Tests of NLO BFKL resummation

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- Analysis of BFKL-NLO in Mellin space

Work done in collaboration with Robi Peschanski, Laurent Schoeffel: hep-ph/0411338, accepted by Nucl.Phys.

Motivation

- Effective BFKL LO phenomenology: LO BFKL successful for F_2(proton) for $x \leq 10^{-2}$, but with effective $\alpha_S \sim 0.1$!
- Tests of BFKL NLO: Theoretical problems, Expected NLO improvement fails (without resummation), creating spurious divergences
- Resummed NLO BFKL kernels: Theoretically improved, but practical complexity
- BFKL NLO phenomenology: Direct tests of BFKL NLO resummations using data, 2 aspects: F_2 NLO fits inspired by LO experience, direct tests in Mellin space

"Effective" LO BFKL phenomenology

- Expression of the proton structure functions at LO:

$$(F_T, F_L, G) = \int \frac{d\gamma}{2i\pi} \left(\frac{Q^2}{Q_0^2}\right)^{\gamma} e^{\frac{\alpha_S N_C}{\pi}\chi(\gamma)Y} (h_T(\gamma), h_L(\gamma), 1)\, \eta(\gamma)$$

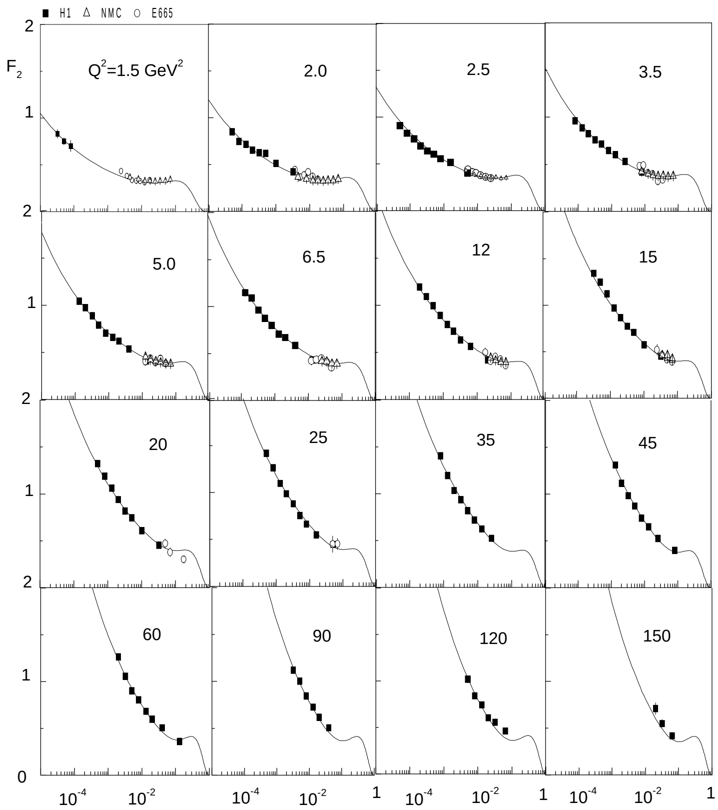
- Impact factors:
  - Coupling to γ: $h(\gamma)$ perturbative QCD,
  - Coupling to proton: $\eta(\gamma)$, non perturbative, leads to unknown normalisation after saddle point approximation, to be determined by fitting the data
  - $\chi_{LO}(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)$

“Effective” LO BFKL phenomenology (cont.)

- Simple saddle point approximation:

$$F_2 = C e^{\alpha_S \chi(\frac{1}{2}) Y} \frac{Q}{Q_0} e^{-\frac{log^2(Q/Q_0}{2\alpha_S \chi''(1/2) Y}}$$

- 3-parameter fit to proton F_2 (α_S, Q_0, C) $\chi^2/dof \sim 1$)



"Effective" LO BFKL phenomenology (cont.)

- H. Navelet, R. Peschanski, C. Royon, S. Wallon, 1994, A.I.Lengyel, M.U.T. Machado, hep-ph 0304195: nice description of F_2 data using LO BFKL
- Low value of α in fit: $\alpha = 0.08$ instead of $\alpha_S = 0.2$ in the Q^2 HERA range
- What about NLO? we know that higher order BFKL corrections are important, what is the impact?
- Understand why LO BFKL works so well despite of high corrections to BFKL equations

“Effective” NLO BFKL phenomenology

- Idea: perform the same saddle point approximation as at LO using χ_{NLO} given by BFKL NLO

- Saddle point approximation
$$F_2 = C e^{\alpha_{RGE}\chi_{eff}(\gamma_c,\alpha_{RGE})Y} \left(Q^2/Q_0^2\right)^{\gamma_c}$$
$$e^{-\frac{log^2(Q/Q_0)}{2\alpha_{RGE}\chi''_{eff}(\gamma_c,\alpha_{RGE})Y}}$$

  γ_C and χ_{eff}: (properties of BFKL NLO if small-x structure function is dominated by the perturbative Green function) (ω is the Mellin transform of Y)
$$\frac{d\chi_{eff}}{d\gamma}(\gamma_C, \alpha_{RGE}(Q^2)) = 0$$
$$\chi_{eff}(\gamma, \alpha_{RGE}) = \omega(\gamma, \alpha_{RGE})/\alpha_{RGE}$$

- 2 parameters fit: C and Q_0, (α given by RGE at NLO)

