Parton Uncertainties and the Stability of NLO Global Analysis

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Abstract. In global analysis of QCD, both experimental and theoretical errors contribute to the uncertainty of the results. A recent study of the stability of NLO global analysis is described.

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In the global analysis of QCD and parton distribution functions (PDFs), data from many different experiments are combined to extract the PDFs and to test the predictions of perturbative QCD. Both experimental and theoretical errors contribute to the uncertainties of the results. Assessment of the uncertainties of PDFs is a nontrivial issue [1-4]: a standard statistical analysis is not adequate because of systematic errors.

The parton structure of the nucleon is a nonperturbative aspect of QCD. The PDFs $f_i(x,Q)$ are not calculated from theory, but must be extracted from data. The functions are parameterized at a low momentum scale Q_0 of order 1 GeV, using functional forms that have reasonable behavior as $x\to0$ and $x\to1$, and with a set of free parameters $\{a_n\}$ that can be adjusted to fit the full set of data chosen for the global analysis. The dependence of the PDFs on momentum scale Q is assumed to be well-described by the DGLAP evolution equations in next-to-leading order (NLO) perturbation theory. (The NNLO approximation is also used in some cases [5-7].)

GLOBAL ANALYSIS AND UNCERTAINTIES OF PDFs

Data from disparate processes are used in the global analysis of QCD. Deep-inelastic scattering (DIS) gives crucial information on the PDFs. The Drell-Yan process yields complementary information. Measurements of inclusive jet production at the Tevatron collider provide important additional constraints on the gluon distribution. By combining experimental data from these processes, and theoretical calculations of the cross sections, we construct a consistent set of PDFs.

In global analysis, we inevitably face two questions: of compatibility and of stability. Are data sets from different experiments compatible? Are the final results of the global analysis stable and robust? A simple parable illustrates the question of compatibility. Suppose two experimental groups have measured a quantity θ. Each experiment has both statistical and systematic errors. Because of the errors, there is a systematic difference between the experiments. The data sets are consistent, provided that the systematic errors are taken into account. But the combined result must be a compromise, with a large uncertainty associated with the systematic errors. This simple example illustrates what happens in global analysis of QCD. Data from different experiments are only consistent when systematic errors are taken into account. Therefore the PDFs that are extracted from the analysis must be a compromise between experiments with systematic differences. The best fit to one data set will not be the best fit to another data set. The final PDFs must fit all data acceptably. They then have large uncertainties comparable to the systematic differences between experiments.

To assess the uncertainties of the PDFs constructed from a global analysis is a painstaking process. Complete methods have been devised to study the PDF uncertainties [1,2]. These methods are applied to the comparison between perturbative QCD theory and current data, and to predictions for future experiments.

THE CTEQ STABILITY STUDY

In order for the results of a global analysis to be trustworthy, they must be robust. The fitting parameters are adjusted such that the theory matches all chosen data sets acceptably, i.e., within the experimental errors. “Stability” means that small changes in the inputs, e.g., the selected data or the theoretical assumptions, will not produce large changes in the outputs, e.g., the PDF parameters or PDF-dependent predictions.

The stability of next-to-leading-order (NLO) global analysis has been challenged in an interesting study by the MRST group [4]. They imposed cuts on the data selected for the global analysis, requiring $Q>Q_{cut}$ and $x>x_{cut}$, and asked whether the resulting PDFs are stable with respect to small variations of the cutoffs, Q_{cut} or x_{cut}. They found surprisingly large changes in the PDFs, for their parameterization, as the cutoff x_{cut} on x was raised from 0 to 0.005. For example, the central prediction for the cross section σ_W(LHC) (for inclusive production of $W^{\pm}$ at the LHC) decreased by 20 percent from $x_{cut}=0$ (the default MRST PDFs) to $x_{cut}=0.005$ (the “conservative” PDF analysis). Since the default PDF uncertainty on σ_W(LHC) is estimated to be approximately ±5%, the large dependence on x_{cut} raises a question of the stability of the NLO analysis. Is the apparent instability a breakdown of the NLO approximation, or a consequence of increased PDF uncertainty, or an artifact of the parameterization, or due to some other reason?

The CTEQ global analysis group has carried out a study of the stability of the NLO global analysis for the CTEQ parameterization of PDFs, in order to clarify the question of stability [8]. Table 1 shows the results. Three choices of exclusionary cuts are compared: standard and strong (similar to the MRST default and conservative cuts, respectively) and an intermediate case. N_{pts} is the number of data points used in each global analysis, equal to 1926 for the standard cuts and 1588 for the strong cuts. The value of χ^2 for the data that is included, changes very little from standard to strong cuts; χ^2 decreases only from 1583 to 1573 for the 1588 data points that pass the strong cuts. Hence the NLO global analysis is in fact stable with respect to the exclusion of low-x ($x < 0.005$) data. Table 1 also shows the central predictions for the cross section σ_W(LHC) (times the branching ratio B for $W^{\pm}$ to decay to leptons) which changes by only 1.5 percent from standard to strong cuts.

