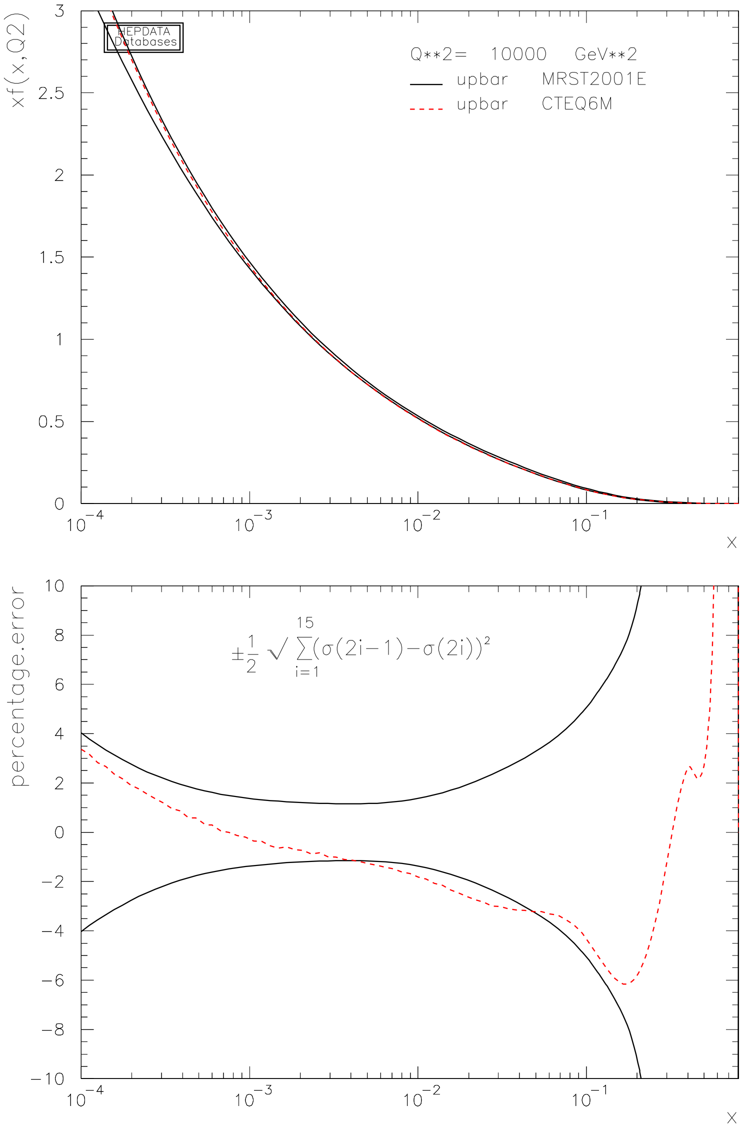
The kinematic range for particle production at the LHC is shown.

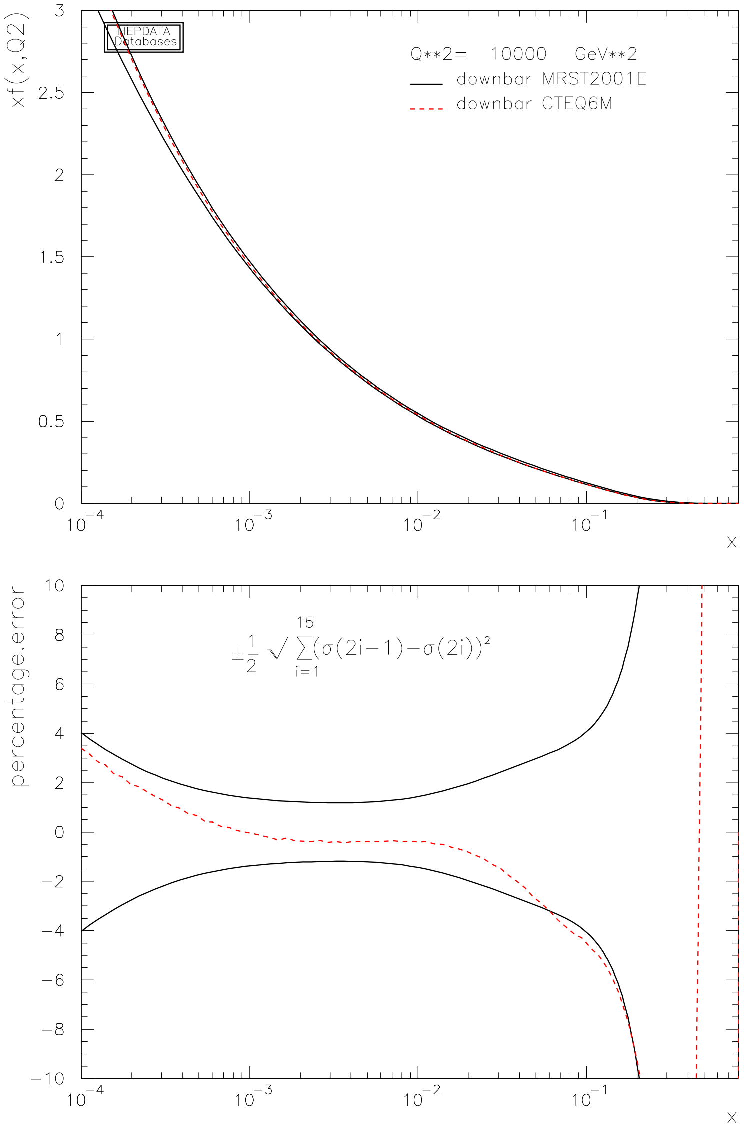
Smallish $x \sim 0.001 - 0.01$ parton distributions therefore vital for understanding the standard production processes at the LHC.

However, even smaller (and higher) x required when one moves away from zero rapidity, e.g. when calculating total cross-section.



Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty in W, Z (Higgs?) at the LHC.





Current best (MRST) estimate

$$\delta\sigma_{W,Z}^{\mathrm{NLO}}(\text{expt pdf}) = \pm 2\%$$

but note that there is a greater uncertainty in the NLO prediction, due to possible problems at small x in the global fit to DIS data.

This is because the large rapidity W and Z total cross-sections sample very small x

$\sigma(W^+)/\sigma(W^-)$ is **gold-plated**

$$R_{\pm} = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)}$$

since sea is u, d symmetric at small x, and using MRST2001E

$$\delta R_{\pm}(\text{expt. pdf}) = \pm 1.4\%$$

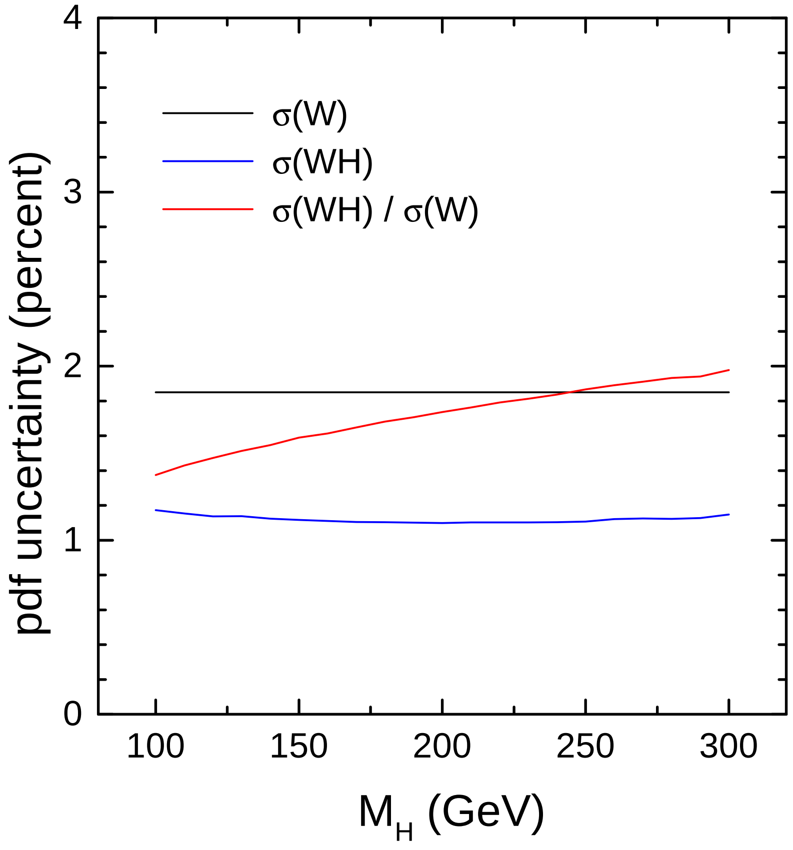
Assuming all other uncertainties cancel, this is probably the most accurate SM cross-section test at LHC.

Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of x are often anticorrelated rather than correlated, partially due to sum rules.



pdf uncertainties on W, WH
cross sections at LHC (MRST2001E)

Similarly no obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross-section, except maybe for very particular, and rather large, M_H.

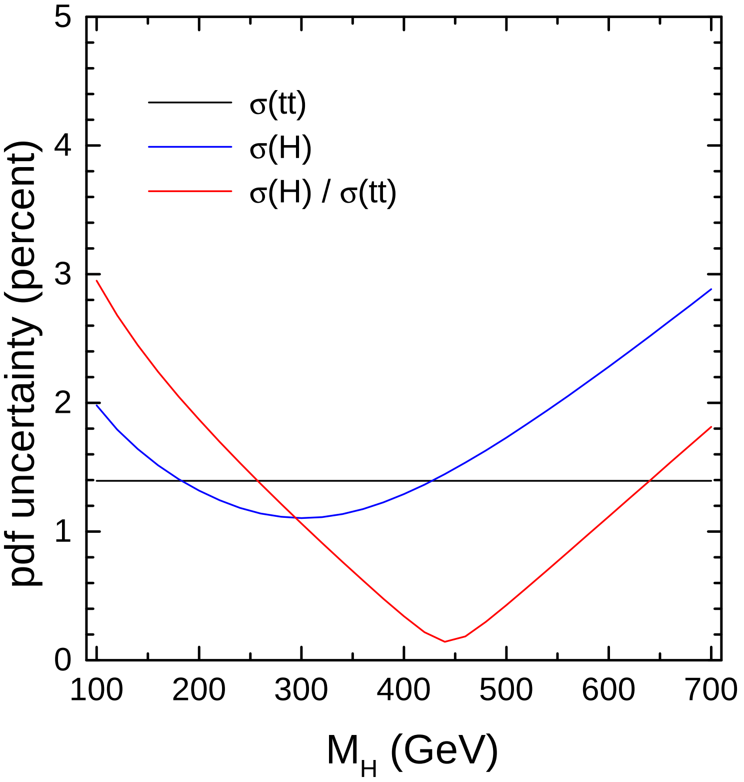
However, a light (SM or MSSM) Higgs dominantly produced via $gg \to H$ and the cross-section has small pdf uncertainty because $g(x)$ at small x is well constrained by HERA DIS data.

Current best (MRST) estimate, for $M_H = 120$ GeV: $\delta\sigma_H^{\rm NLO}$(expt pdf) $= \pm 2 - 3\%$ with less sensitivity to small x than $\sigma(W)$.

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

$\delta\sigma_H^{\rm NNLL}$(scale variation) $= \pm 8\%$

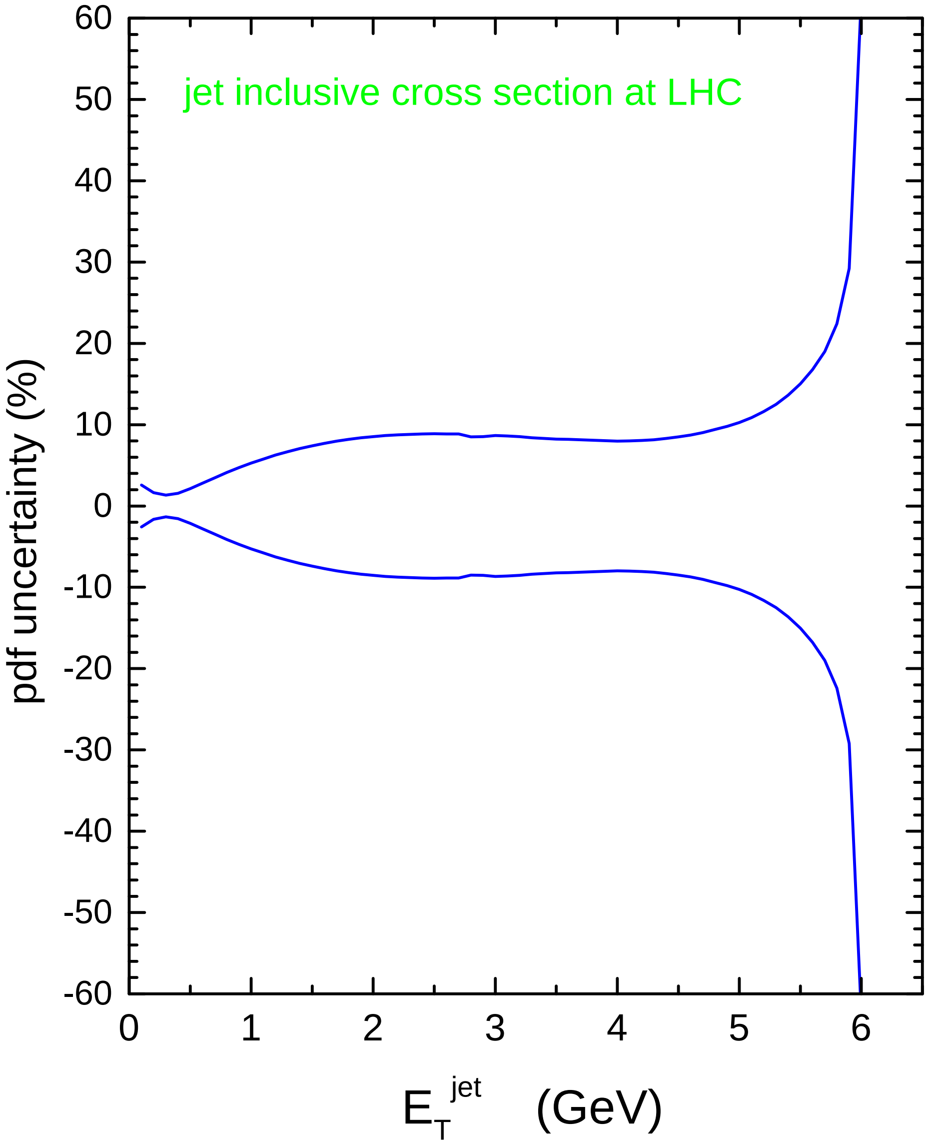
pdf uncertainties on top, (gg→) H cross sections at LHC (MRST2001E)



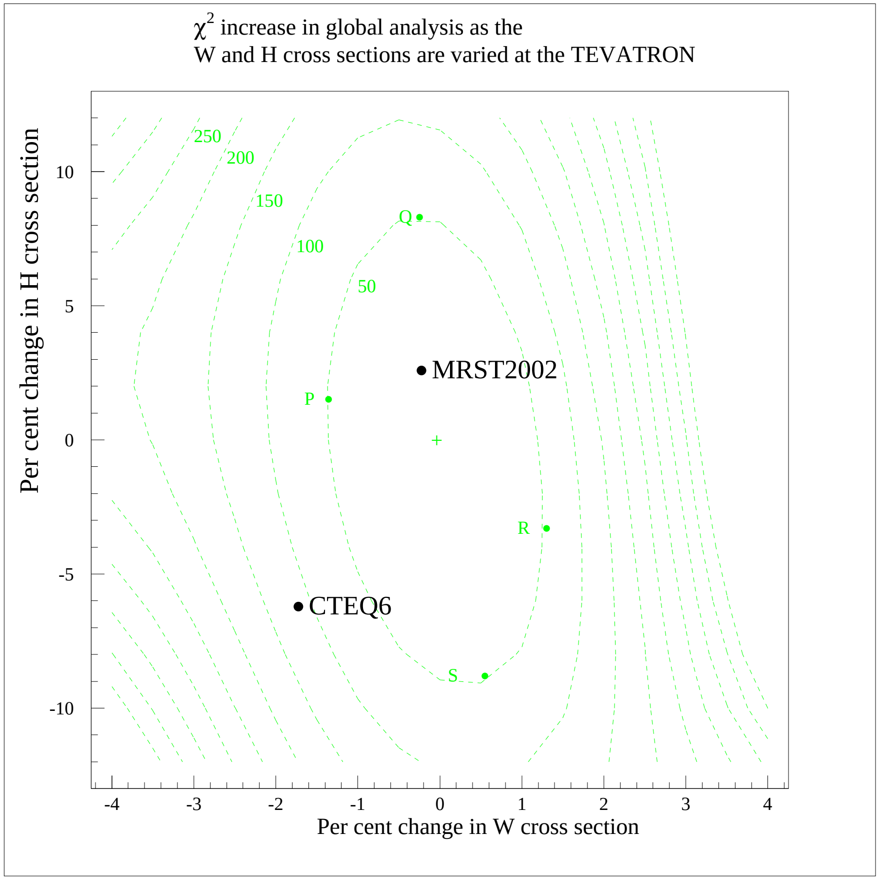
High-E_T Jets

In constrast, the error on predictions for very high-E_T jets at the LHC is dominated by the parton uncertainties.