- Difficulties: χ complicate formula at NLO and scheme dependent

Strategy for NLO fits

- First step: Knowledge of $\chi_{NLO}(\gamma, \omega, \alpha)$ from BFKL equation and resummation schemes
- Second step: Use implicit equation $\chi(\gamma, \omega) = \omega/\alpha$ to compute numerically ω as a function of γ for different schemes and values of α
- Third step: Numerical determination of saddle point values γ_C as a function of α as well as the values of χ and χ''
- Fourth step: Perform the BFKL-NLO fit to HERA F_2 data with two free parameters C and Q_0^2

Remarks on resummation schemes

- NLO BFKL kernels need resummation: to remove additional spurious singularities in γ and $(1-\gamma)$
- NLO BFKL kernel:

$$\chi_{NLO}(\gamma,\omega) = \chi^{(0)}(\gamma,\omega) + \alpha(\chi_1(\gamma) - \chi_1^{(0)}(\gamma))$$

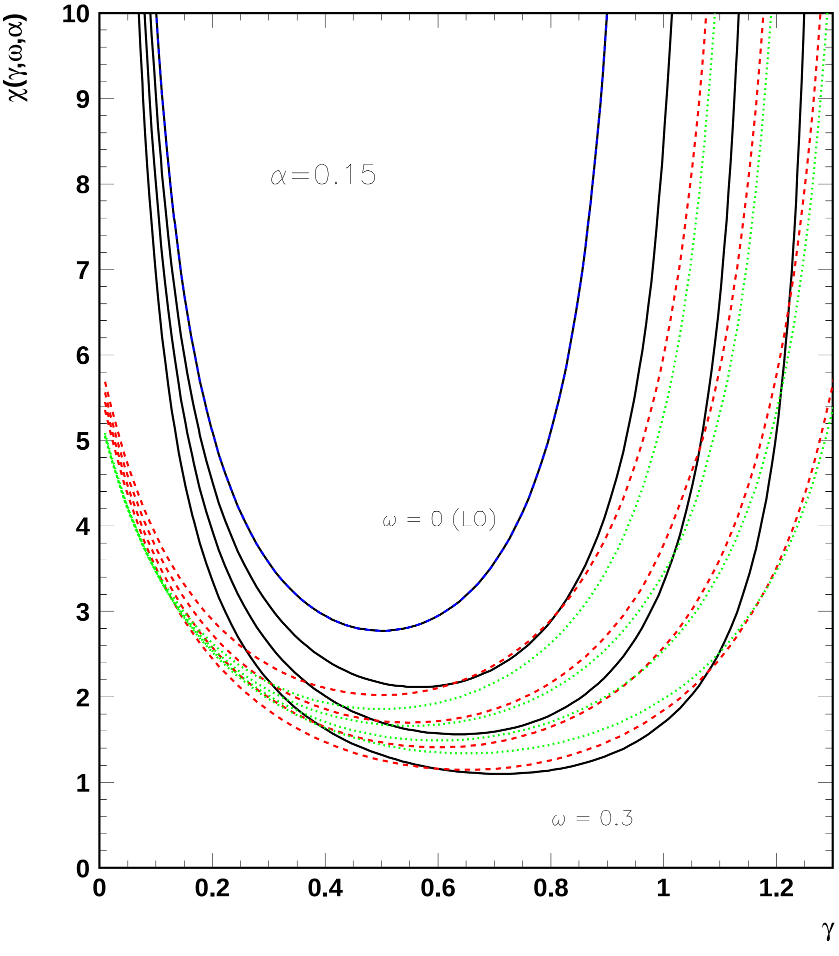
- $\chi_1(\gamma)$: calculated, NLO BFKL eigenvalues (Lipatov, Fadin, Camici, Ciafaloni)
- $\chi^{(0)}$ and $\chi_1(0)$: ambiguity of resummation at higher order than NLO, different ways to remove these singularities, not imposed by BFKL equation, Salam, Ciafaloni, Colferai
- Transformation of the energy scale: $\gamma \rightarrow \gamma - \omega/2$, Salam

Remarks on resummation schemes (Cont.)

- Different schemes tested in the following:
  - Schemes 1 and 2 (Salam): $\chi^{(0)}$ not changed changes in $\chi_1(0)$ only, does not work...
  - Scheme 3 (Salam): $\chi^{(0)}(\gamma,\omega) = (1-\alpha A)(2\Psi(1) - \Psi(\gamma + \frac{\omega}{2} + \alpha B) - \Psi(1-\gamma+\frac{\omega}{2}+\alpha B)$ and A and B are determined so that $\chi_1(\gamma) - \chi_1^{(0)}(\gamma)$ is regular when $\gamma \leftarrow 0$
  - Scheme 4 (Salam): other form of $\chi^{(0)}$
  - Scheme CCS (Ciafaloni et al., hep-ph 0307188)

