Understanding Parton Distributions from Lattice QCD

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Lattice Hadron Physics Collaboration

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http://talks.drubryantrenner.org/dis2005_4-29-05.pdf

Recent Lattice Calculations of Structure Functions

Three Representative Observables

- Transverse Quark Distributions
- Momentum Fraction
- Axial Charge

Generalized Parton Distributions

Generalized Form Factors

- for example, unpolarized twist two operators

$$O_q^{\mu_1 \cdots \mu_n} = \bar{q} i D^{(\mu_1} \ldots i D^{\mu_{n-1}} \gamma^{\mu_n)} q$$

- off-forward matrix elements of the twist two operators [1]

$$\langle P', S' | O_q^{\mu_1 \cdots \mu_n} | P, S \rangle = \overline{U}(P', S') [\sum_{\substack{i=0 \\ \text{even}}}^{n-1} A_{ni}^q(t) K_{ni}^A(P', P)$$
$$+ \sum_{\substack{i=0 \\ \text{even}}}^{n-1} B_{ni}^q(t) K_{ni}^B(P', P) + \delta_{\text{even}}^n C_n^q(t) K_n^C(P', P)] U(P, S)$$

[1] X. D. Ji hep-ph/9807358

Parton Distributions and Form Factors

- moments of parton distributions - $\langle P | O_q^{\mu_1 \cdots \mu_n} | P \rangle$

$$A_{n0}^q(0) = \int_{-1}^{1} dx \; x^{n-1} \, q(x)$$

- form factors - $\mathcal{O}_q^{\mu} = \overline{q} \gamma^{\mu} q$

$$A_{10}^q(t) = F_1^q(t) \quad \text{and} \quad B_{10}^q(t) = F_2^q(t)$$

Quark Angular Momenta and Transverse Quark Distributions

- quark angular momenta [1]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

$$\Delta\Sigma^q = \tilde{A}^q_{10}(0) \qquad J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right) \qquad L^q = J^q - \frac{1}{2}\Delta\Sigma^q$$

- transverse quark distributions [2]

$$\int_{-1}^{1} dx\; x^{n-1}\, q(x, \vec{b}_\perp) \;=\; \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A^q_{n0}(-\vec{\Delta}^2_\perp)$$

[1] X. D. Ji hep-ph/9603249
[2] M. Burkardt hep-ph/0005108

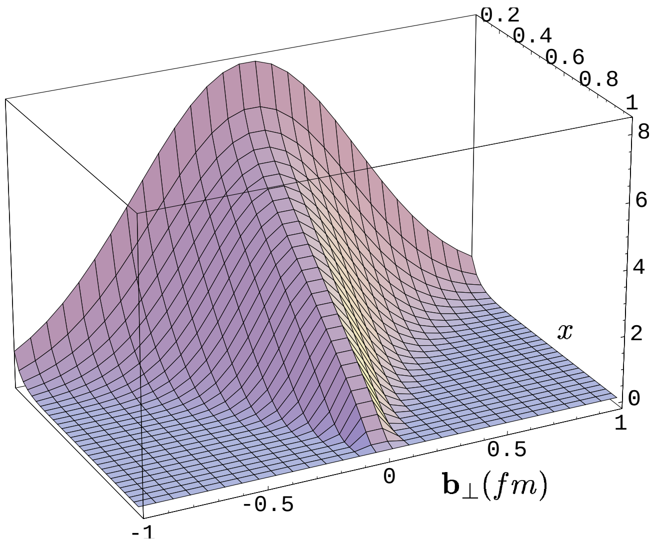
Full QCD Calculations

Collaboration	m_π (MeV)	Quark Action
LHPC/SESAM	> 650	Wilson
QCDSF/UKQCD	> 550	Clover Improved Wilson
RBCK	> 500	Domain Wall
LHPC/MILC	> 350	Staggered/Domain Wall

- a variety of lattice spacings and volumes
- dominant systematic error is still the extrapolation in m_π

What do we learn about the transverse quark structure of the nucleon?