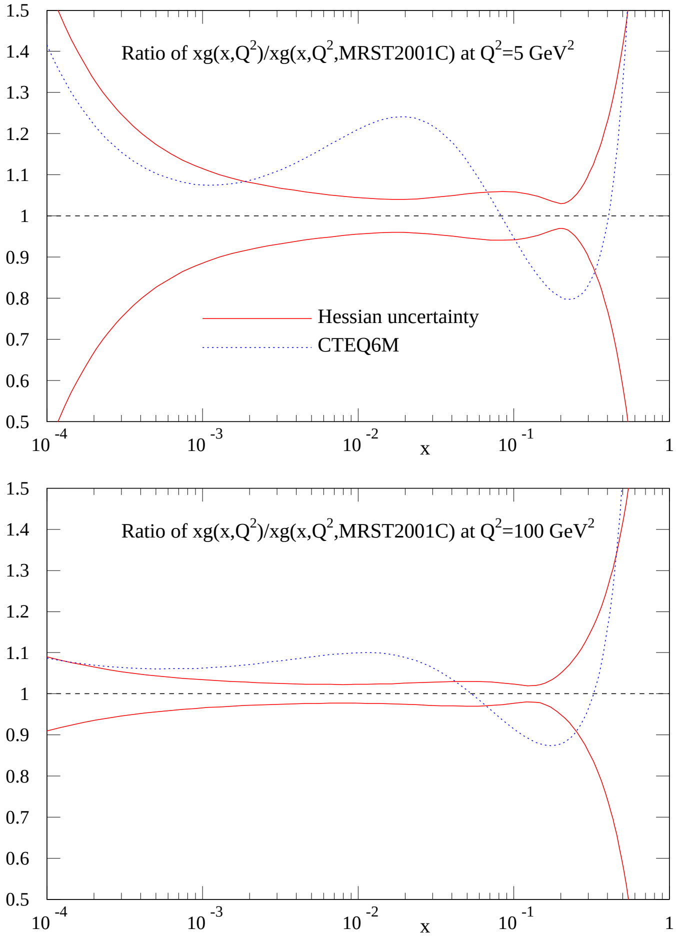
Sensitive to relatively poorly known high-x gluon.



Different approaches to fits generally lead to similar uncertainty for measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit and correctness of NLO QCD.



Uncertainty of gluon from Hessian method



Many can be as important as experimental errors on data used (or more so).

Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross-sections for Drell-Yan pairs (e^+e^-) with `PYTHIA` 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

PDF set	Comment	xsec [pb]	PDF uncertainty %
$81 < M < 101$ GeV			
CTEQ6	LHAPDF	1065 ± 46	4.4
MRST2002	LHAPDF	1091 ± ...	3
Fermi2002	LHAPDF	853 ± 18	2.2

Comparison of $\sigma_W \cdot B_{l\nu}$ for MRST2002 and Alekhin partons.

PDF set	Comment	xsec [nb]	PDF uncertainty
Alekhin	Tevatron	2.73	± 0.05 (tot)
MRST2002	Tevatron	2.59	± 0.03 (expt)
CTEQ6	Tevatron	2.54	± 0.10 (expt)
Alekhin	LHC	215	± 6 (tot)
MRST2002	LHC	204	± 4 (expt)
CTEQ6	LHC	205	± 8 (expt)

In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.

Problems in the fit.

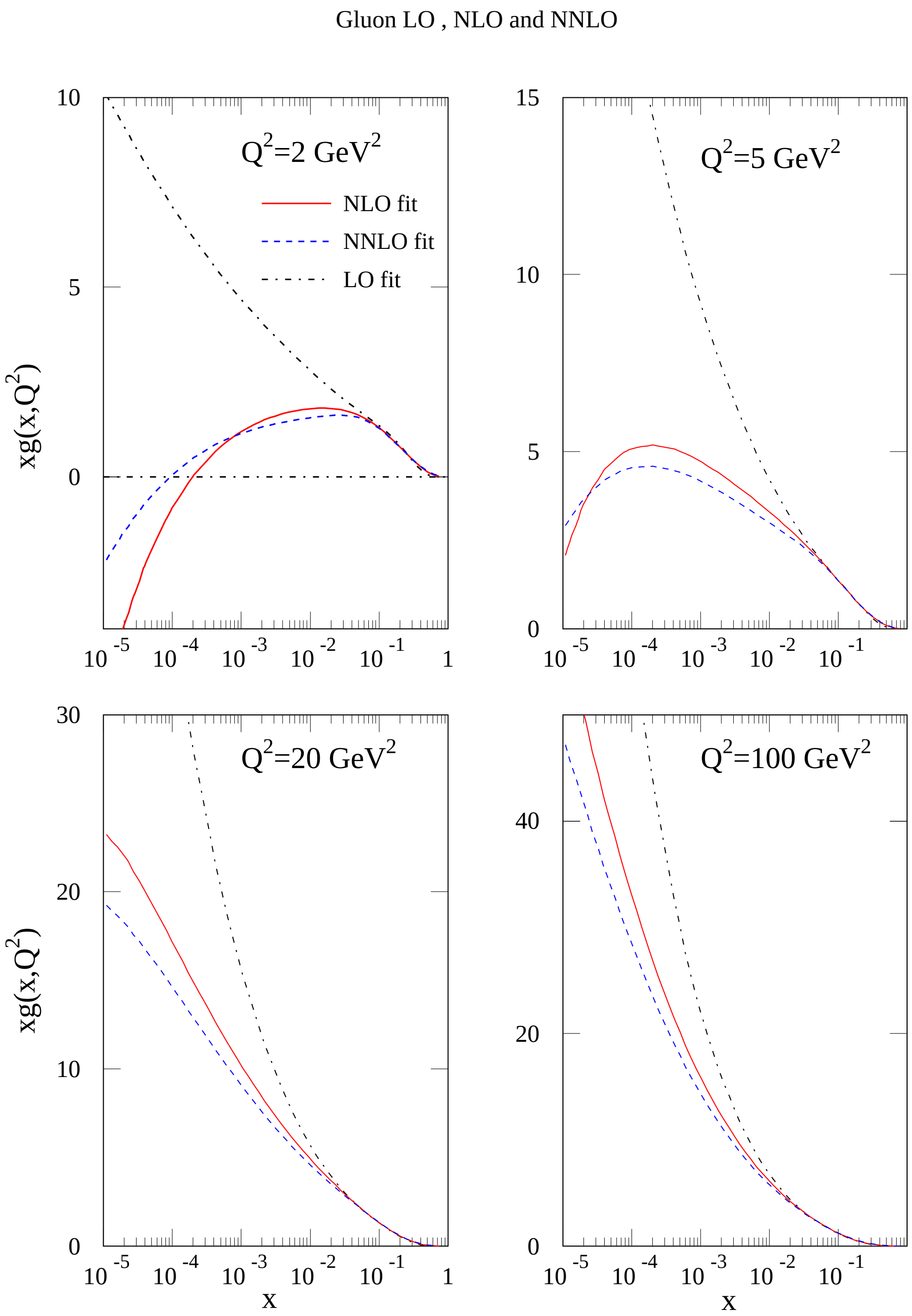
Variations from different approaches partially due to inadequacy of theory.

Failings of NLO QCD indicated by some areas where fit quality could be improved.

Good fit to HERA data, but some problems at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$.

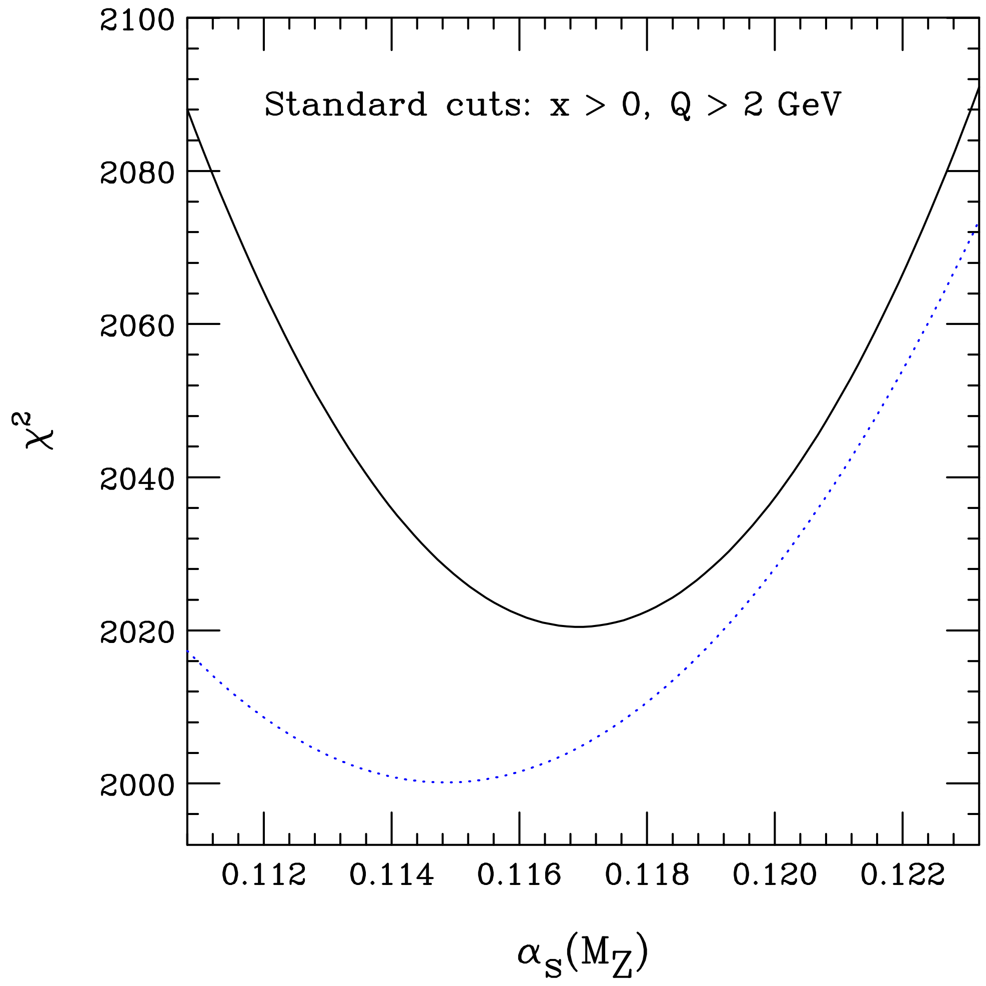
Data require gluon to be negative at low Q^2, e.g. MRST $Q_0^2 = 1\mathrm{GeV}^2$. Needed by all data (e.g. Tevatron jets) not just low Q^2 low x data.

$xg(x,Q^2)$ going from LO → NLO → NNLO.



Other groups find similar problems with gluon at low x.

CTEQ have valence-like input gluon at $Q_0^2 = 1.69\text{GeV}^2$ which would like (at least a little) to be negative. (Blue line – negative gluon allowed, **black** line – positive definite gluon.)

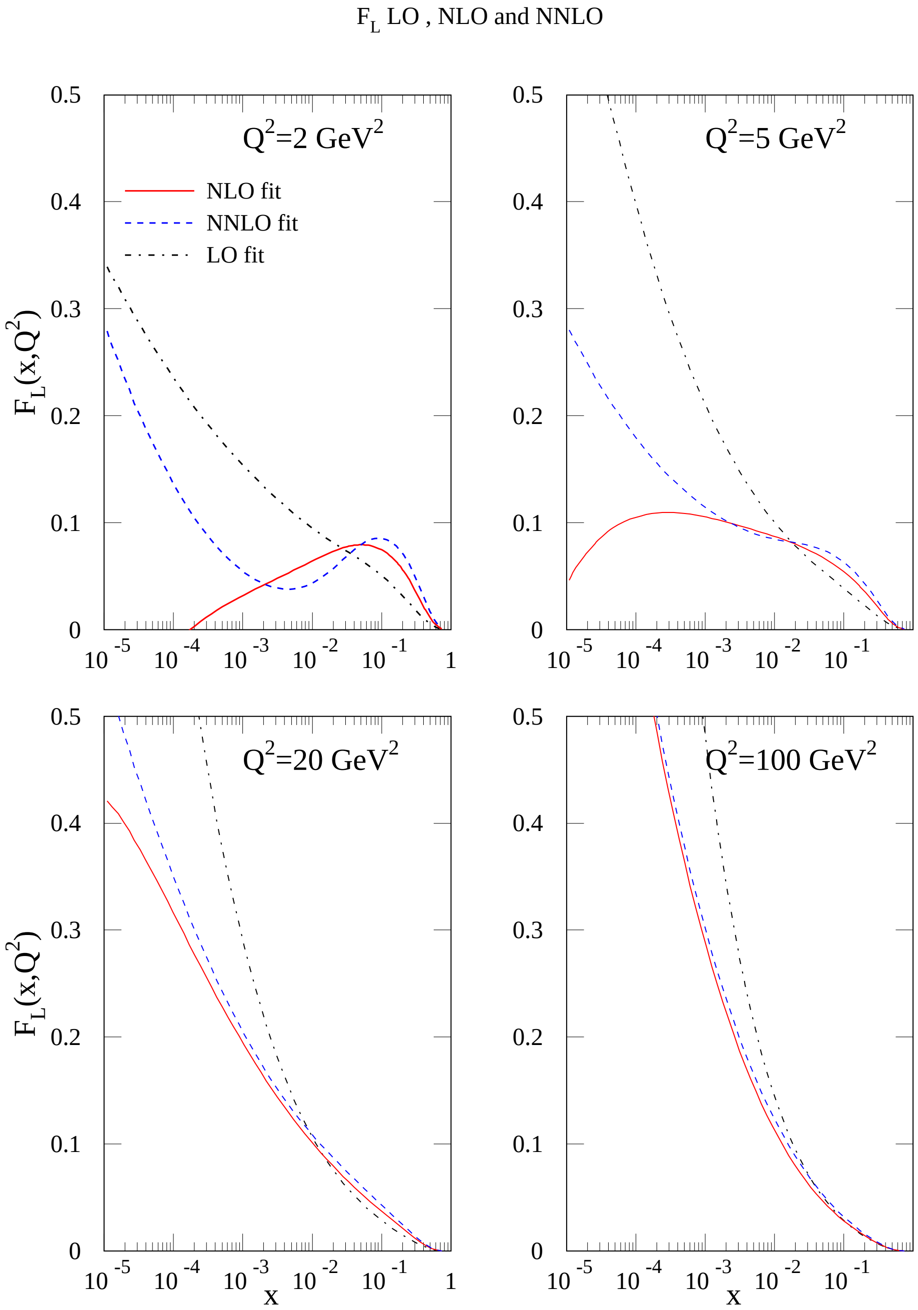


Also instability in physical, gluon dominated, quantity $F_L(x,Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Gluon at NLO $\rightarrow$ $F_L(x,Q^2)$ dangerously small at smallest x, Q^2.