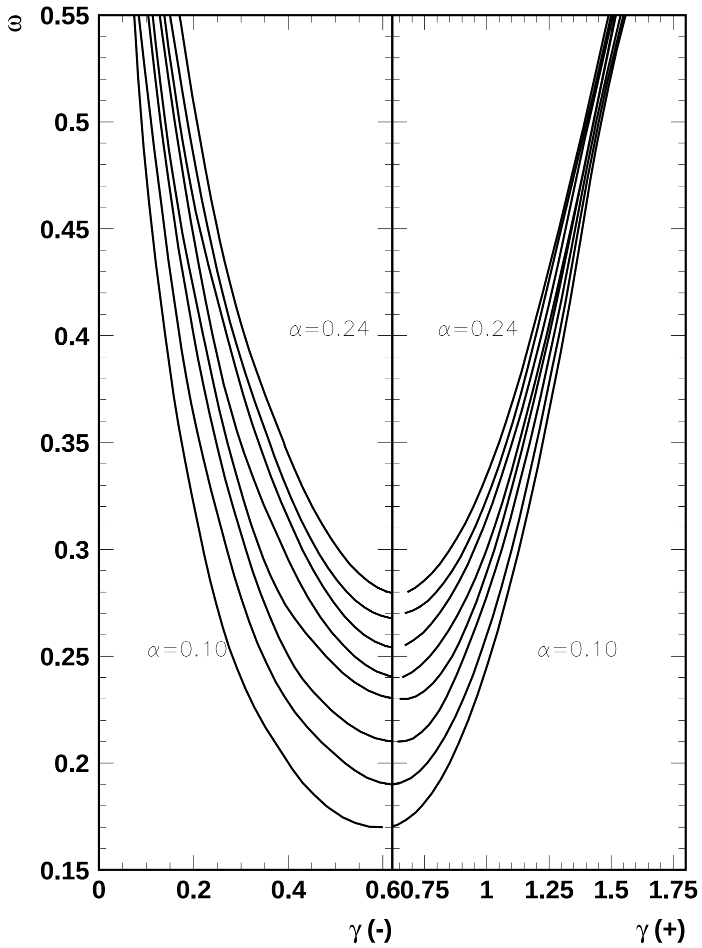
χ_{NLO} for different schemes

- $\chi_{NLO}(\gamma, \omega, \alpha)$ for different resummation schemes (CCS, S3 and S4)
- As an example $\alpha = 0.15$ and four values of ω: betweeb 0 and 0.3



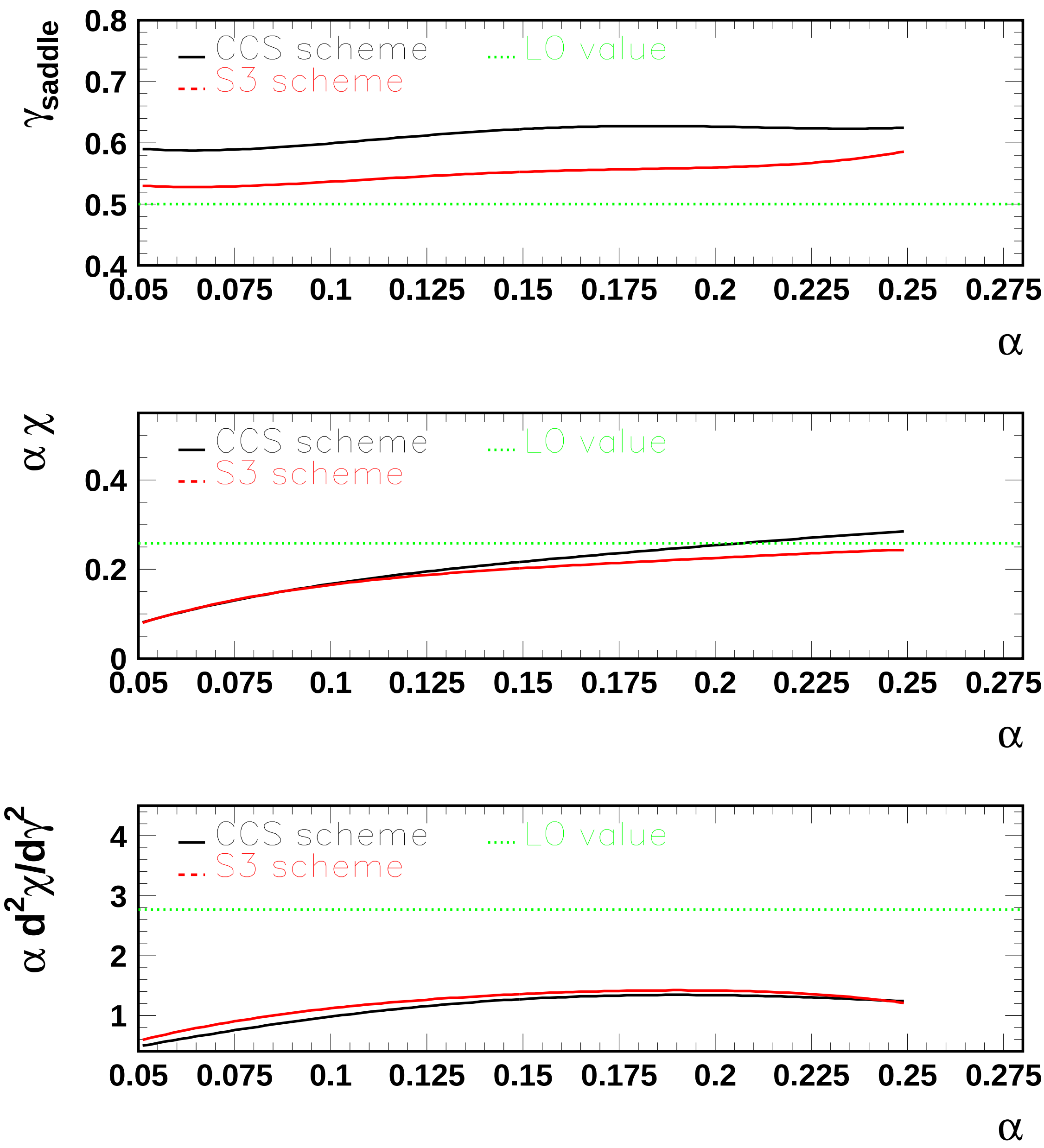
ω as a function of γ

- ω as a function of γ for one scheme (CCS) as an example, solution of $\chi(\gamma, \omega) = \omega/\alpha$
- α varies between 0.1 and 0.22 in steps of 0.02