Transverse Distributions



$$A_{n0}^q(-\vec{\Delta}_\perp^2) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} \, q(x, \vec{b}_\perp)$$

$$\langle b_\perp^2 \rangle_{(n)}^q = -4 \, \frac{A_{n0}^{q\,\prime}(0)}{A_{n0}^q(0)}$$

- at $x = 1$ a single quark carries all the momentum

$$\lim_{x \to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

- higher moments A_{n0}^q weight $x \sim 1$ more heavily

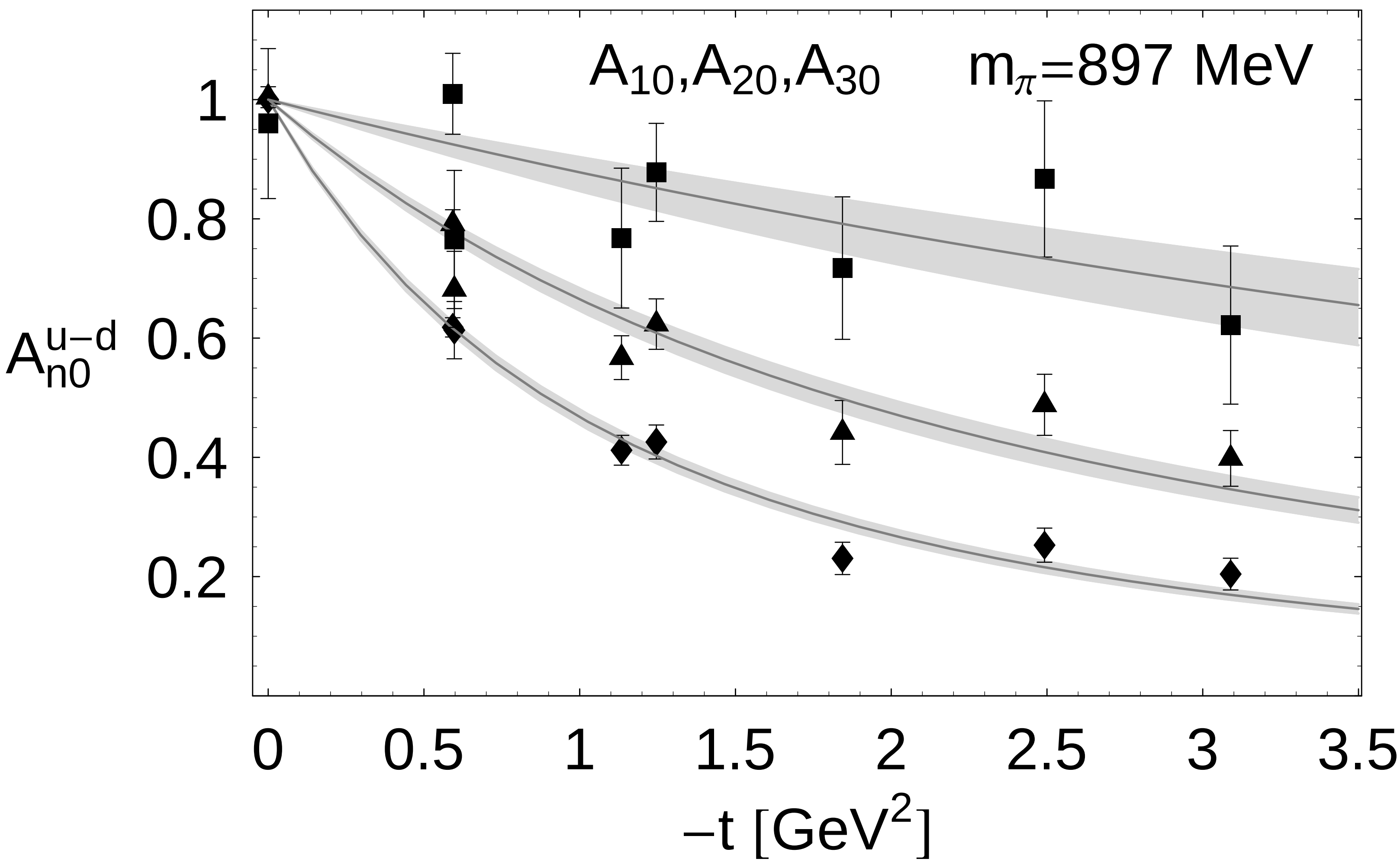
$$\lim_{n \to \infty} A_{n0}^q(t) \propto \int d^2b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \delta^2(\vec{b}_\perp) = \text{constant}$$