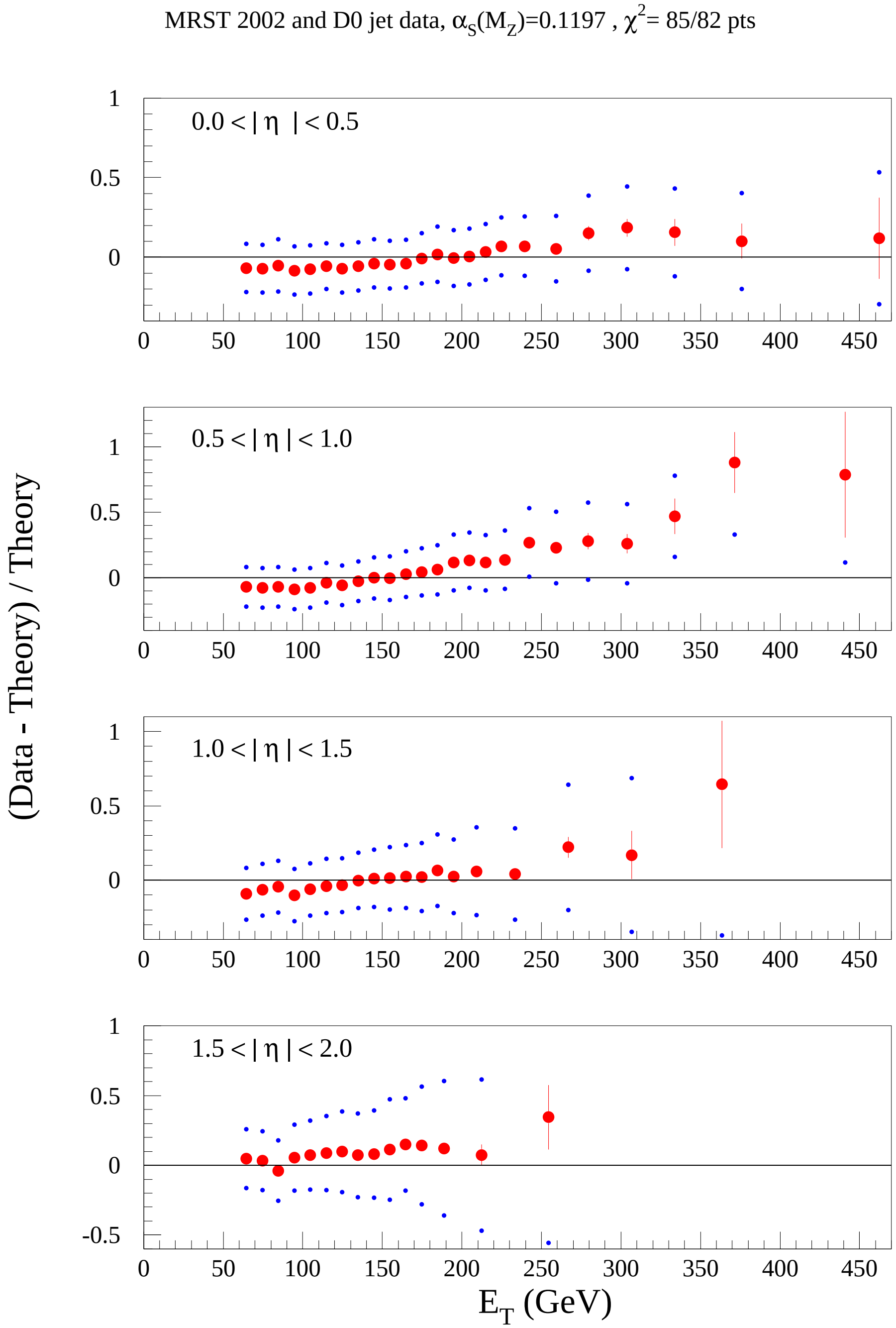
Note very large effect of exact NNLO coefficient function.

Possible sign of required $\ln(1/x)$ corrections.



As an example of the effect of assumptions, MRST found only a reasonable fit to jet data, but need to use the large systematic errors.

Better for CTEQ6 due to different cuts on other data, and different type of high-x parameterization. Usually worse for other partons (jets not in fits). General tension between HERA and NMC data and jets.



Comparison to CDF1B jet data.

Can explicitly see data move relative to theory using correlated systematic errors.

MRST 2001 and CDF1B jet data

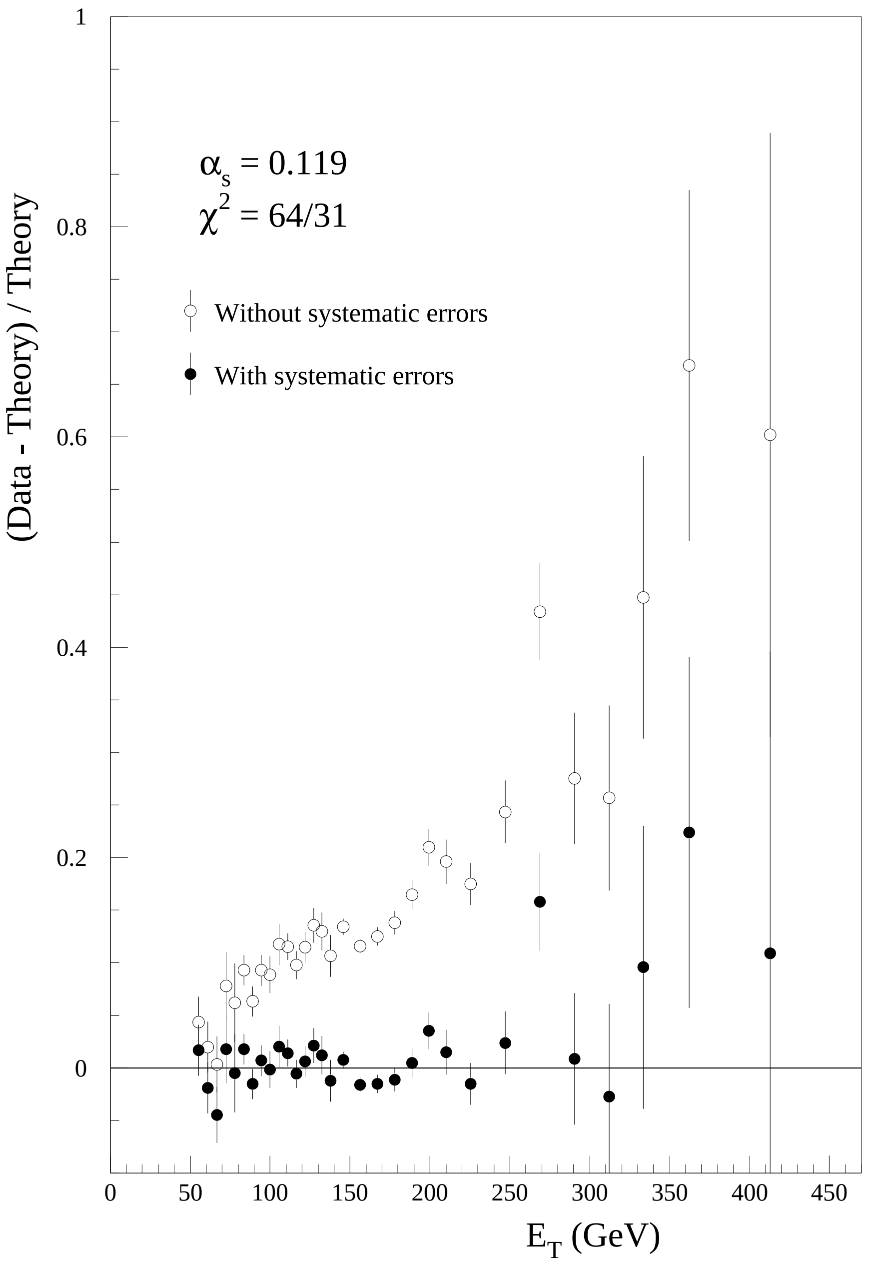


Illustration of problem with jets.

Using simple spectator counting rules, at high x,

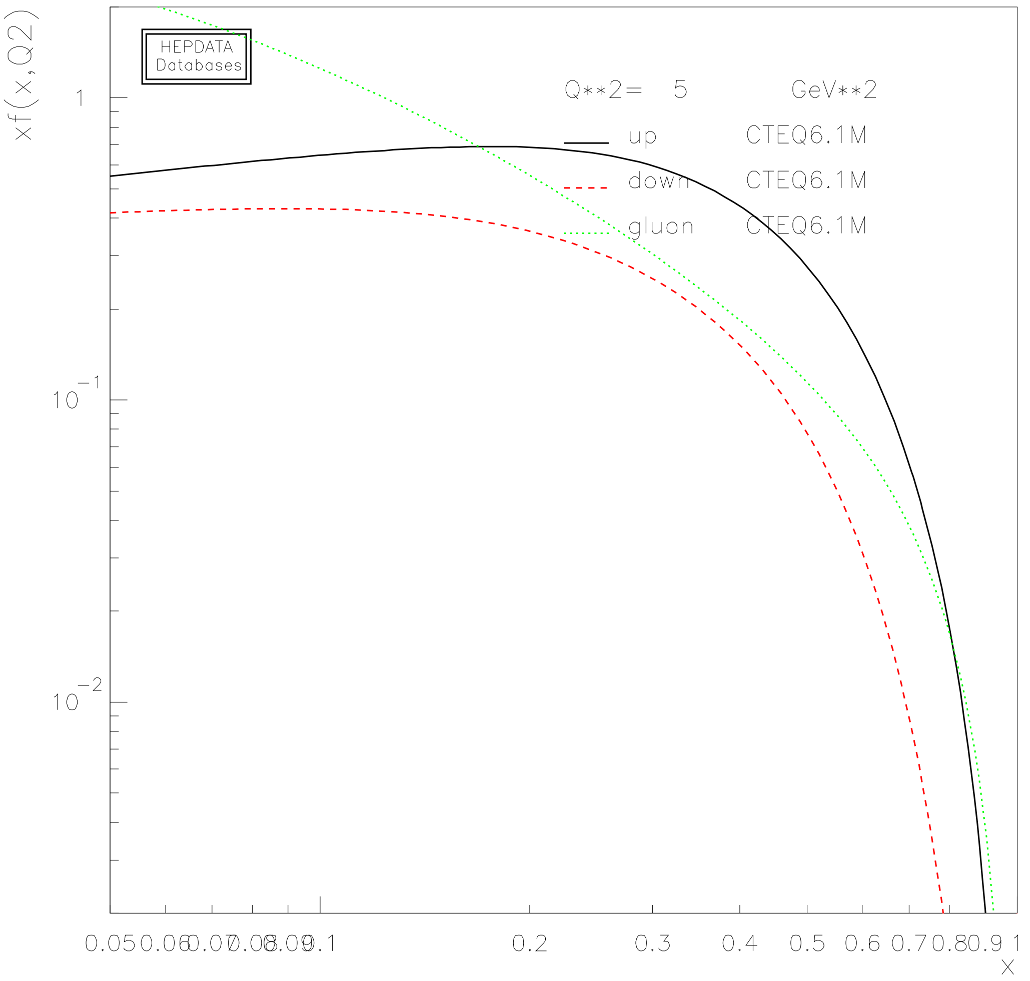
$q_V(x) \sim (1-x)^3, \;\; g(x) \sim (1-x)^5$

Clearly not true for CTEQ6.1M partons which give good jet fit.

Gluon is hardest as $x \to 1$.

MRST parameterizations don't allow such a hard gluon. Fits not as good as one would ideally like.

Worse at NNLO since high-x quarks smaller → even bigger gluon.



New approach to high-x gluon.

In DIS scheme $F_2(x,Q^2) \equiv \sum_i^{N_f} e_i^2\, xq_i(x,Q^2)$.

Under change of scheme from $\overline{MS}$ to DIS schemes we have:

$$q^{DIS}(x) = q^{\overline{MS}} + C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}} + C_{2,g}^{\overline{MS}} \otimes g^{\overline{MS}},$$

$$g^{DIS} = g^{\overline{MS}} - C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}} - C_{2,g}^{\overline{MS}} \otimes g^{\overline{MS}}.$$

Designed to maintain 100% momentum.

Scheme transformation should dominate high-x gluon if valence quarks naturally biggest at high x.

If $g^{\overline{MS}} \sim (1-x)^5$, then becomes negative in DIS scheme. Or if $g^{DIS} \sim (1-x)^5$, then transformation determines very high-x limit.

DIS scheme is certainly more physical for quarks. $\overline{MS}$ scheme not really physical at all.

Assume high-x gluon is smaller than high-x quarks in DIS scheme. Therefore in $\overline{MS}$ scheme

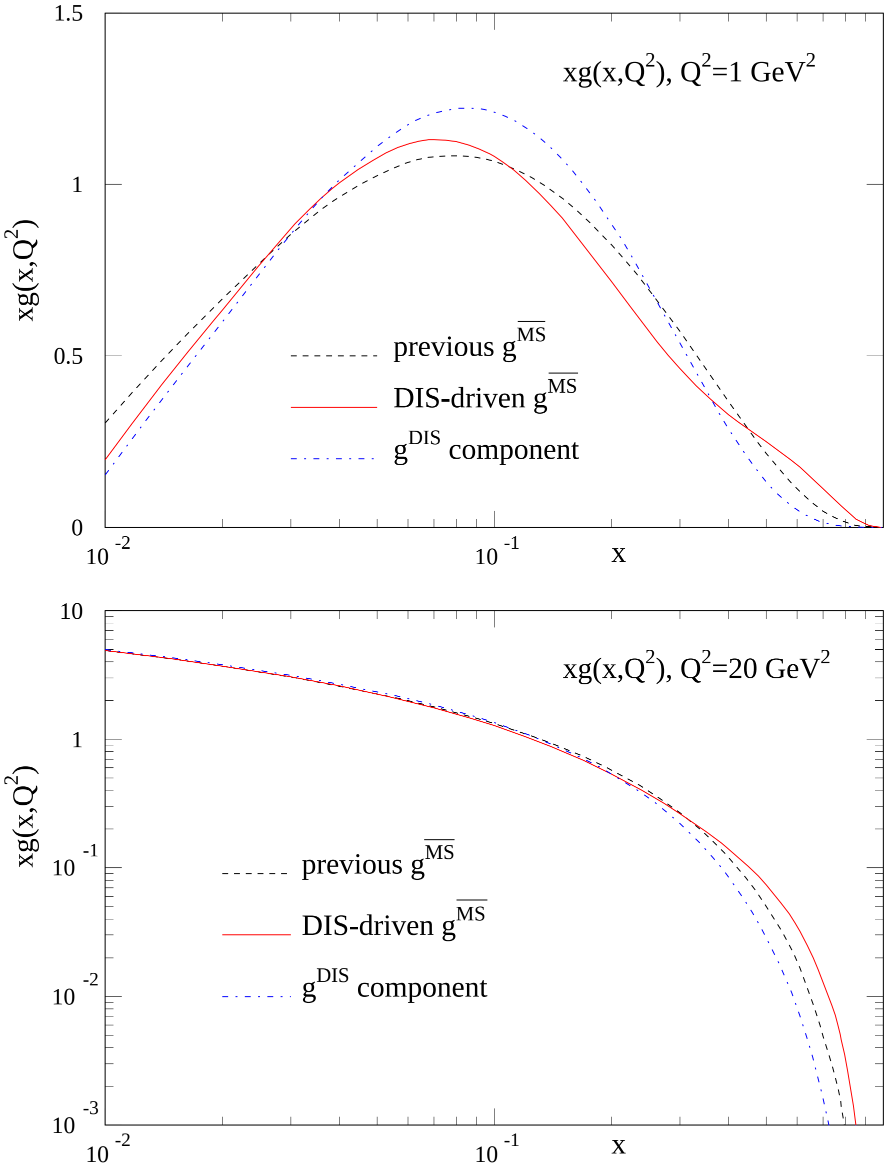
$$g^{\overline{MS}} = g^{DIS} + C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}},$$

so high-x gluon determined from quarks.

Works extremely well. χ^2 for jets $154 \rightarrow 116$.

Total $\Delta\chi^2 = -26$.

DIS scheme gluon more natural at high x.



Works even better at NNLO.

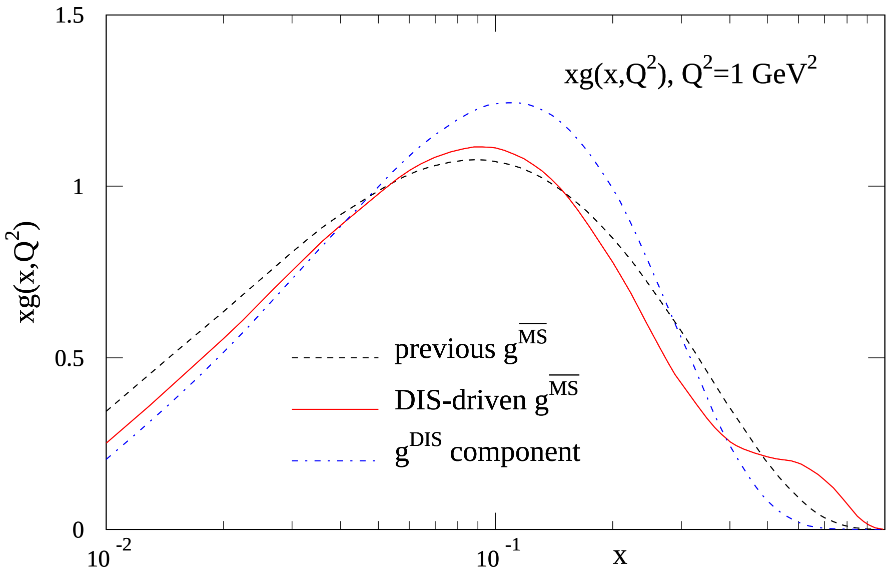
$C_{2,q}^{\overline{MS},(2)} \otimes q^{\overline{MS}}$ positive and significant at very high $x \rightarrow$ high-x gluon even more determined from quarks.