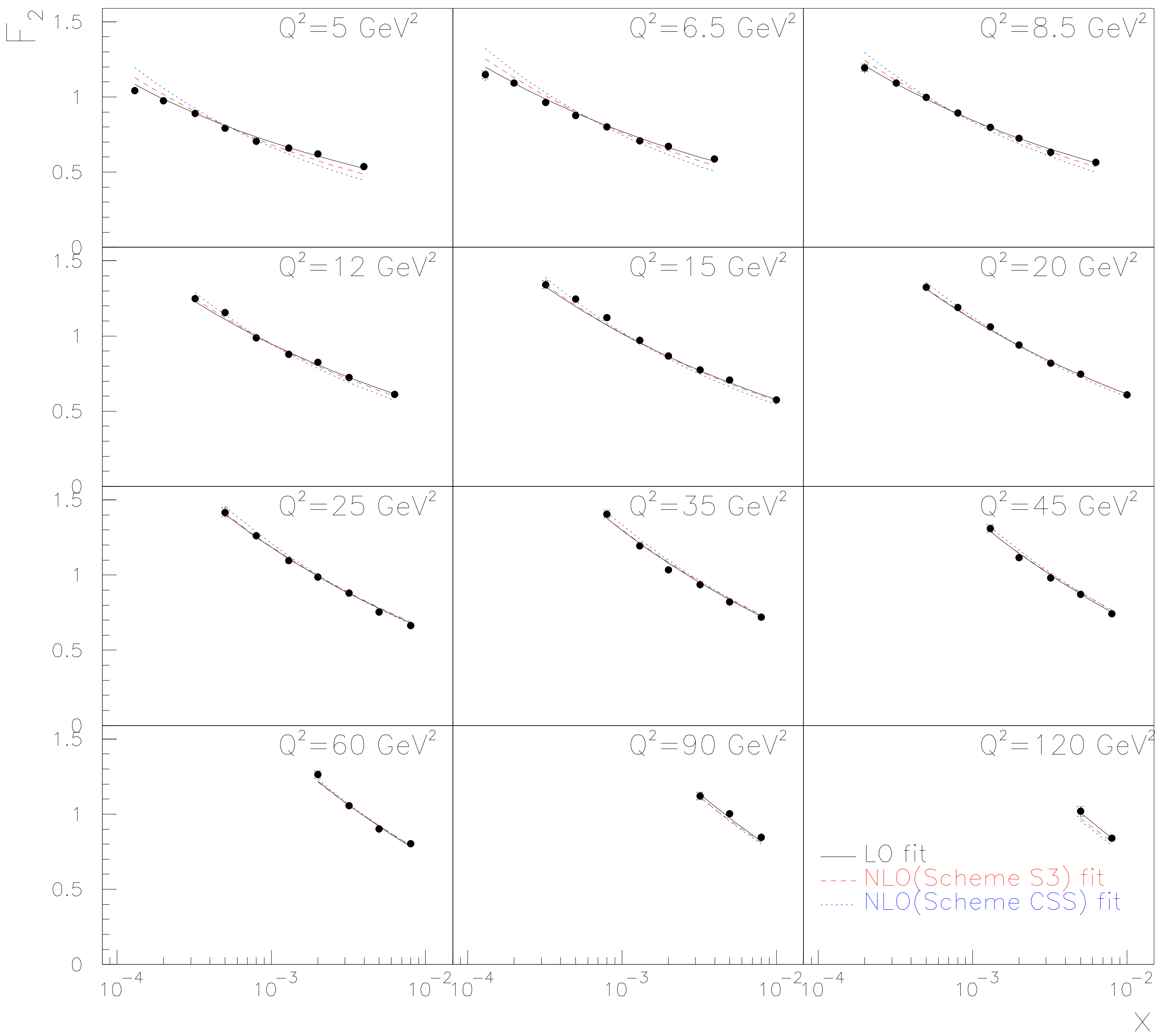
Determination of saddle point γ_C

Using saddle point equation, determination of γ_C to perform fits to F_2 (working for $\alpha = 0.2$)