- slopes of A_{n0}^q should decrease as n increases

graph from M. Burkardt hep-ph/0207047

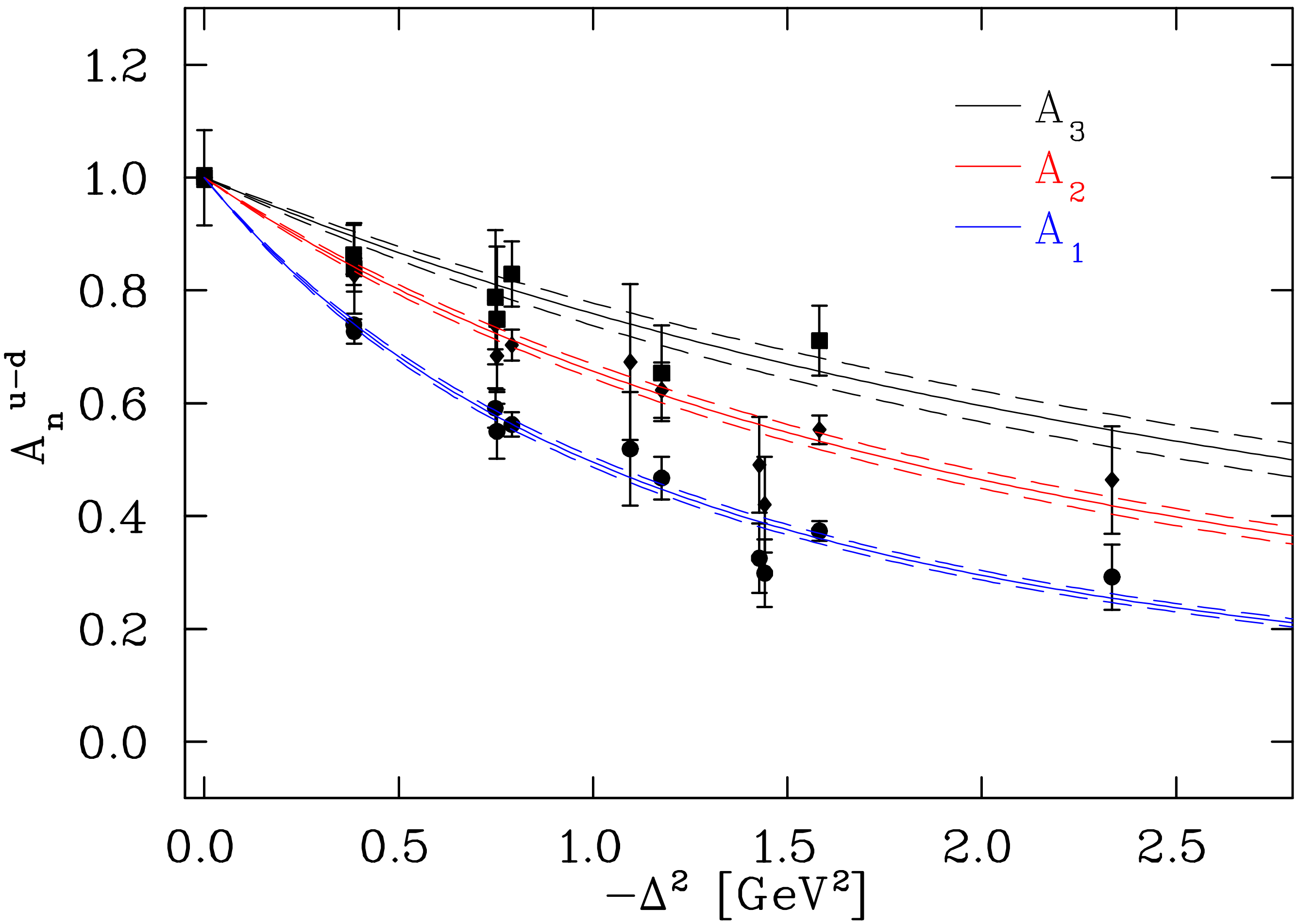
Transverse Distributions: Q^2 Dependence (LHPC)