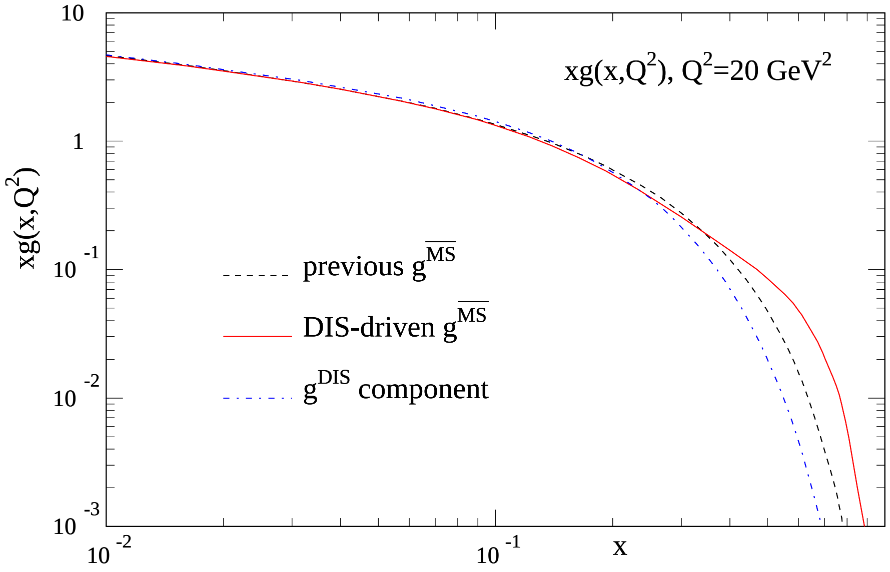
Now χ^2 for jets $164 \rightarrow 117$.

Total $\Delta\chi^2 = -79$.

DIS scheme gluon again more natural at high x.

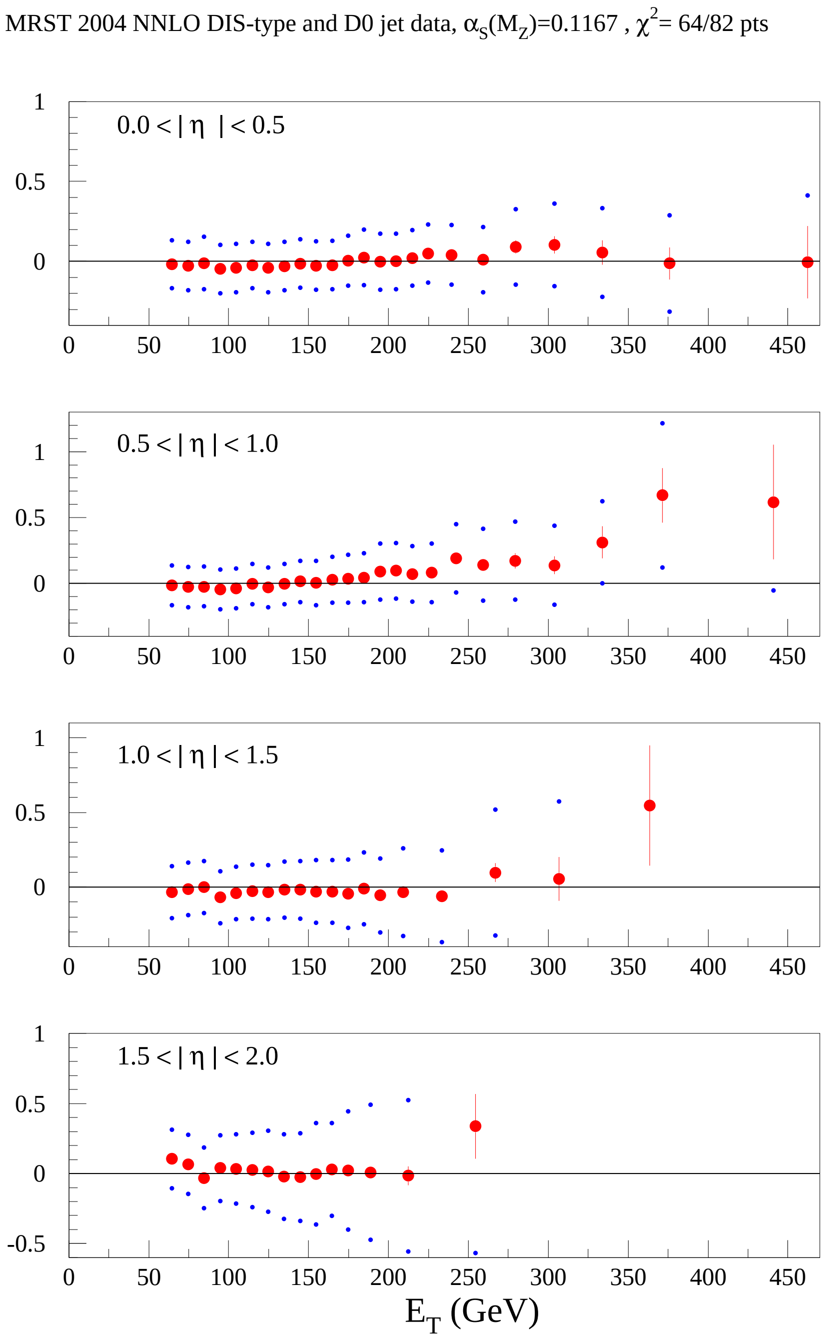
In $\overline{MS}$ scheme high-x gluon unphysical and determined entirely by quarks?





Comparison to D0 jet data for scheme change-inspired partons.

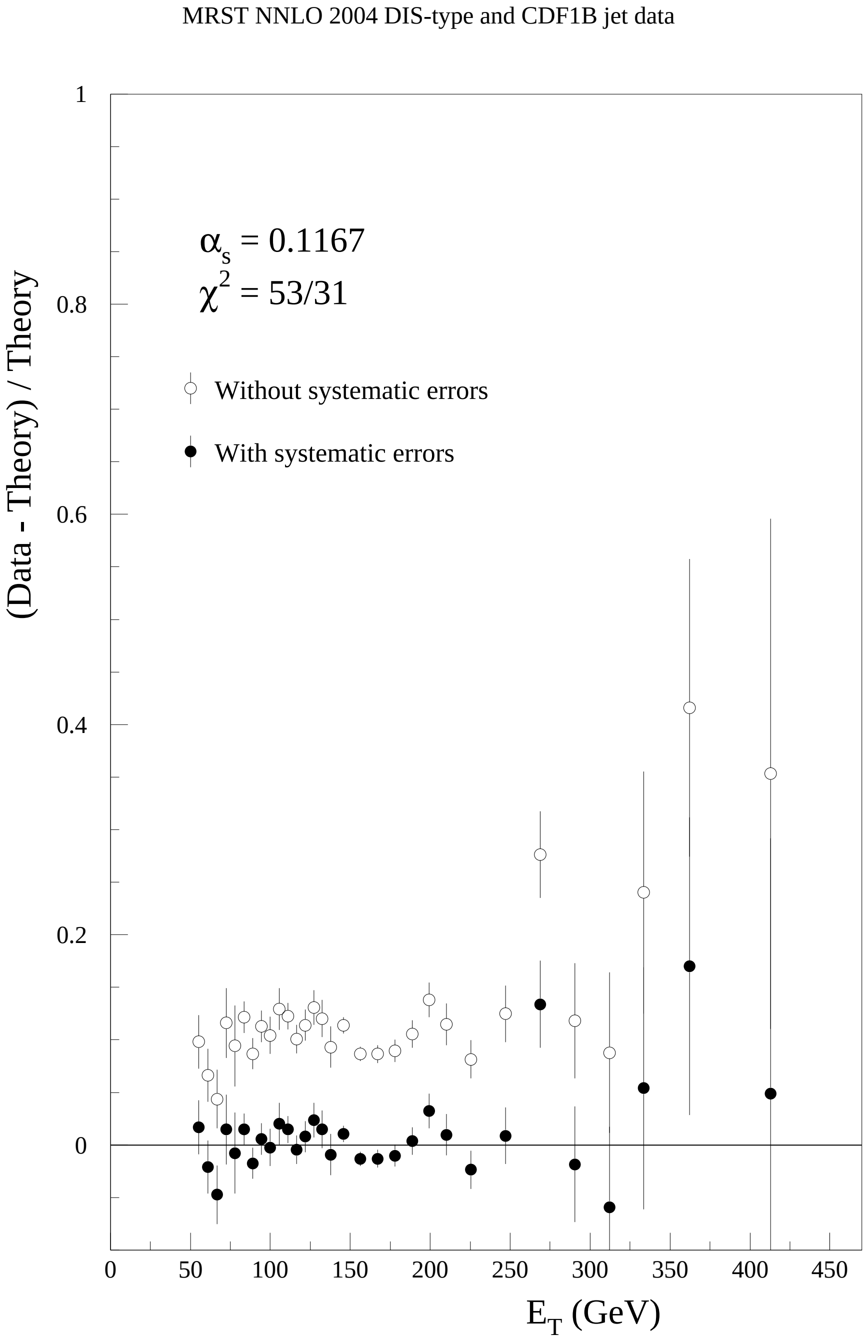
Shape much better.



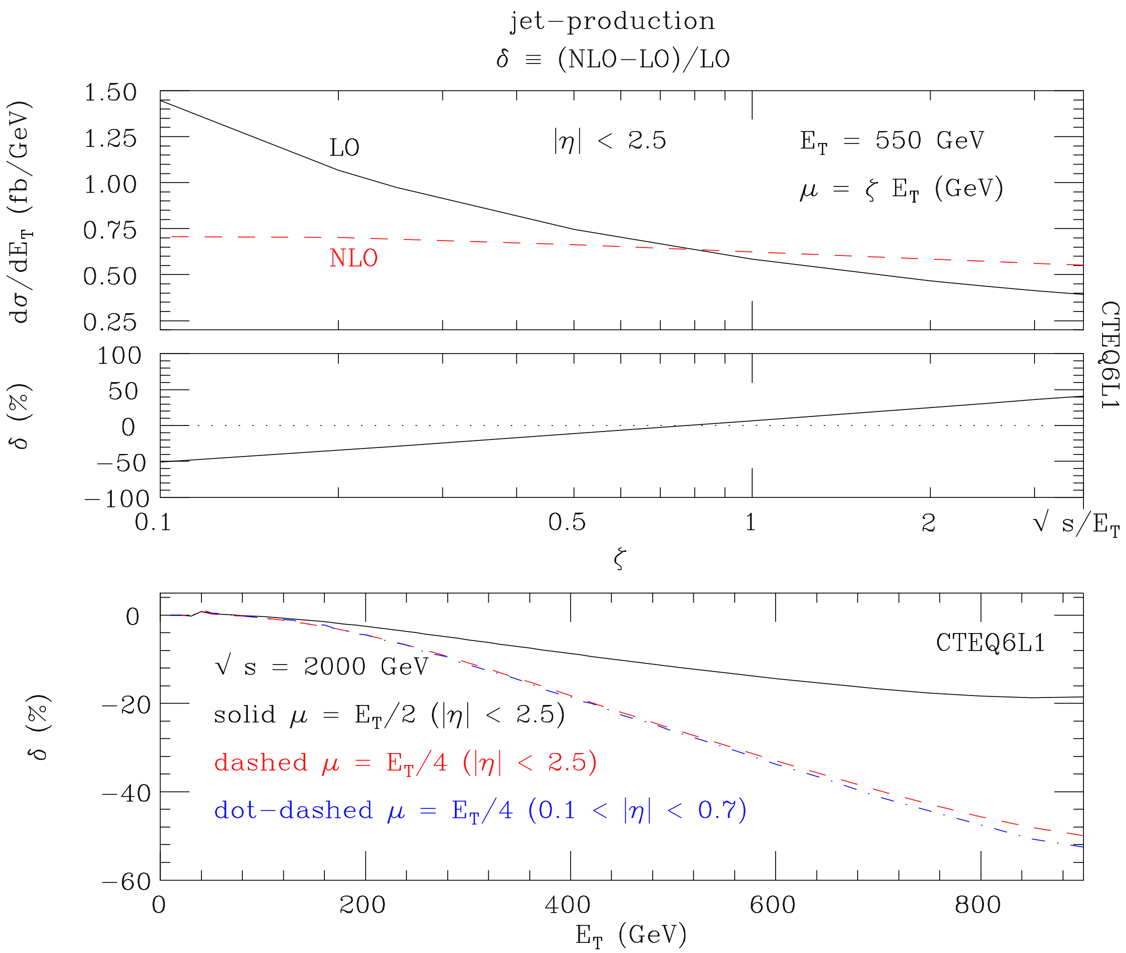
Comparison to CDF1B jet data for scheme change-inspired partons.

Shape now correct. Normalization shift of theory relative to data.

6% normalization difference between CDF and $D0$.



Weak corrections



Calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{2}{3}C_F\frac{\alpha_W}{\pi}\log^2(E_T^2/M_W^2))$.

Dominated by quark-(anti)quark processes.

They suggest $\approx 12\%$ correction at $E_T = 450\mathrm{GeV}$. Question validity of recent partons.

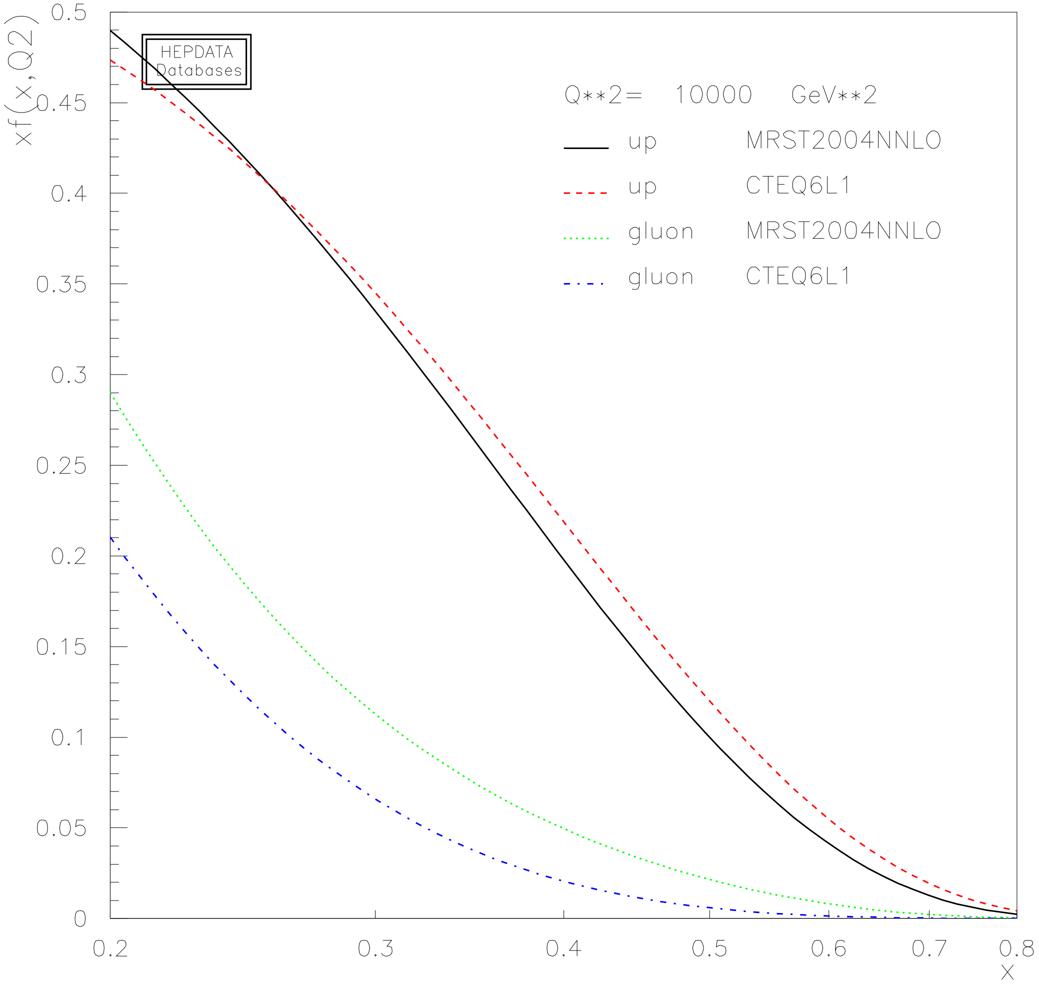
Not quite as big in reality. Use LO partons with big high-x quarks, very small gluon → high-E_T cross-section almost all quarks.