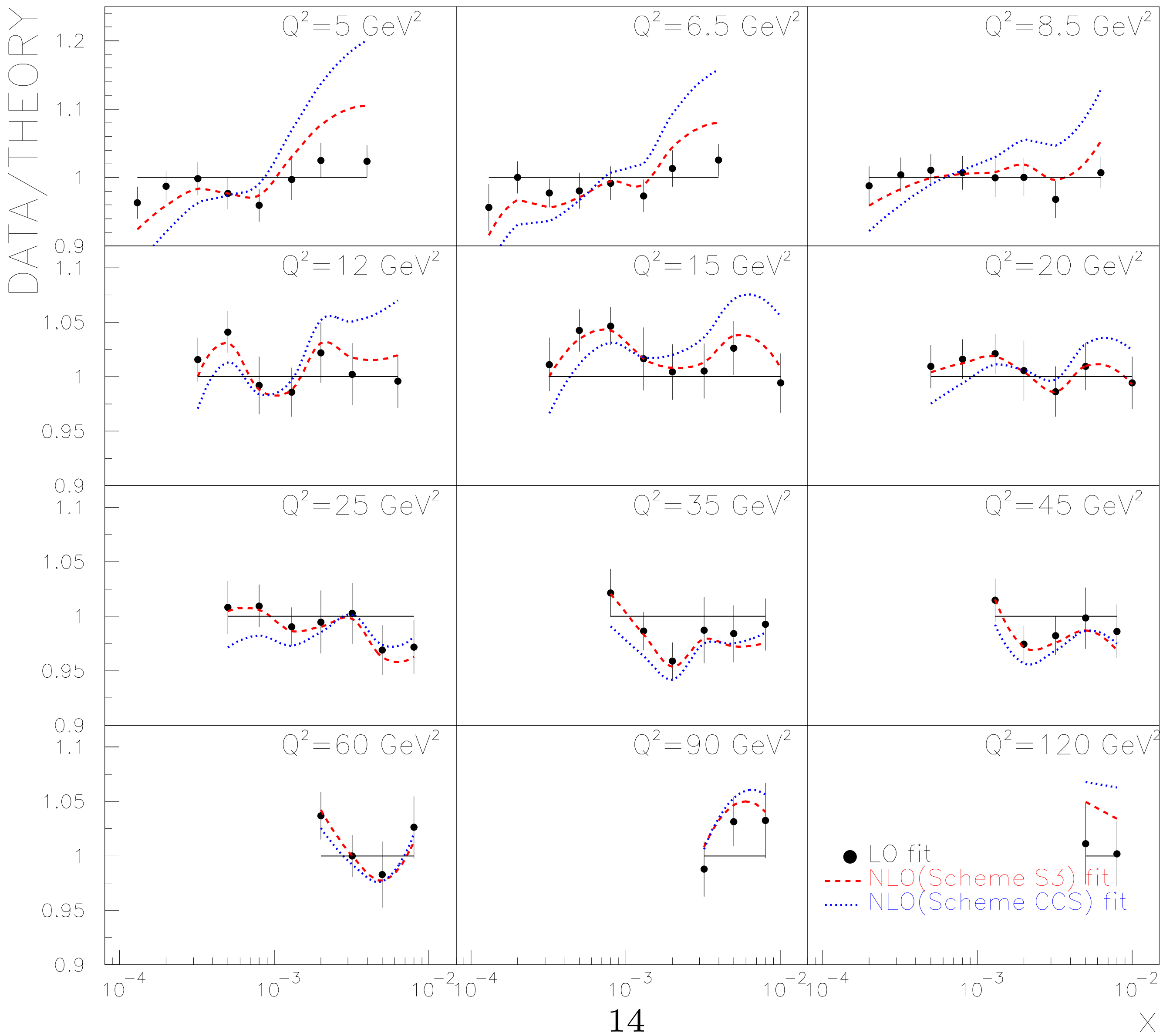
BFKL NLO fit to H1 data

Result of the NLO BFKL fit shown for CSS and S3 schemes: disagreement at low Q^2



Data / theory

- Data / theory: points: LO fit, curves: NLO fit for S3 and CSS
- Why such a difference: analysis in Mellin space

