

# Understanding Parton Distributions from Lattice QCD

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**Abstract.** I examine the past lattice QCD calculations of three representative observables, the transverse quark distribution, momentum fraction, and axial charge, and emphasize the prospects for not only quantitative comparison with experiment but also qualitative understanding of QCD.

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## INTRODUCTION

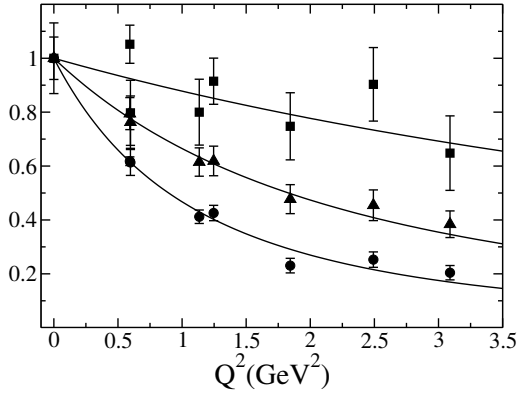
Lattice QCD calculations provide the opportunity for both quantitative comparison with experimental measurements and for advancing our qualitative understanding of QCD. I examine several observables which exemplify this range of opportunities. The lattice calculation of the moments of parton distributions is an essential step in achieving quantitative agreement with experimental results. Additionally our understanding of QCD can be further expanded by calculating the remaining moments of the generalized parton distributions which determine the three dimensional distribution of the transverse position and longitudinal momentum of quarks and gluons within the nucleon.

*Generalized Form Factors.* The generalized form factors provide an alternative but equivalent language to the generalized parton distributions. They encode the ordinary form factors and parton distributions as well as the nucleon spin decomposition [1] and transverse quark distributions [2] and hence provide a unifying language to describe calculations of nucleon structure. Each tower of twist two operators has a corresponding set of generalized form factors. As an example the unpolarized operators  $O_q^{\mu_1 \dots \mu_n} = \bar{q} i D^{(\mu_1} \dots i D^{\mu_{n-1}} \gamma^{\mu_n)} q$  define the generalized form factors  $A_{ni}^q$ ,  $B_{ni}^q$ , and  $C_n^q$  via

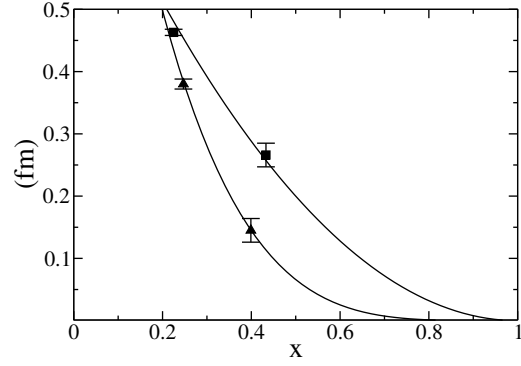
$$\langle P' | O_q^{\mu_1 \dots \mu_n} | P \rangle = \bar{U}(P') \left[ \sum_{i=0, \text{even}}^{n-1} \left( A_{ni}^q(t) K_{ni}^A + B_{ni}^q(t) K_{ni}^B \right) + \delta_{\text{even}}^n C_n^q(t) K_n^C \right] U(P) \quad (1)$$

where  $K$  are known functions of  $P$  and  $P'$ . A complete set of results can be found in [3].

*Lattice Calculations.* There have been several full QCD calculations of nucleon structure to date. Results from these and other calculations [4-14] were shown at the conference. Each calculation uses different actions as well as differing lattice spacings and volumes. However the dominate systematic error in lattice calculations of nucleon observables is still due to the chiral extrapolation.



**FIGURE 1.** squares are  $A_{30}^{u-d}$ , triangles are  $A_{20}^{u-d}$ , circles are  $A_{10}^{u-d}$  [4]



**FIGURE 2.** squares are  $\langle b_{\perp}^2 \rangle_{(n)}^{u+d}$ , triangles are  $\langle b_{\perp}^2 \rangle_{(n)}^{u-d}$  for  $n = 1, 3$  [4, 6]

## TRANSVERSE QUARK DISTRIBUTIONS

The transverse quark distribution  $q(x, \vec{b}_{\perp})$  gives the probability to find a quark of flavor  $q$  carrying a fraction  $x$  of the nucleon's longitudinal momentum at a displacement  $\vec{b}_{\perp}$  from the center of the nucleon. The moments of the transverse quark distribution are given by

$$q_n(\vec{b}_{\perp}) = \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\vec{b}_{\perp} \cdot \vec{\Delta}_{\perp}} A_{n0}^q(-\vec{\Delta}_{\perp}^2). \quad (2)$$

Current calculations are restricted to the lowest three moments, however the following shows several ways to examine the transverse structure using just the low moments.

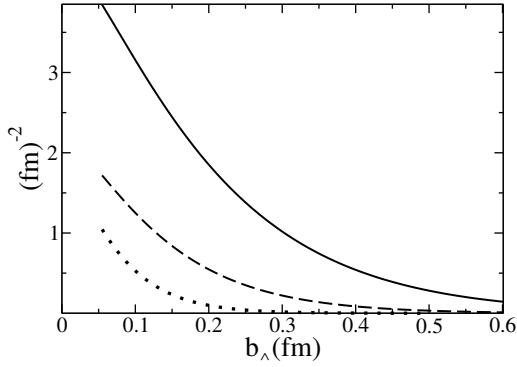
*$Q^2$  Dependence.* The slope of the generalized form factors in the forward limit is of particular interest because it determines the rms radius of the  $n^{\text{th}}$  moment of  $q(x, \vec{b}_{\perp})$ ,

$$\langle b_{\perp}^2 \rangle_n = \frac{\int d^2 b_{\perp} b_{\perp}^2 \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_{\perp})} = \frac{-4}{A_{n0}^q(0)} \frac{dA_{n0}^q(0)}{dQ^2}. \quad (3)$$