Not the case with most recent partons (look at $x = 0.5$).

$qq \;\; qg \;\; gg$ matrix elements in ratio $5 \;\; 6 \;\; 30$.

Even at highest E_T gluon contributes $\sim 30\%$.

Estimate max suppression reduced to $\approx 8\%$.



Phenomenological impact not huge.

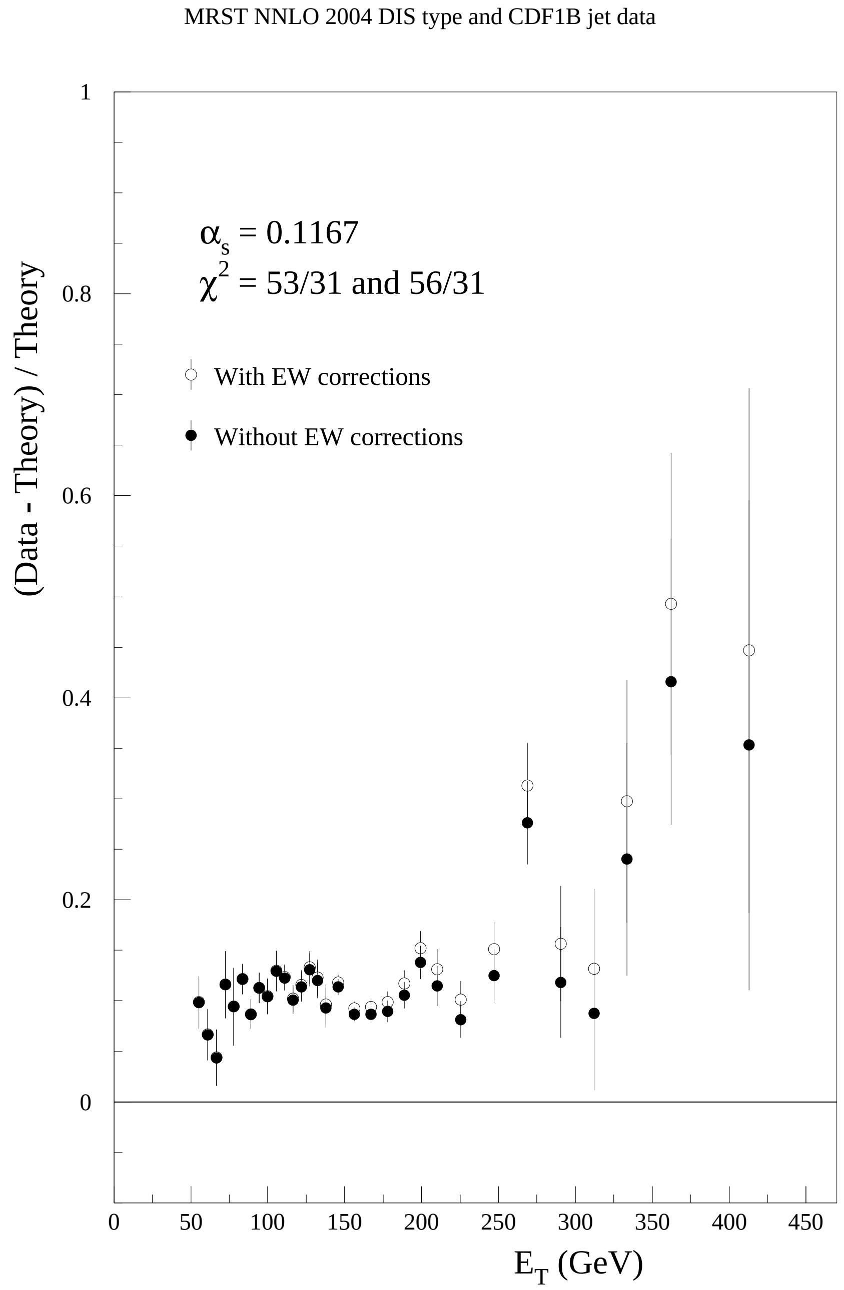
Movement of both CDF and D0 data relatively small.

Total χ^2 goes from $117/113$ to $131/113$ (without refitting).

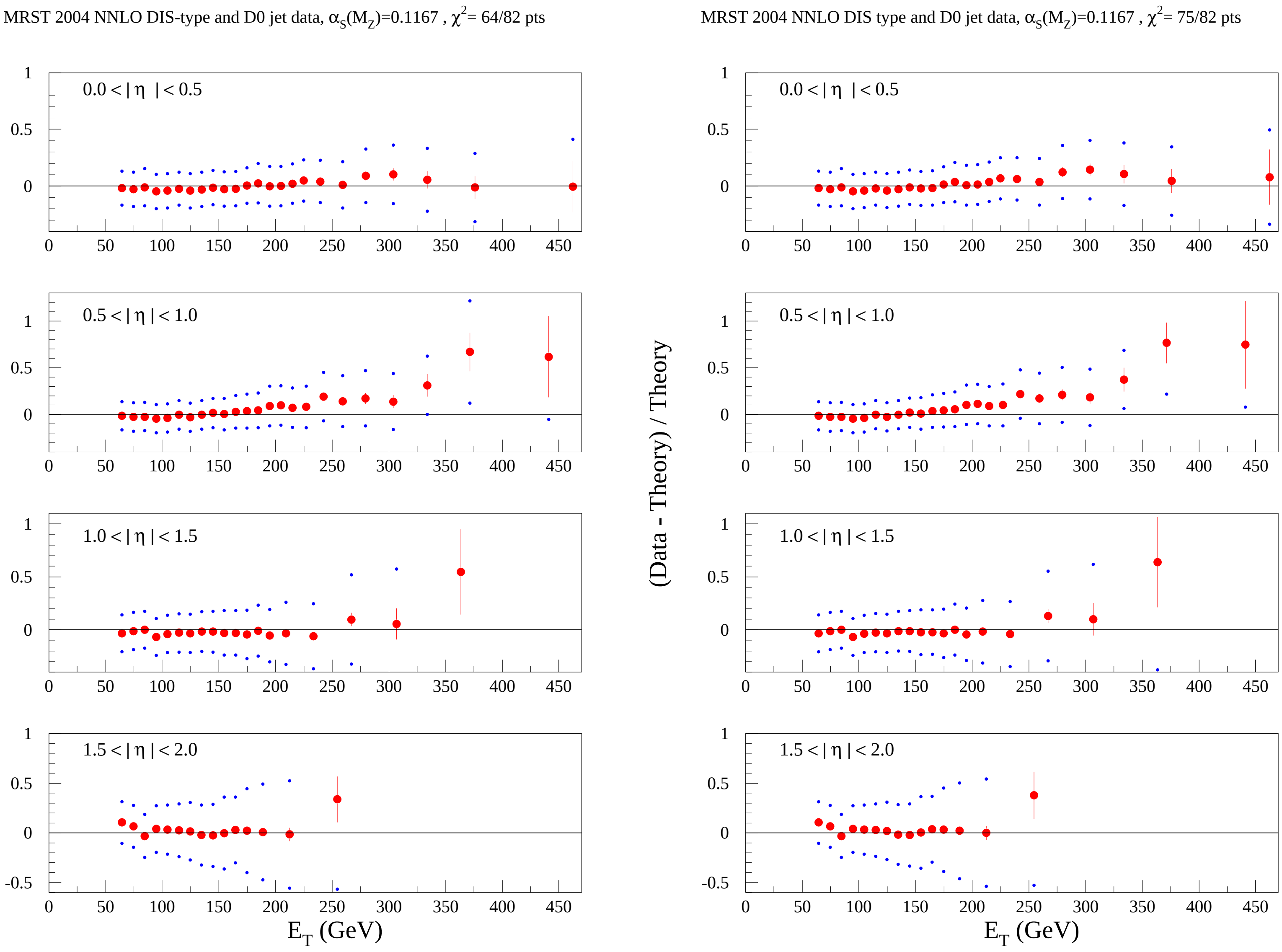
Significant but not a disaster by any means.

More important at higher E_T.

But positive real corrections to be added (depend on jet definitions).



Change in fit to D0 data.



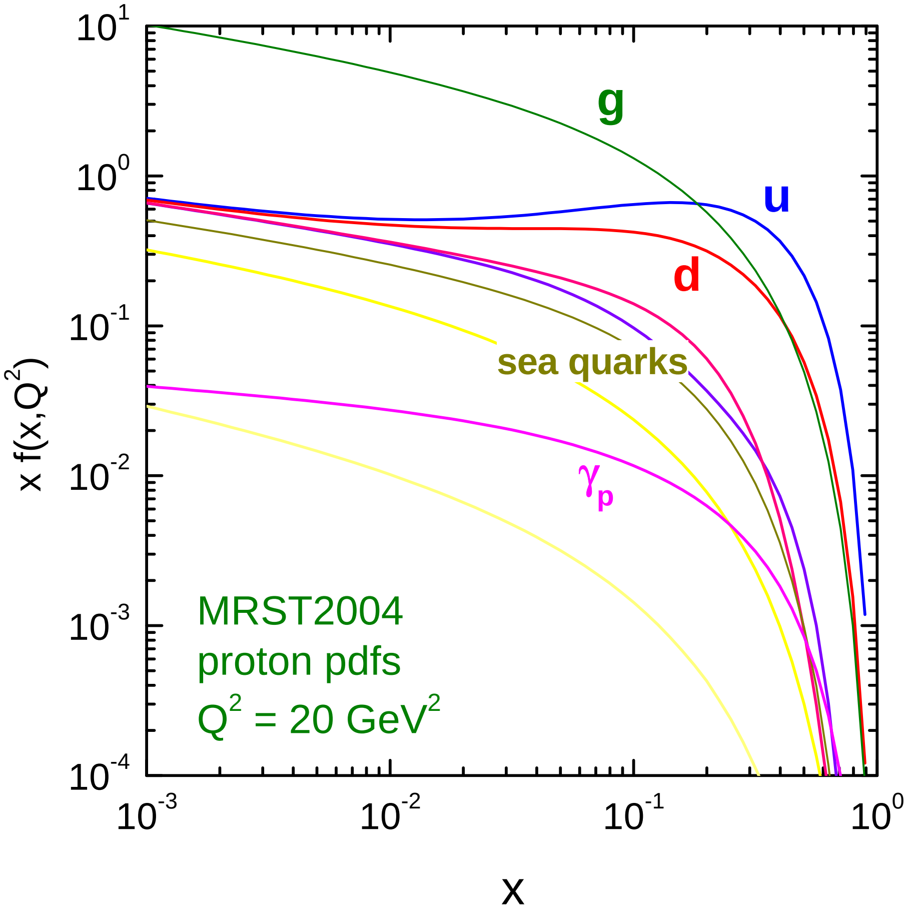
QED **Effects**.

New study by MRST.

Effect on quark distributions is entirely negligible at small x where gluon contribution dominates DGLAP evolution.

At large x, photon radiation from quarks leads to faster evolution, roughly equivalent to a slight shift in α_S : $\Delta\alpha_S(M_Z^2) \simeq +0.0003$

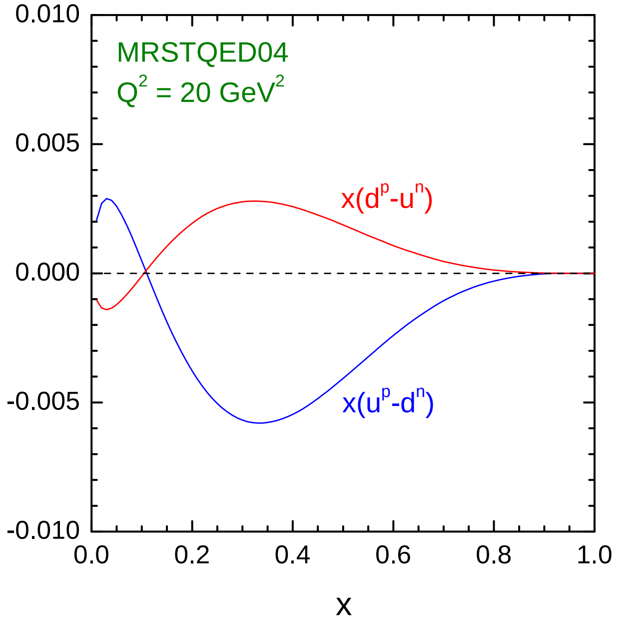
Overall QED effects much smaller than many sources of uncertainty.



However, QED effects to lead to small isospin violation.

$u_V^p(x)$ quarks radiate more photons than $d_V^n(x)$ quarks.

To rough approximation



$$\gamma(x,Q^2) = \sum_j e_j^2 \frac{\alpha}{2\pi} \ln(Q^2/m_q^2) \int_x^1 \frac{dy}{y} P_{\gamma q}(y) q_j(\frac{x}{y}, Q^2).$$

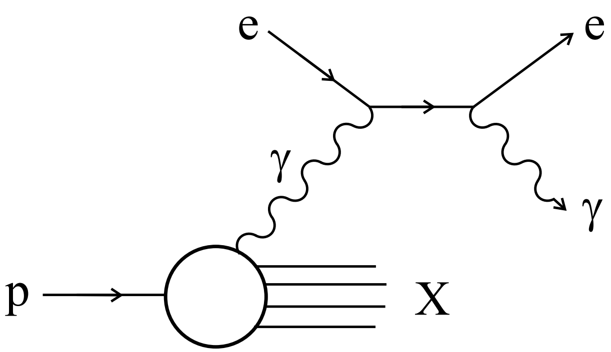
So more photon momentum in proton than neutron due to high-x up quarks radiating more than high-x down quarks.

Momentum conservation $\rightarrow$ $u_V^p(x) < d_V^n(x)$ at high .

Hence, $[\delta U_\mathrm{v}] < 0$ as required by NuTeV anomaly.

Estimates for $m_u = 6$ MeV and $m_d = 10$ MeV imply similar to isospin violation observed by best fit! Reduces NuTeV anomaly to about $1/2$.

Model supported by wide angle photon scattering, i.e. $ep \rightarrow e\gamma X$ where final state electron and photon have equal and opposite large transverse momentum.



ZEUS has recently published a measurement of this cross-section ($x_\gamma \approx 0.005$):

$$\sigma(ep \rightarrow e\gamma X) \;=\; 5.64 \;\pm\; 0.58\,\text{(stat.)}\;\begin{matrix}+0.47\\-0.72\end{matrix}\;\text{(syst.) pb.}$$

Neither PYTHIA nor HERWIG can explain the observed rate – underestimating the cross-section by factors of 2 and 8 respectively.

Using the proton's photon parton distribution we find

$$\sigma(ep \rightarrow e\gamma X) \;=\; 6.2 \;\pm\; 1.2\;\text{pb.}$$

Using constituent quark masses our prediction nearly halves.

NNLO

Splitting functions now complete. (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates → no real change in NNLO partons. Improve quality of fit very slightly (MRST), and reduces $\alpha_S \rightarrow 0.116$.

To do absolutely correct NNLO fit we need not only exact NNLO splitting functions.

Also require rigorous heavy quark thresholds (partons **discontinuous** at NNLO - see Heavy Flavours talk), NNLO Drell-Yan cross-sections, and a complete treatment of uncertainties. All in hand.