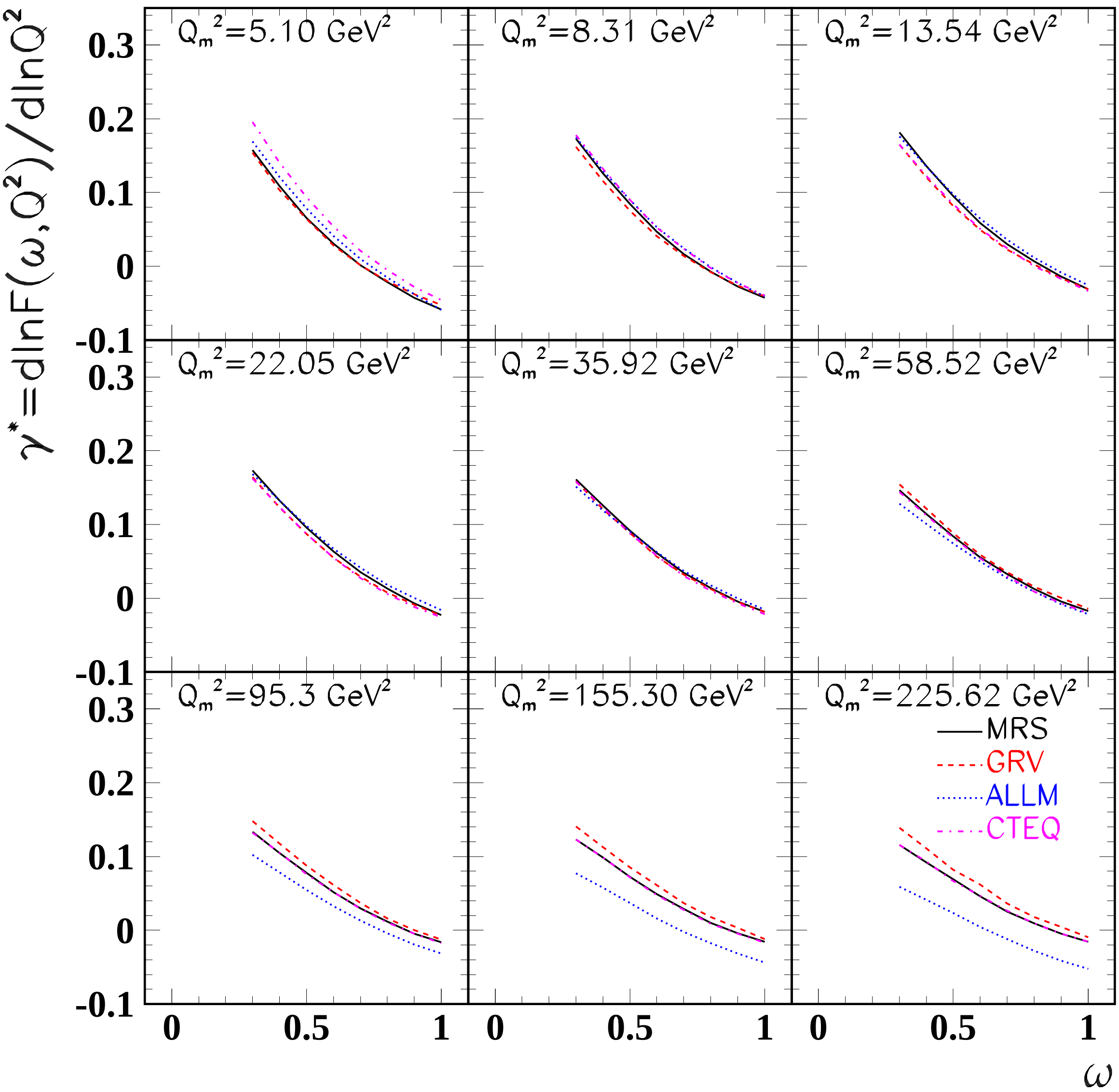


Determination of γ^*, γ_C

- In Mellin space, γ^* defined as $d\log F_2(\omega, Q^2)/d\log Q^2$ is the saddle point of the structure function
- It is possible to check that γ^* verifies the property $\chi(\gamma^*(\omega, Q^2), \alpha_{RGE}(Q^2)) = \omega/\alpha_{RGE}(Q^2)$
- Idea: Use different parametrisations of F_2, and perform a Mellin transform of these parametrisations
- 3 parametrisations based on DGLAP evolution equation: Martin, Roberts, Stirling (MRS 2001), Glück, Reya, Vogt (GRV 98), CTEQ (CTEQ 6.1)
- 1 additional parametrisation based on a Regge analysis of proton structure function data: ALLM

Determination of γ^*

$$\gamma^* = \frac{dlogF_2(\omega, Q^2)}{dlogQ^2}$$



Phenomenological test of LO and NLO BFKL

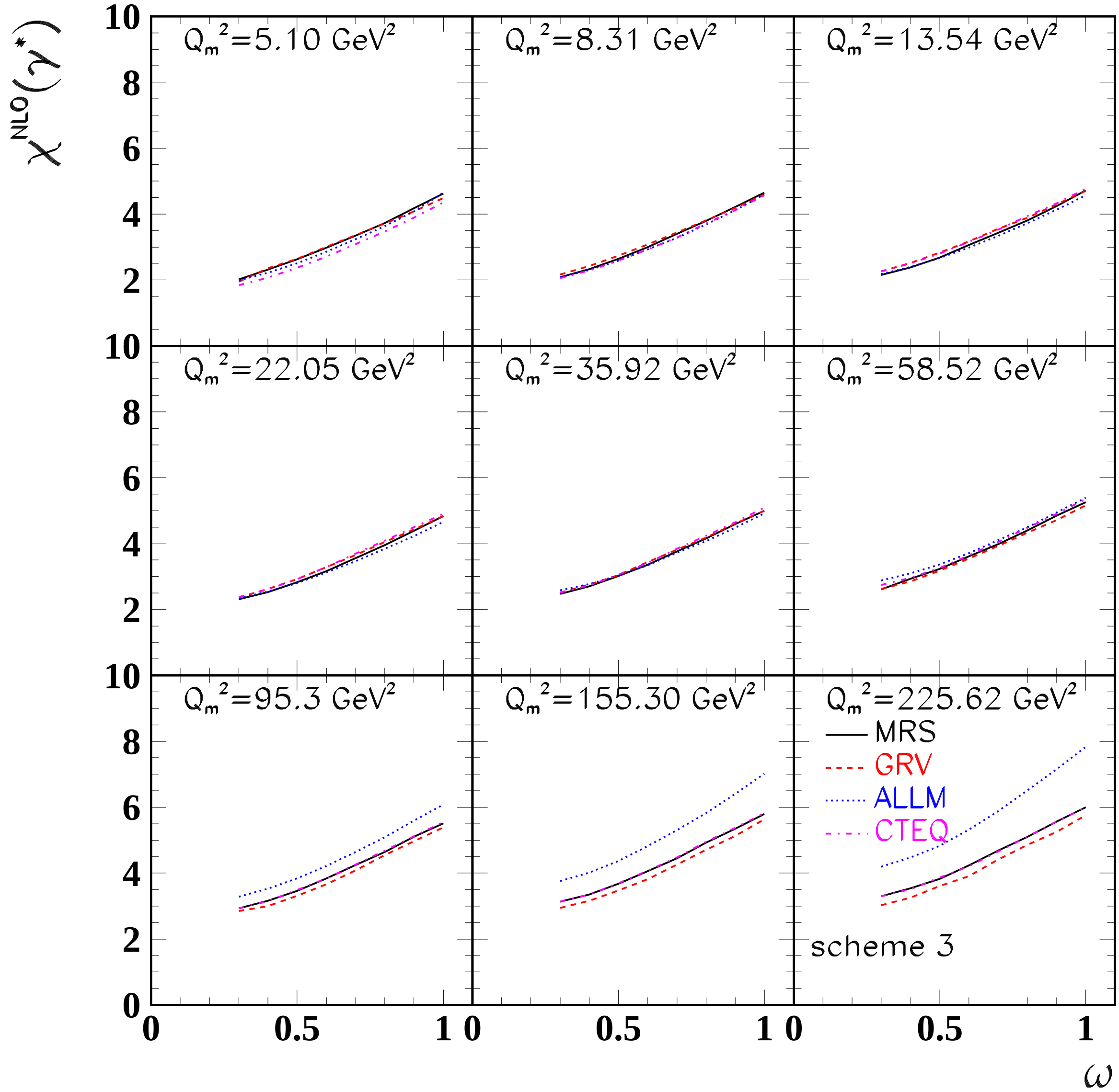
$$\frac{\omega}{\alpha_S} = \chi(\gamma^*(\omega), \omega, Q^2)$$