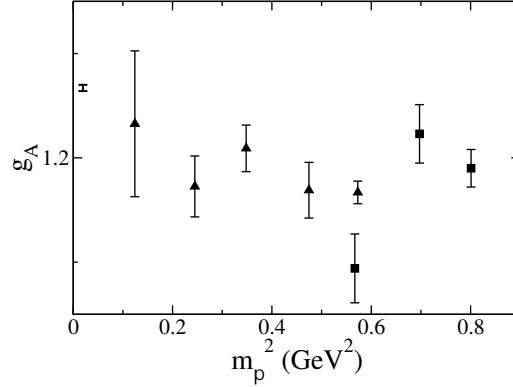
The moments in Eq. 3 are dominated by  $x$  near 1 for large  $n$ . In such a limit the quark carries all the longitudinal momentum and is kinematically constrained to reside at the center of the nucleon [4]. Thus higher moments determine the transverse size of larger  $x$  quarks which are distributed more narrowly in  $b_{\perp}$ . Consequently the qualitative expectation, illustrated in Fig. 1, is that the slopes should decrease as  $n$  increases for large enough  $n$ . That an expectation for large  $n$  is so clearly seen for the lowest three moments demonstrates that the transverse distribution of quarks within the nucleon depends strongly on the momentum fraction at which the quarks are probed.

*$x$  Dependence.* The transverse rms radius of the nucleon at a fixed  $x$  is

$$\langle b_{\perp}^2 \rangle_x = \frac{\int d^2 b_{\perp} b_{\perp}^2 q(x, \vec{b}_{\perp})}{\int d^2 b_{\perp} q(x, \vec{b}_{\perp})},$$



**FIGURE 3.** solid, dashed, dotted are  $q_n(\vec{b}_\perp)$  for  $n = 1, 2, 3$ ,  $q = u - d$  [6]



**FIGURE 4.**  $g_A$ , triangles [12], squares [15]

whereas lattice calculations determine the transverse radius at a fixed moment  $n$  as shown in Eq. 3. To understand the meaning of the transverse radius at a fixed moment we can think of  $\langle b_\perp^2 \rangle_n$  as a coarse grained transverse size of the nucleon corresponding to a region centered on the average  $x$  of the  $n^{\text{th}}$  moment,

$$\langle x \rangle_n = \frac{\int d^2 b_\perp \int_{-1}^1 dx |x| x^{n-1} q(x, \vec{b}_\perp)}{\int d^2 b_\perp \int_{-1}^1 dx x^{n-1} q(x, \vec{b}_\perp)} = \frac{\langle x^n \rangle + 2(-1)^n \int d^2 b_\perp \int_0^1 dx x^n \bar{q}(x)}{\langle x^{n-1} \rangle}. \quad (4)$$

Lattice QCD is not currently capable of calculating the anti-quark contribution in  $\langle x \rangle_n$ , however the phenomenologically determined parton distributions indicate it is small enough that it does not affect the following qualitative conclusions. Fig. 2 shows the transverse radius versus corresponding momentum fraction for the lowest moments illustrating, as above, that the transverse size of the nucleon depends significantly on the longitudinal momentum of its constituents.

*$b_\perp$  Dependence.* By assuming a dipole ansatz for the generalized form factors the  $b_\perp$  dependence of each moment can be determined from Eq. 2. The details are given in [6], and the results are shown in Fig. 3. Of particular importance are the lowest two moments which determine the transverse distribution of quarks ( $n = 1$ ) and of longitudinal momentum ( $n = 2$ ) within the nucleon.

## MOMENTS OF PARTON DISTRIBUTIONS

Moments of parton distributions provide an opportunity for quantitative comparison between experimental measurements and lattice calculations of nucleon structure. The axial charge and momentum fraction of the nucleon represent the state of affairs with the former observable providing an example of a potential success of current calculations and the latter an example of a challenge to future calculations.

$\langle x \rangle_{u-d}$ . Extensive quenched calculations of  $\langle x \rangle_{u-d}$  [7] have shown very little dependence on  $m_\pi$  while overestimating the experimental result by nearly a factor of two. The

first unquenched calculations [15] confirmed the earlier quenched results and lead to the suggestion that sizable chiral corrections could accommodate both lattice calculations and experimental measurements [8]. Recent calculations with lighter quark masses [5, 9] have not resolved this discrepancy, however one recent quenched calculation [16] shows a significant but unconfirmed shift toward the experimental result.

$g_A$ . Lattice calculations of the nucleon axial coupling are beginning to mature. In particular recent calculations with chiral actions allow for a non-perturbative renormalization of  $g_A$ . This observable has been shown to have sizable finite size corrections for light quark masses [10, 11], however current calculations have reached large enough volumes that such effects appear under control. Furthermore simple linear extrapolations, in  $m_\pi^2$ , of the lightest calculations [12] give estimates of 1.23(2) to 1.26(2) (using 3 to 6 of the lightest points) the latter of which agrees with the experimental measurement within the statistical errors. However detailed study of the chiral behavior is needed to reliably estimate the systematic errors in such extrapolations.

## CONCLUSIONS

Lattice calculations of nucleon structure are beginning to realize their promise to elucidate QCD and make contact with the experimental programs. Recent calculations are painting a qualitative three dimensional picture of nucleon structure revealing a significant  $x$  dependence of the transverse size of the nucleon. Quantitative calculations of moments of parton distributions are progressing, in particular the calculation of  $g_A$  may soon reach a few percent accuracy.

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