Table 1: Results of the CTEQ stability study

Cuts	Q_{min}	x_{min}	N_{pts}	$\chi^2(1926)$	$\chi^2(1770)$	$\chi^2(1588)$	σ_W B
Standard	2 GeV	0	1926	2023	1850	1583	20.02 nb
Intermed	2.5 GeV	0.001	1770	-	1849	1579	20.10 nb
Strong	3.16 “	0.005	1588	-	-	1573	20.34 nb

Figure 1 shows σ_W.B graphically, from the CTEQ (+) and MRST (•) studies. (Also shown (×) are the CTEQ results for a parameterization in which the gluon distribution is allowed to be negative for small x at $Q=Q_0$.) The NLO prediction of σ_W(LHC) is stable for the CTEQ parameterization.

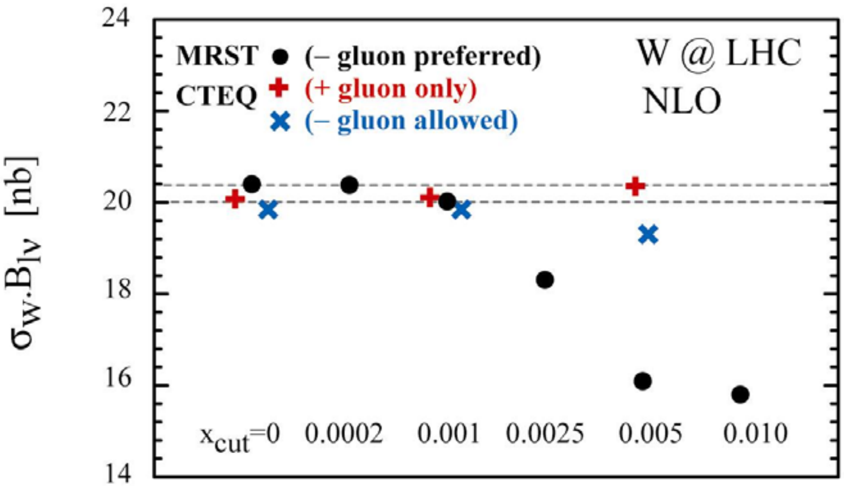

FIGURE 1. The cross section for $W^{\pm}$ production at the LHC, based on global analyses of the PDFs, as a function of the cutoff x_{cut} on x.

LAGRANGE MULTIPLIER METHOD AND THE GLUON PDF

To gain more insight into the stability of the NLO global analysis, we also used the Lagrange Multiplier method (LM) to study the uncertainty of σ_W(LHC) as a function of exclusionary cuts on input data [8]. In general, the LM method calculates the minimum χ^2 as a function of any chosen constrained variable X that depends on the PDFs [1]. We applied the method to the cross section σ_W(LHC) for $W^{\pm}$ production at the LHC, separately for the three choices of exclusionary cuts in Table 1. Thus for each case we obtained the parabola of the best χ^2 versus σ_W(LHC).

The results of the LM analysis are (i) that the position of the absolute minimum of the χ^2 parabola changes very little from standard to strong cuts; but (ii) that the width of the χ^2 parabola increases significantly from standard to strong cuts. In other words, the central prediction, which is the value of σ_W at the absolute minimum of the χ^2

parabola, is stable; that result is already seen in Table 1 and Figure 1. However, the uncertainty of the prediction increases significantly from standard to strong cuts. The latter result makes sense. By excluding 338 data points (those with $x < 0.005$) we have lost a lot of information about the PDFs. Any prediction that is sensitive to the parton structure at small x will have a much larger uncertainty for the strong cuts. From this viewpoint, the stability of the central prediction is not such a significant issue; more important is the growth of uncertainty.

It is easy to see that σ_W(LHC) is sensitive to the gluon distribution function. In the LO parton process $pp \to u\bar{d} \to W^+$, the $\bar{d}$ is a sea quark of the second proton; sea quarks are closely related to the gluon distribution. Or, the NLO process $ug \to dW^+$ depends directly on the gluon distribution. The gluon distribution still has a large uncertainty at present, so the cross section σ_W(LHC) must be uncertain and sensitive to unconstrained assumptions about the form of the gluon PDF—a theoretical uncertainty. The CTEQ and MRST parameterizations for the gluon PDF are quite different [8]. The difference between CTEQ and MRST on the stability of the NLO global analysis, illustrated in Figure 1, must originate in the difference between their parameterizations, i.e., part of the theoretical uncertainty of PDFs.

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REFERENCES

1. D. Stump et al., Phys. Rev. D **65**, 014012 (2002) [arXiv:hep-ph/0101051].
2. J. Pumplin et al., Phys. Rev. D **65**, 014013 (2002) [arXiv:hep-ph/0101032].
3. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **28**, 455 (2003).
4. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **35**, 325 (2004).
5. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B **531**, 216 (2002).
6. S. Alekhin, Phys. Rev. D **68**, 014002 (2003); arXiv:hep-ph/0311184.
7. A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B **691**, 129 (2004).
8. J. Huston, J. Pumplin, D. Stump and W.K. Tung, Stability of NLO Global Analysis and Implications for Hadron Collider Physics, to appear in JHEP [arXiv:hep-ph/0502080].