- γ^* determined from the parametrisation in different Q^2 and ω bins
- Compute $\chi(\gamma^*(\omega), \omega, Q^2)$ at LO and NLO
- Perform a linear fit of $\chi(\gamma^*(\omega), \omega, Q^2)$ as a function of ω in the different Q^2 bins, and check properties

$\chi(\gamma^*)$ at NLO - scheme 3

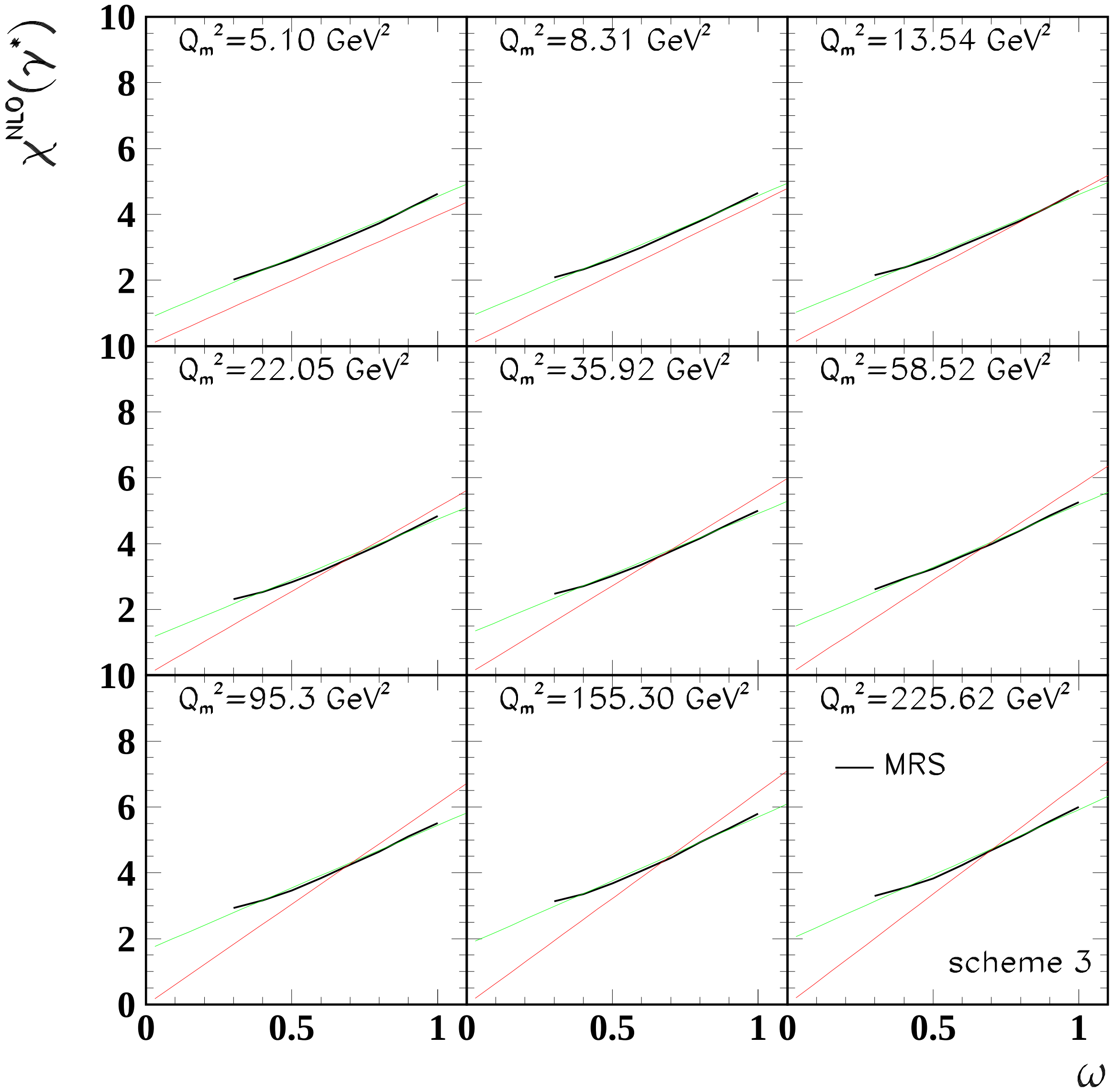
Consistency check:

$$\frac{\omega}{\alpha_S} = \chi(\gamma^*(\omega), \omega, Q^2)$$



$\chi(\gamma^*)$ at NLO - scheme 3

- black: MRS, green: linear fit to MRS, red: consistency check
- Consistency check fails!