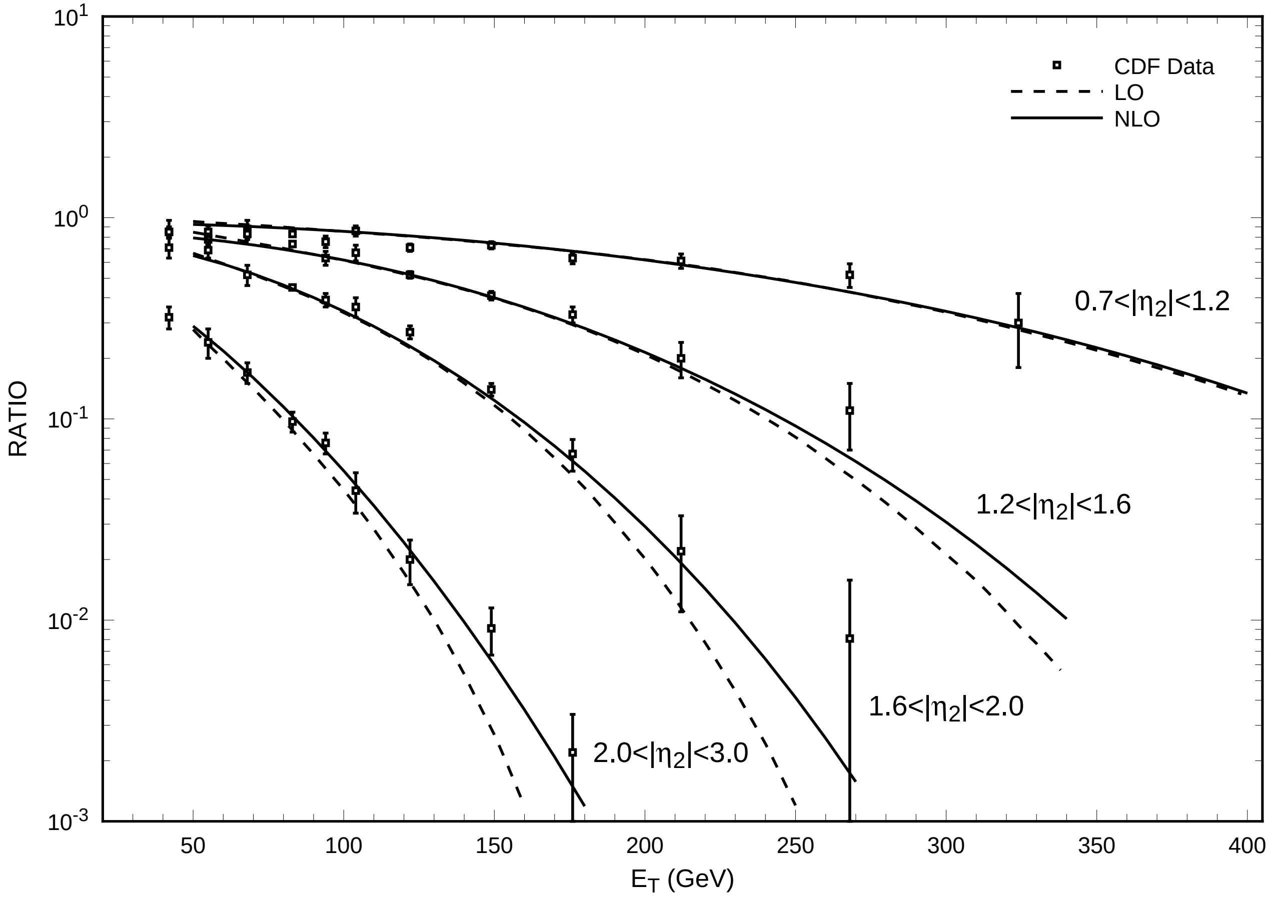
Essentially full NNLO determination of partons very soon.

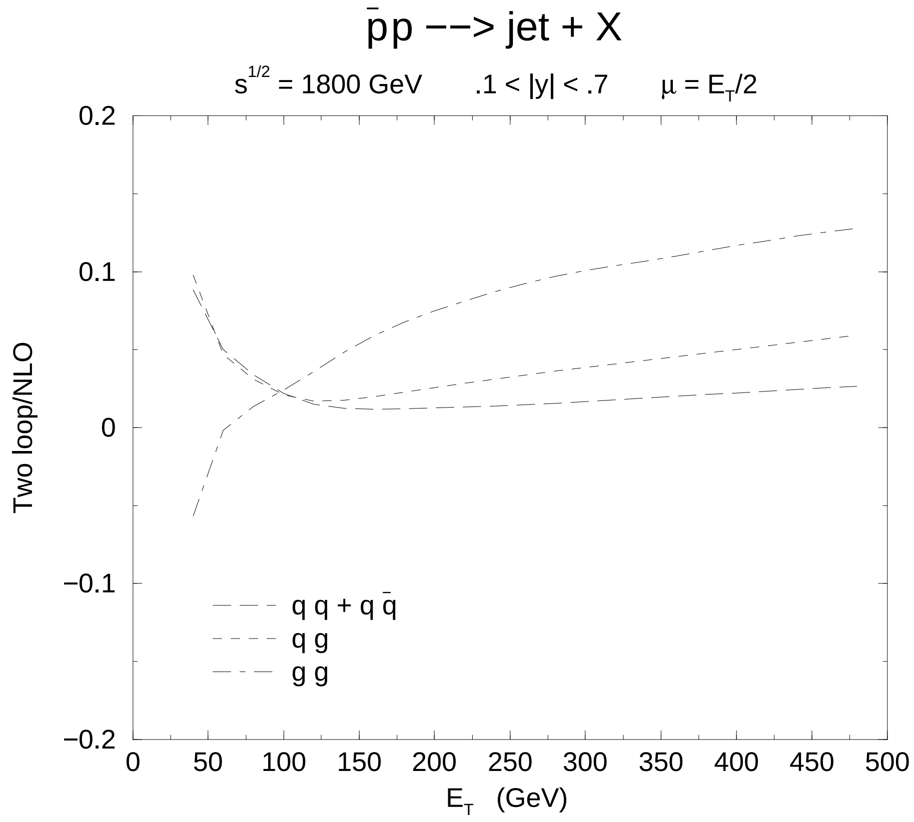
Only NNLO jet cross-sections missing. Is this important?

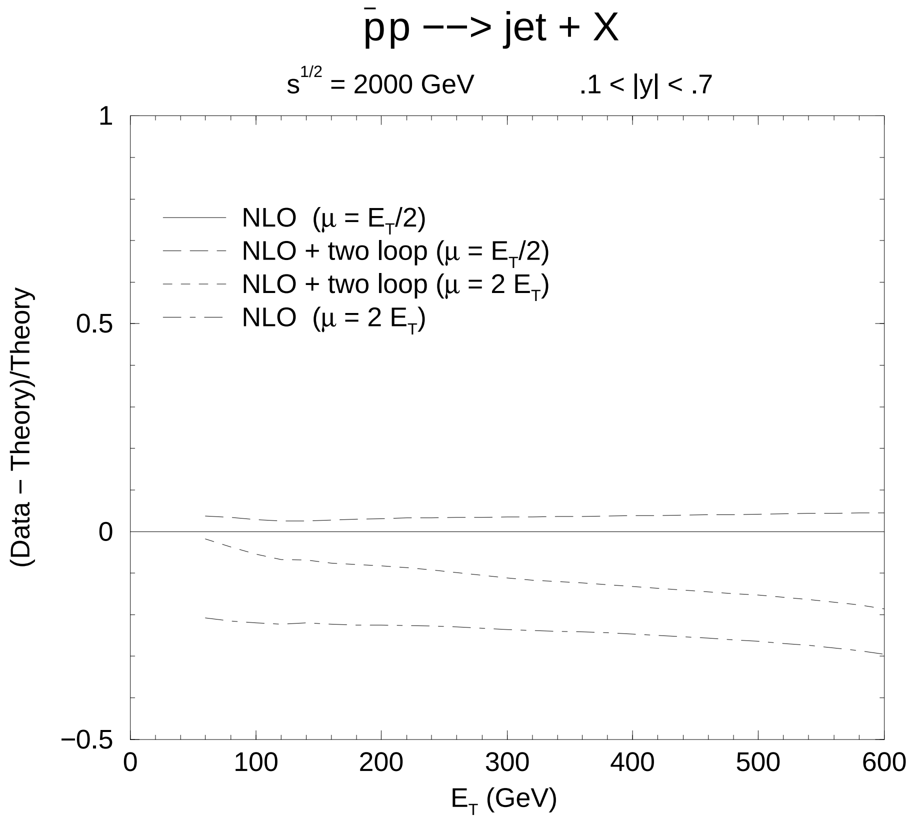
Probably not!

NLO corrections themselves not large, except at high rapidities.

At central rapidities $\leq 10\%$. Similar to correlated errors.







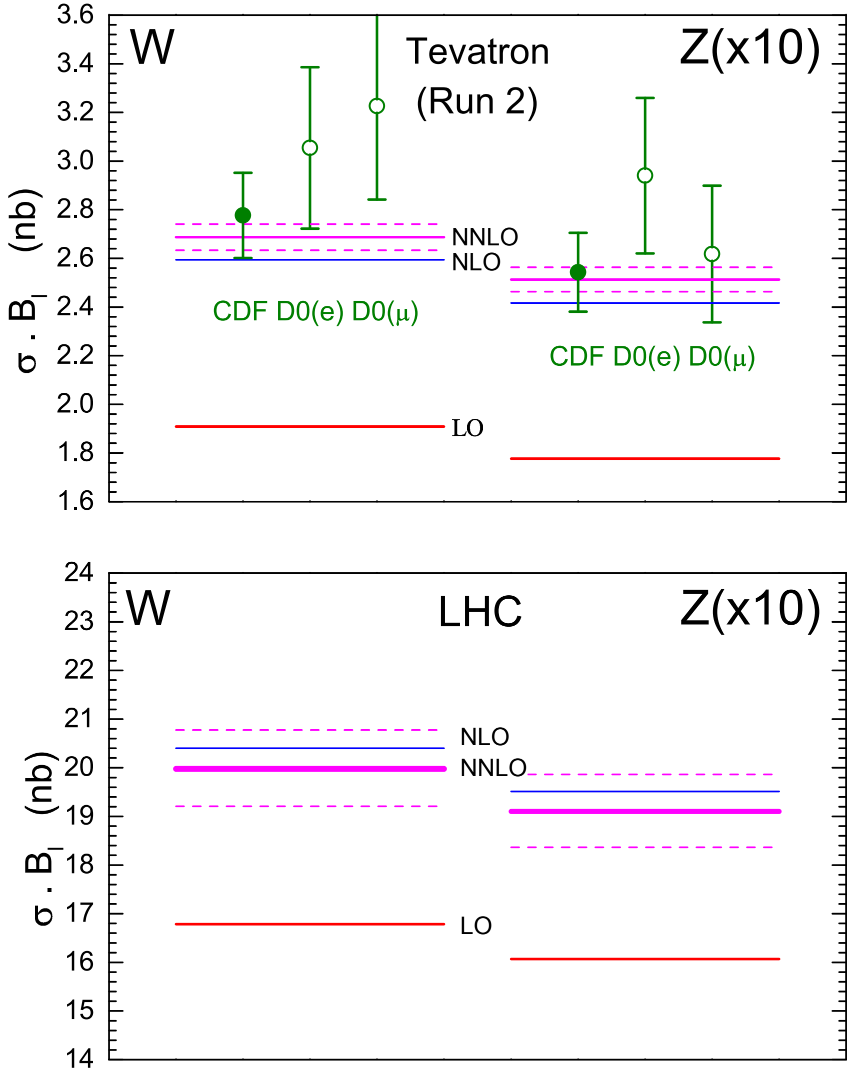
Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms. Expected to be major component of total NNLO correction.

→ Flat $3 - 4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data.

Mistakes from ignoring jets in fits bigger than mistakes made at NNLO by not knowing exact hard cross-section.

Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data. Much worse for gluon dominated quantities e.g. $F_L(x,Q^2)$, as seen. Unstable at small x and Q^2.



partons: MRST2002
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

Approach to Look for Safe Theoretical Regions.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons both stabilize.

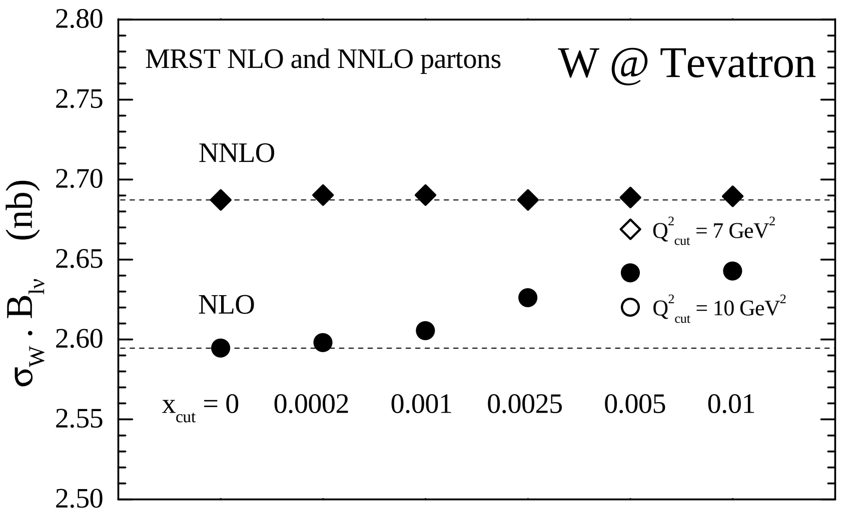
Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$.

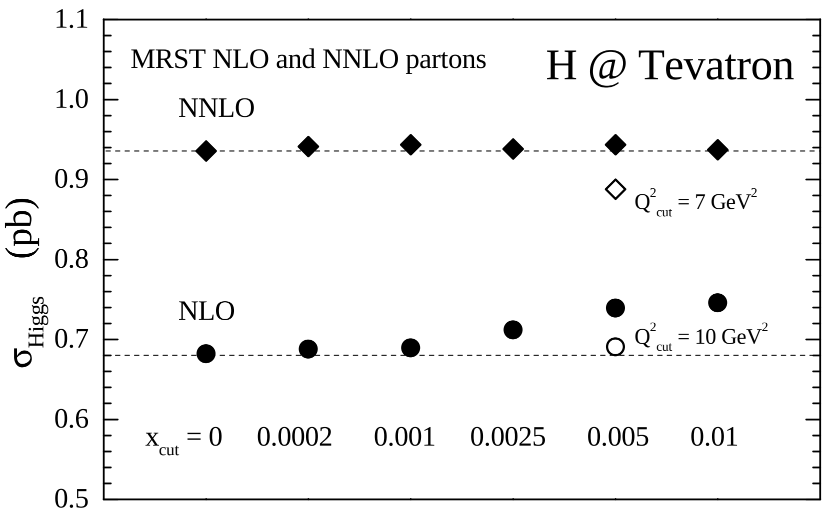
Raising x_{cut} from 0 to 0.005 continuous improvement. At each step moderate x gluon becomes more positive.

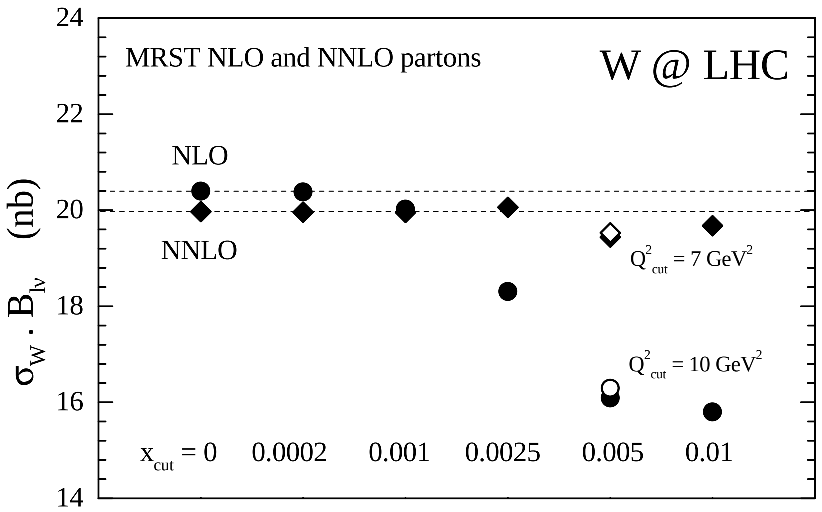
→ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

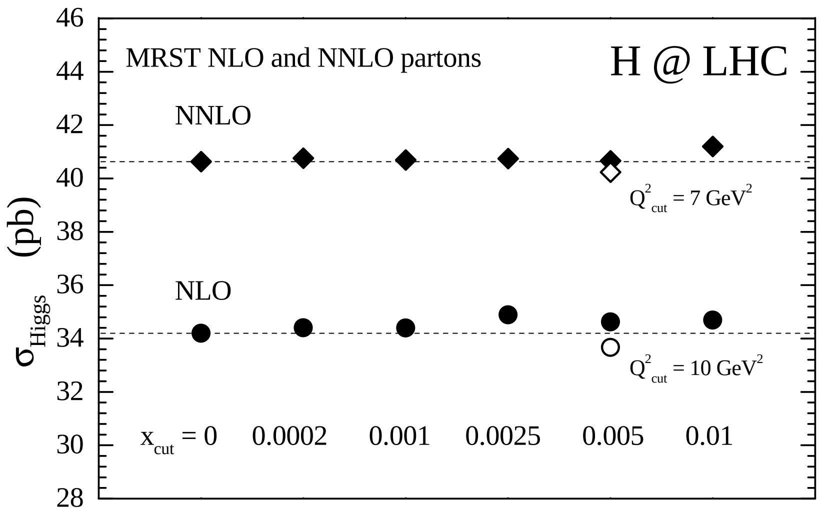
Also NNLO conservative partons. Similar cuts and improvement in fit quality (bit smaller), but change in partons considerably less. Aleady includes important theoretical corrections.