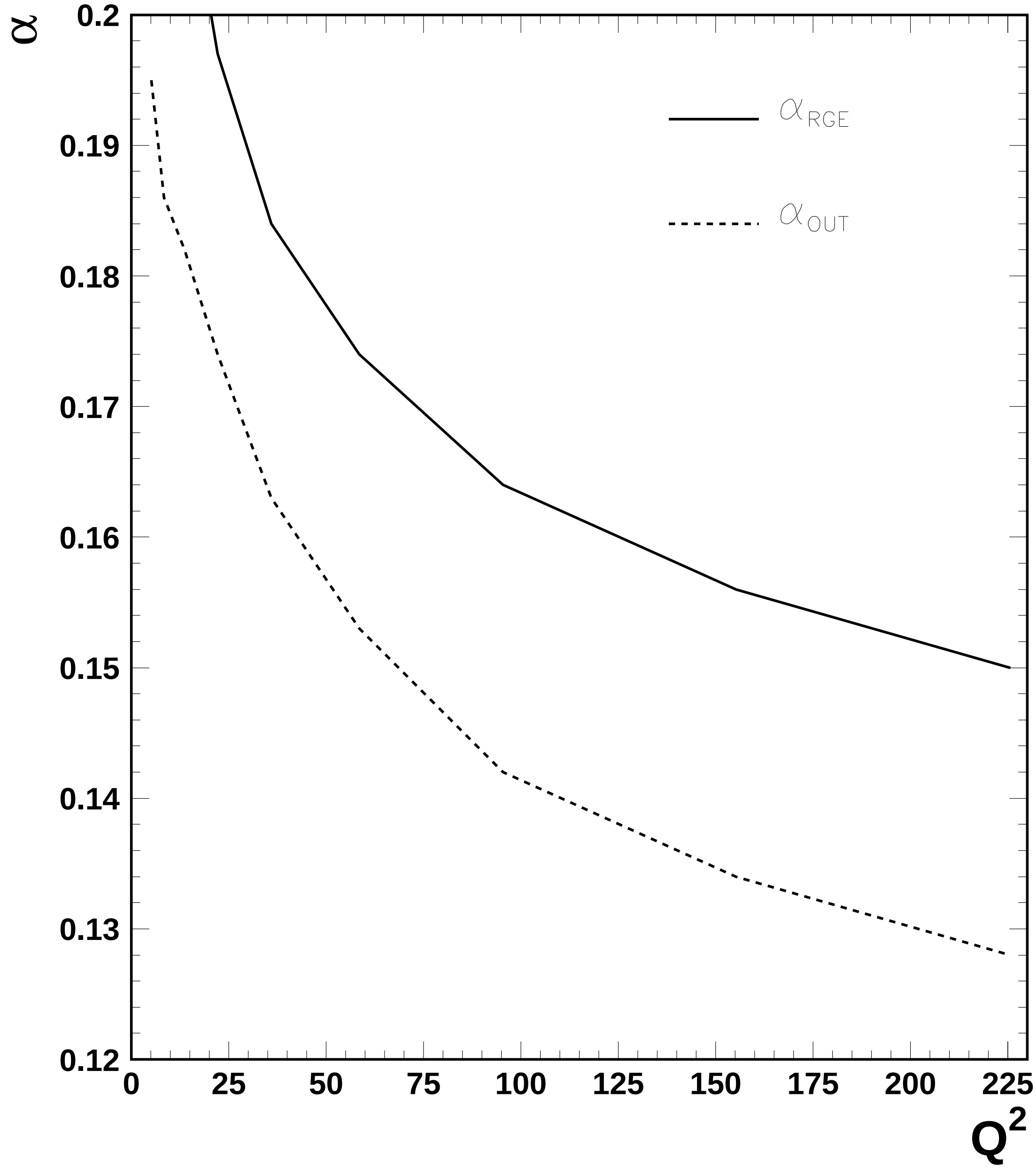
Test of BFKL NLO - Conclusion

$$\frac{\omega}{\alpha_S} = \chi(\gamma^*(\omega), \omega, Q^2)$$

- Not verified for schemes 1 and 2
- Schemes 3 and 4: We find a good linear fit with two caveats:
  - does not go through the origin in (χ, ω) plane
  - the slope is not $\alpha_S(RGE)$: We compare the slope of the linear fit to the value of α_S used in χ. (see next slide)
- Conclusion: Consistency check not successful Sensitivity to NNLO effects? γq coupling (impact factors) not known for BFKL NLO?, saddle point approximation not valid?

α_{out} vs $\alpha_S(RGE)$ - scheme 4

upper curve: $\alpha_S(RGE)$, lower curve: α coming from the linear fit



Conclusion

- LO BFKL fits successful with a low value for α
- Phenomenological studies of NLO BFKL: F_2 fit with saddle point approximation not as good as a t LO $\longrightarrow$ perform study in Mellin space to test BFKL properties
- Test of the property

$$\frac{\omega}{\alpha_S} = \chi(\gamma^*(\omega), \omega, Q^2)$$

- Property partially verified for resummation schemes 3 and 4 (Salam) and CCS (linearity, but does not go through the origin, and slope not equal to $\alpha_S(RGE)$
- Identify the reasons? NNLO effects? impact factors? saddle point approximation?