Variation in predictions with cuts. Indicates range of possible theoretical error.



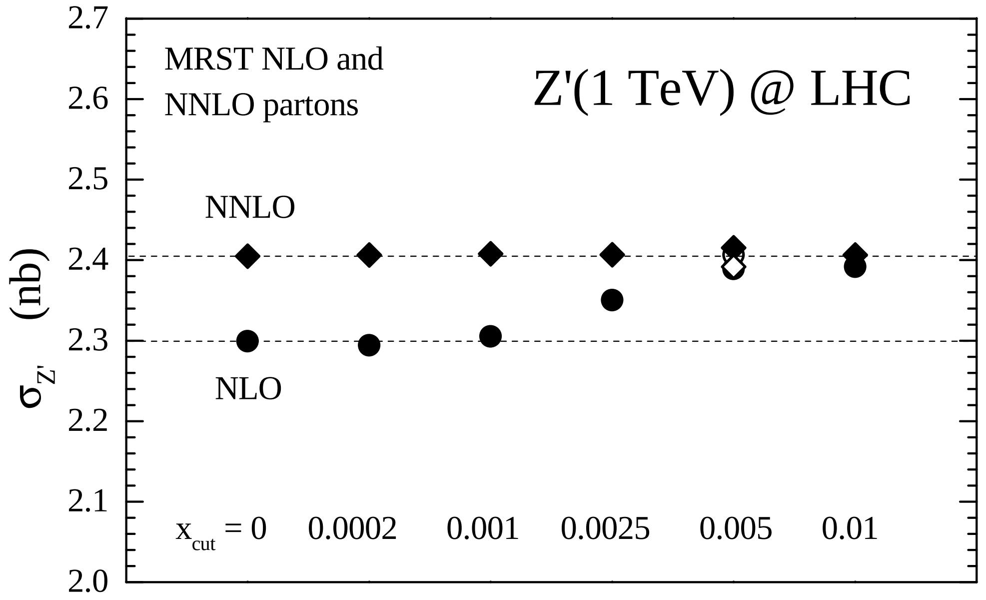


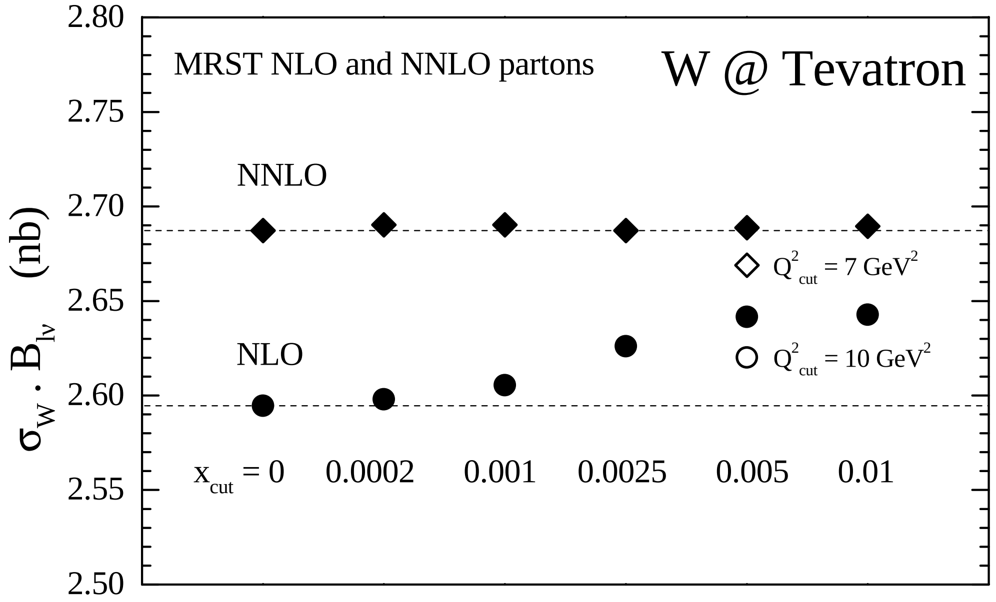




Much more reliable at NNLO. LHC uncertainties $\sim 3-4\%$ including theory uncertainty.
σ_W a good candidate for luminosity determination.

A change in the mass of the vector boson is very similar to a change in centre of mass energy for a fixed mass. Hence the variation with cuts for Z' with mass 1000 GeV is similar to that for W production at the Tevatron rather than the LHC.





CTEQ results

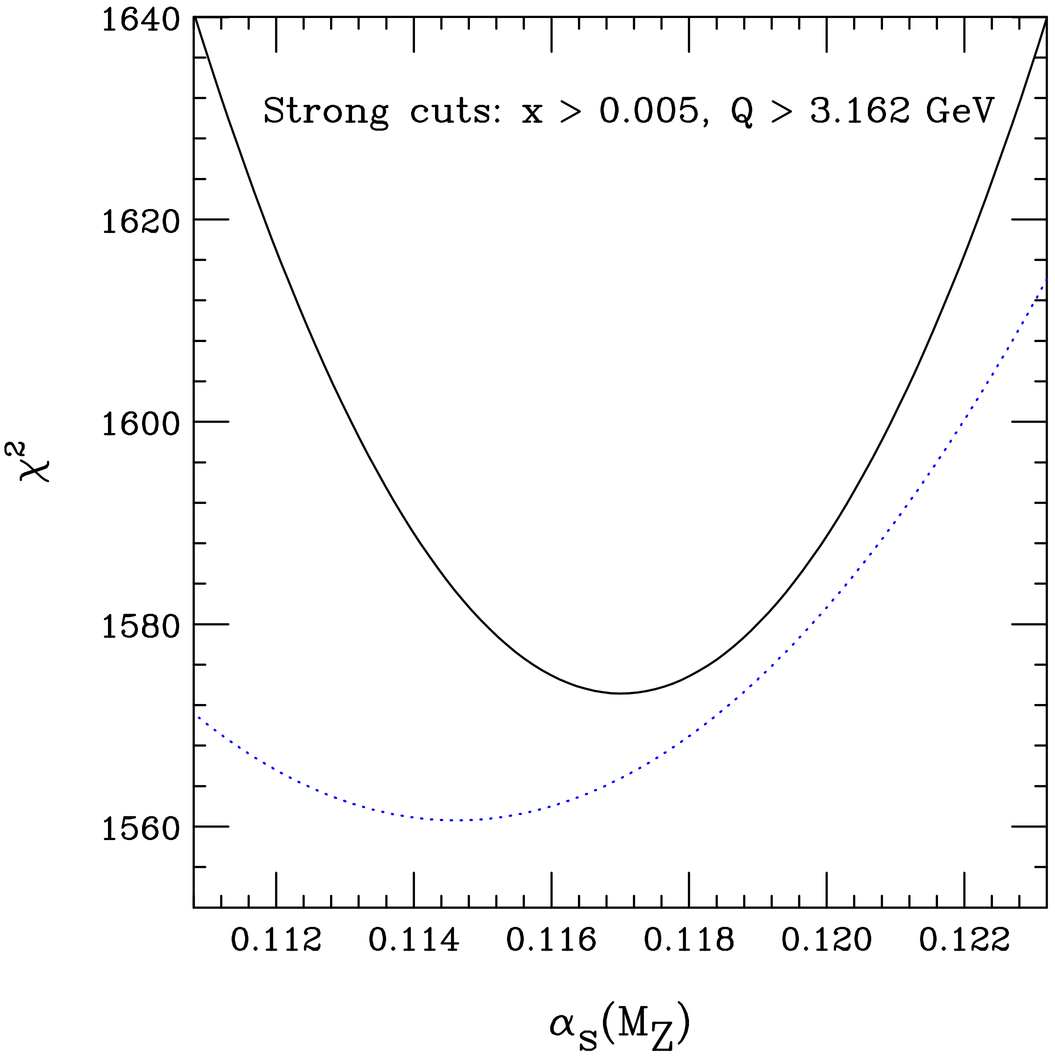
CTEQ see similar type of behaviour with cuts, though not as dramatic.

With conservative cuts on data their input gluon is as keen to have negative component (remember $Q_0^2 = 1.69\mathrm{GeV}^2$), and best value of $\alpha_S(M_Z^2)$ moves lower.

Blue line – negative gluon allowed.

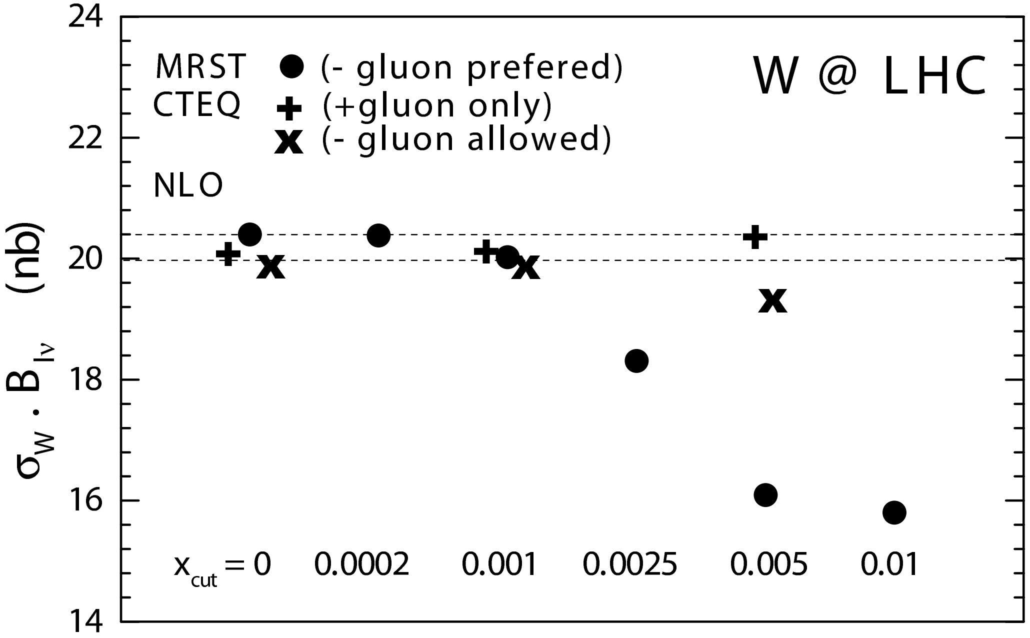
Black line – positive definite gluon.

Verifies negative/small gluon at low x and Q^2 **not** due to data at low x and Q^2.



Prediction stability.

Also find prediction for σ_W at the LHC moves down a little as more cuts imposed. Not as significant as MRST by a long way, it appears.



However, loss of data leads to larger errors, and χ^2 profile is very flat indeed in the downwards direction.

Not really any inconsistency with MRST.

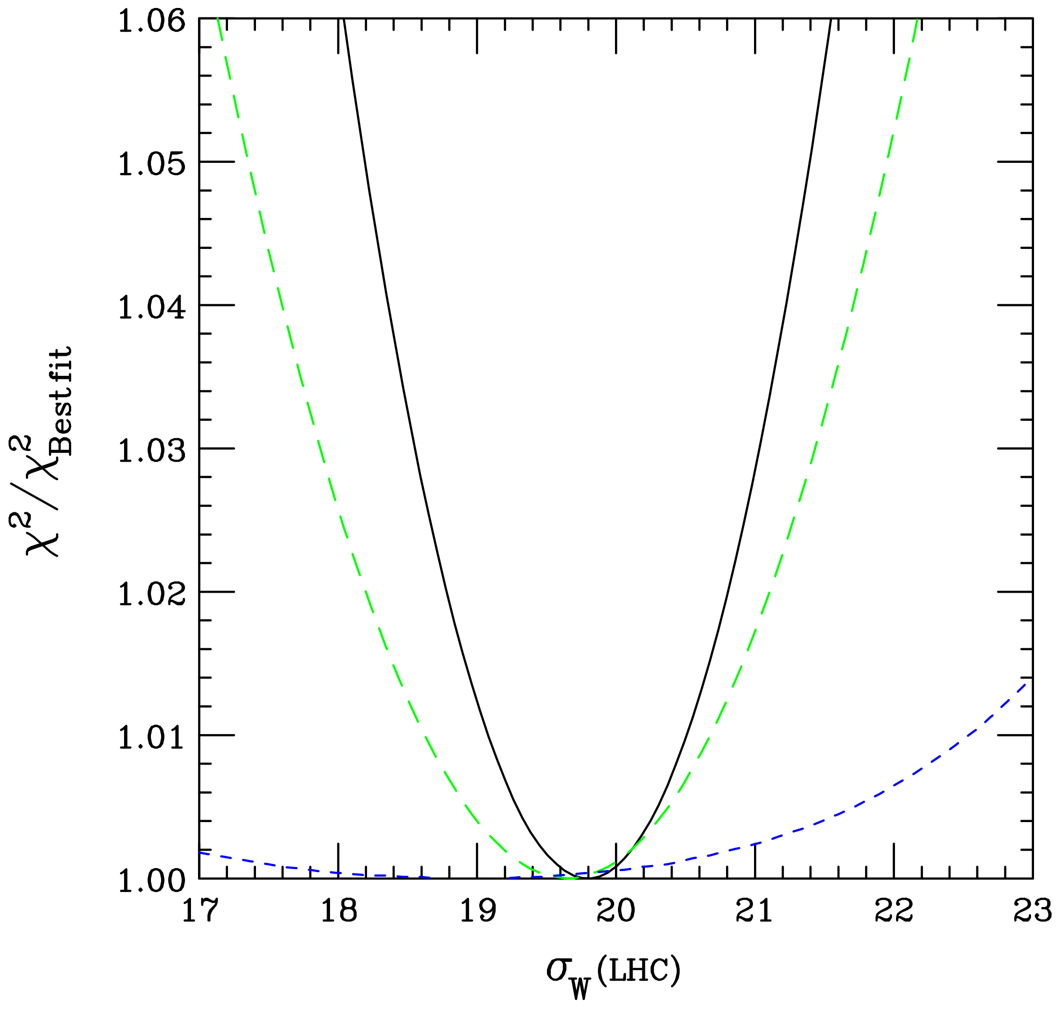
If one is cautious about accuracy of theory at low x and Q^2, conclusion that uncertainty large on small x-sensitive quantities holds. CTEQ claim no reason to be cautious.

blue line - conservative cuts

green line - semi-conservative cuts

black line - normal cuts.

Not so much of an issue at NNLO though.



Conclusions

One can determine the parton distributions and predict cross-sections by performing global fits, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. Uncertainties rather small – $\sim 1-5\%$for most LHC quantities. Ratios often don't reduce uncertainties.

QED corrections small but introduce important isospin asymmetry.

Uncertainty from input assumptions e.g. cuts on data, data used, $etc.$, comparable and potentially larger. Can shift central values of predictions significantly. Assumptions about input form can solve apparent high-E_T jet problem (even with weak corrections).

Errors from higher orders/resummation potentially large. Cutting out low x and/or Q^2 allows improved fit to remaining data, and altered partons. CTEQ see some effects, but much smaller. NNLO much more stable than NLO.

Theory (in general terms) often the dominant source of uncertainty. Much progress – NNLO, resummations .... Lots of work to do on this. Pretty much full NNLO fits imminent. Should beome new standard.

Saturation corrections do not help at NLO or NNLO.

MRST fit with slightly steep input gluon and fairly large shadowing corrections extrapolated to $Q^2 \leq 5\mathrm{GeV}^2$

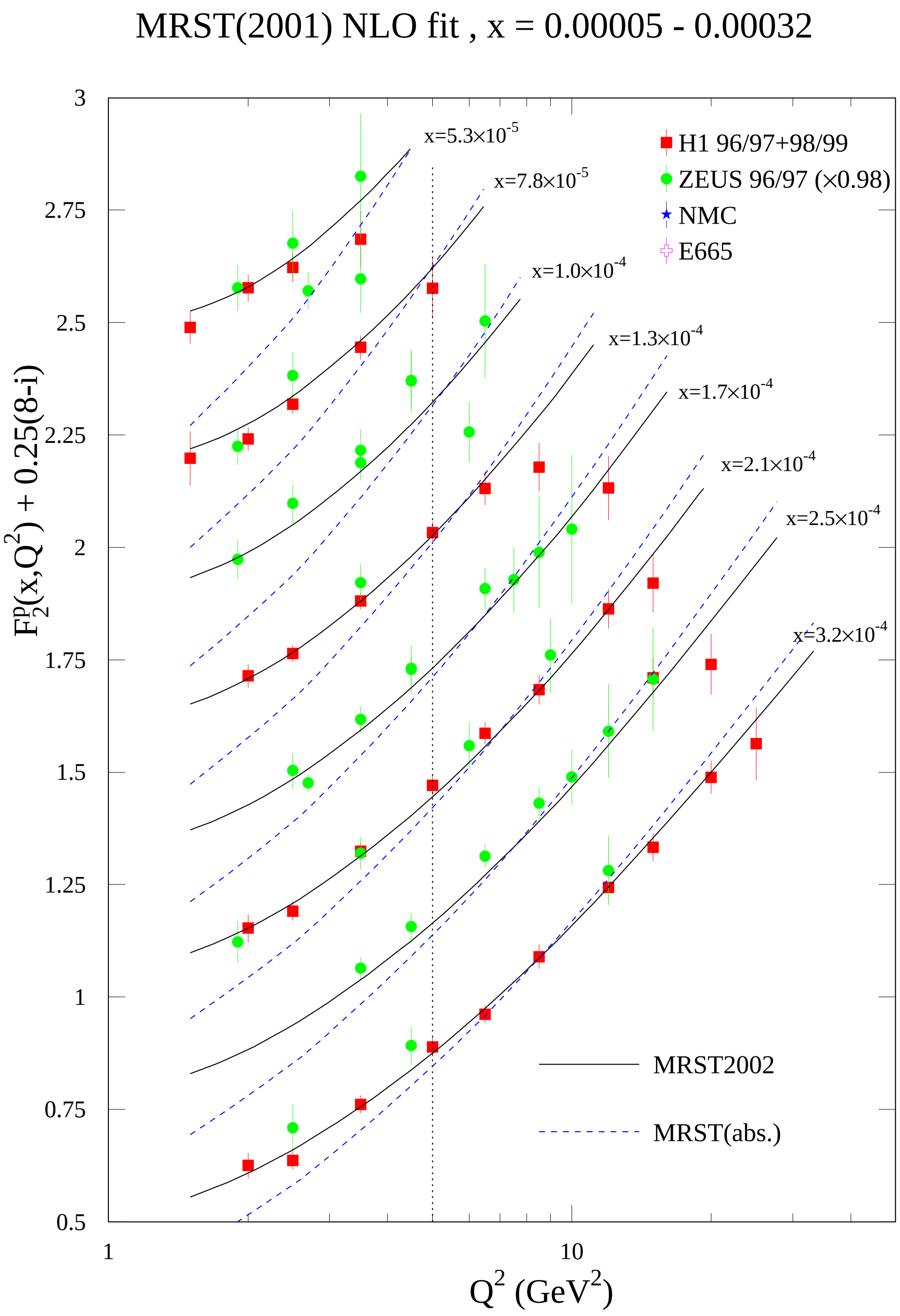
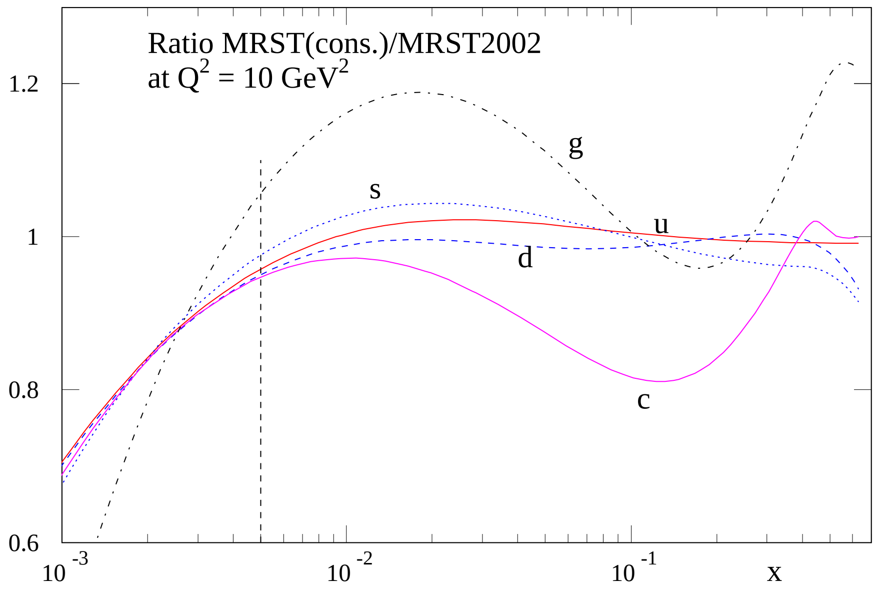


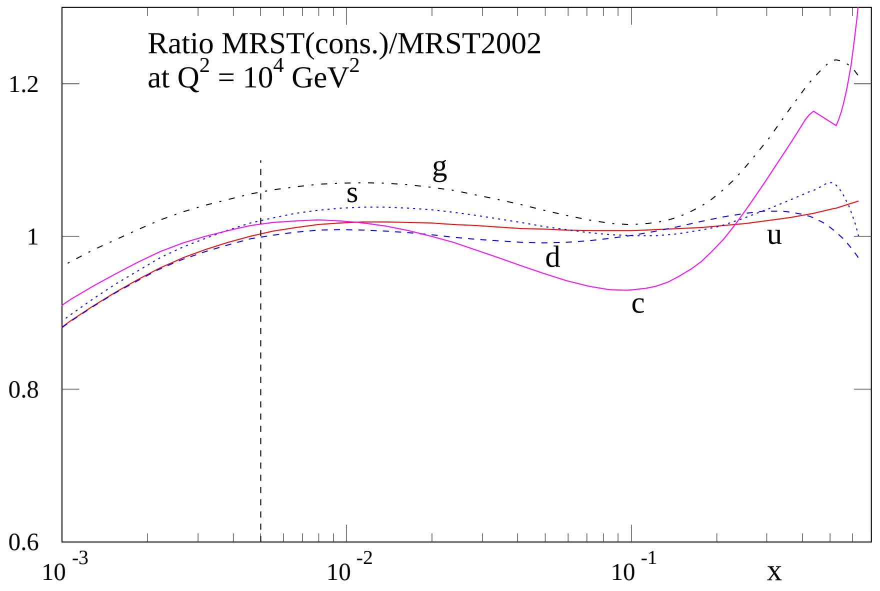
Table 2: Cross sections for Drell-Yan pairs (e^+e^-) with `PYTHIA` 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

PDF set	Comment	xsec
$81 < M < 101$ GeV		
CTEQ5L	PYTHIA internal	1516 ± 5 pb
CTEQ5L	PDFLIB	1536 ± 5 pb
CTEQ6	LHAPDF	1564 ± 5 pb
MRST2001	LHAPDF	1591 ± 5 pb
Fermi2002	LHAPDF	1299 ± 4 pb
$M > 1000$ GeV		
CTEQ5L	PYTHIA internal	6.58 ± 0.02 fb
CTEQ5L	PDFLIB	6.68 ± 0.02 fb
CTEQ6	LHAPDF	6.76 ± 0.02 fb
MRST2001	LHAPDF	7.09 ± 0.02 fb
Fermi2002	LHAPDF	7.94 ± 0.03 fb

Note anti-correlation between deviations at high and low mass, i.e. high and low x. Typical result from sum rules and evolution.

The ratio of the conservative partons to the default partons at NLO. One can see the dip of the conservative partons below $x_{cut} = 0.005$.

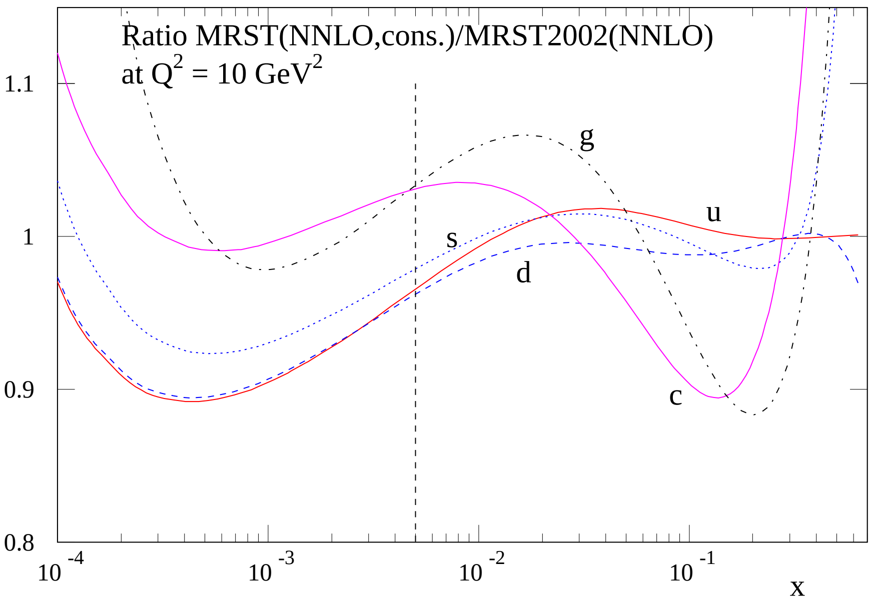


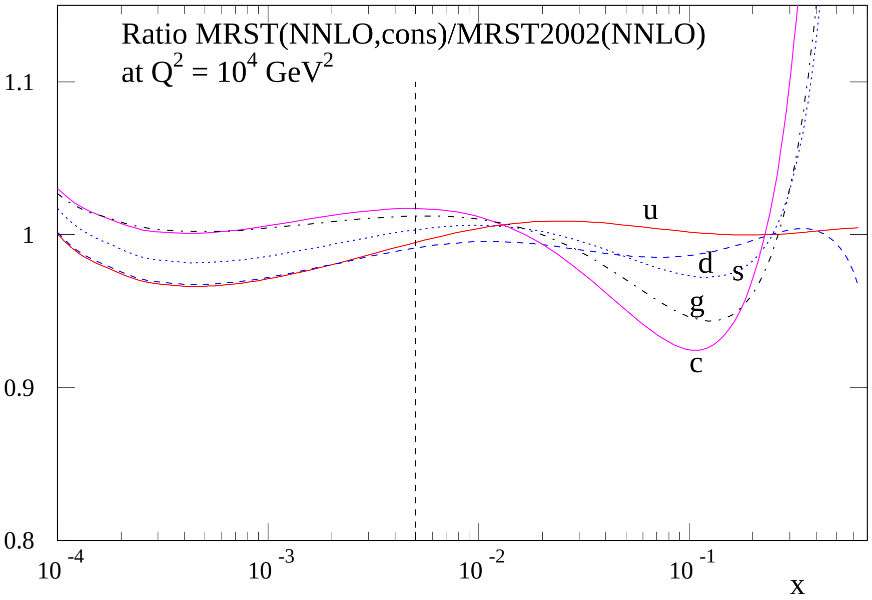


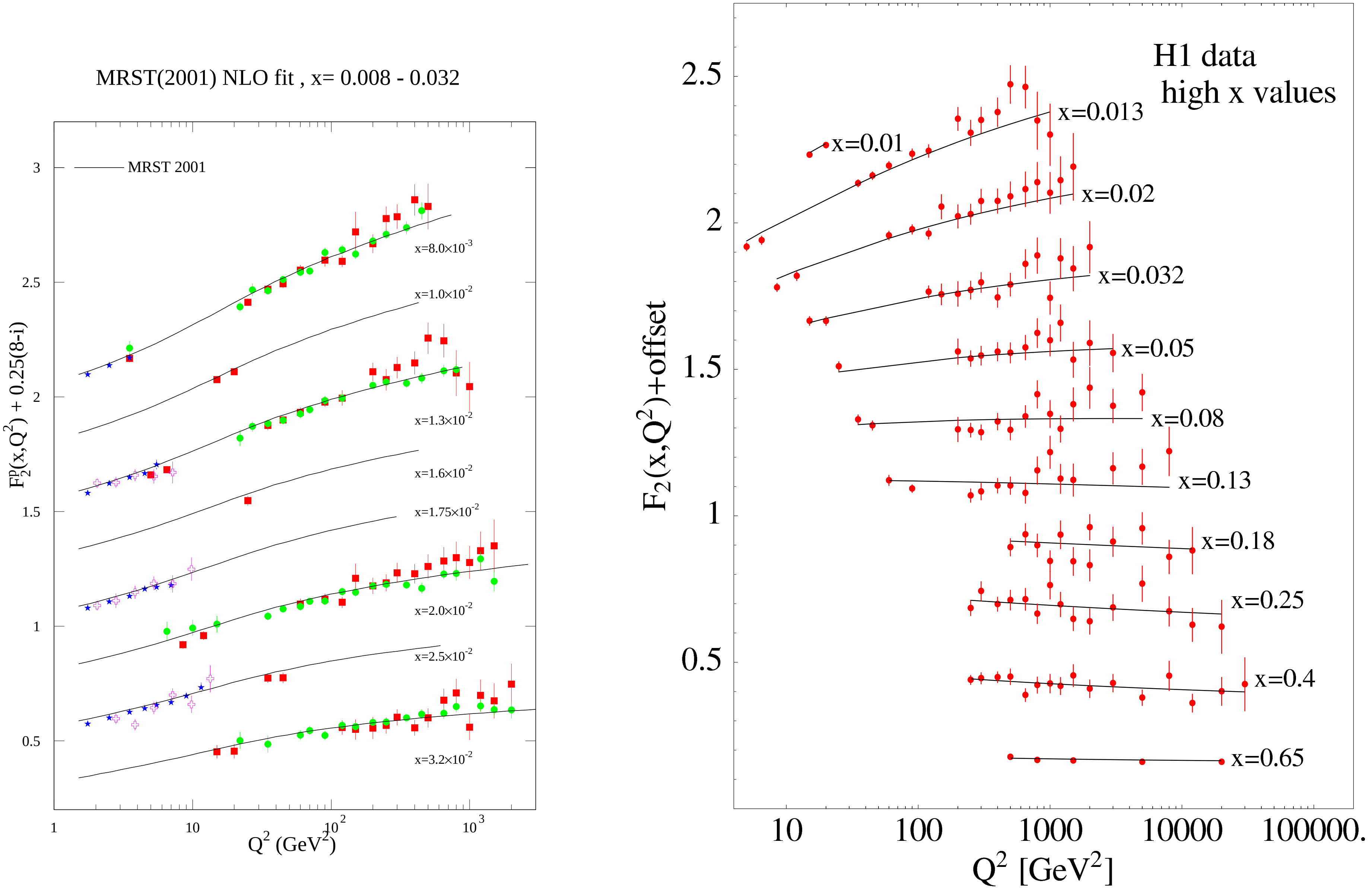
The ratio of the conservative partons to the default partons at NNLO. Now $x_{cut} = 0.005$ and $Q^2_{cut} = 7\mathrm{GeV}^2$. Slight improvement.

$\Delta\chi^2$ still large.

However, now the partons are similar below $x_{cut} = 0.005$. Significant or partially accidental?







Comparison of MRST(2001) $F_2(x,Q^2)$ with HERA, NMC and E665 data (left) and of CTEQ6 $F_2(x,Q^2)$ and H1 data.

Rapidity

Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and the conservative set. The reduction in the total cross-section in the latter case is clearly due to the huge reduction at high y_W and represents the possible type of theoretical uncertainty in this region when working at NLO.

Note a slight increase in cross-section for $y_W = 0$ $(x = 0.006)$. Due to increased evolution of quarks here.