- slope of $A_{10}^{u-d} = -0.93 \pm 0.04$ (GeV)$^{-2}$
- slope of $A_{30}^{u-d} = -0.13 \pm 0.03$ (GeV)$^{-2}$ (factor of 7)



Transverse Distributions: Q^2 Dependence (QCDSF)

- again, slopes of A_{n0}^q decrease as n increases



- for more QCDSF results see Wed's talk by G. Schierholz

graph from QCDSF hep-lat/0409162

Transverse Distributions: x Dependence

- transverse rms radius

$$\left\langle b_\perp^2 \right\rangle_x = \frac{\int d^2b_\perp\, b_\perp^2\, q(x,\vec{b}_\perp)}{\int d^2b_\perp\, q(x,\vec{b}_\perp)}$$

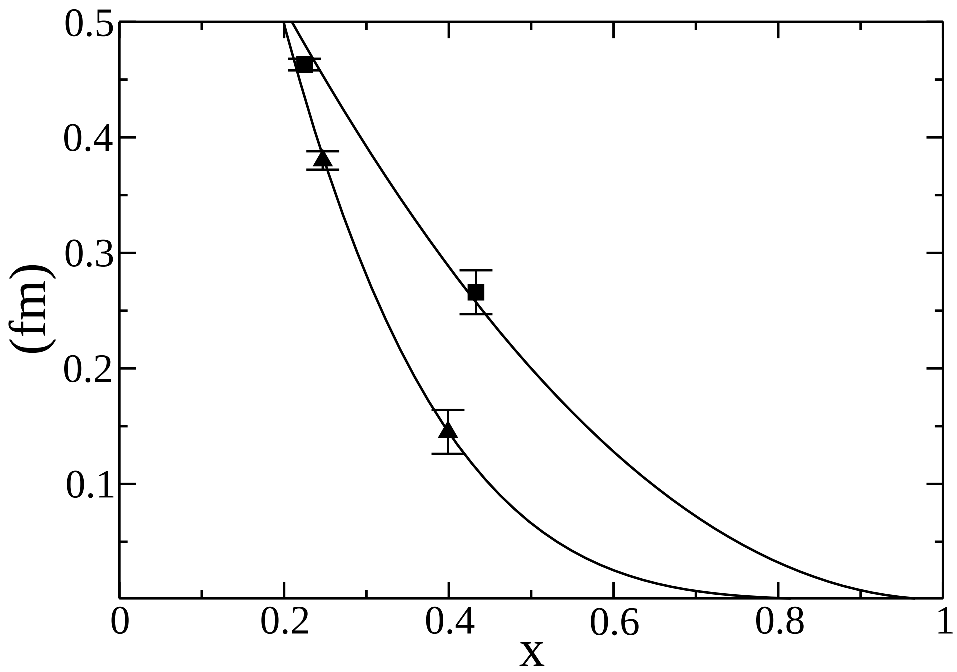
- transverse rms *moment* radius

$$\frac{A'_{n0}(0)}{A_{n0}(0)} = -\frac{1}{4}\left\langle b_\perp^2 \right\rangle_{(n)} \qquad \left\langle b_\perp^2 \right\rangle_{(n)} = \frac{\int d^2b_\perp\, b_\perp^2\, \int_{-1}^{1} dx\, x^{n-1} q(x,\vec{b}_\perp)}{\int d^2b_\perp\, \int_{-1}^{1} dx\, x^{n-1} q(x,\vec{b}_\perp)}$$

- the average x in $\left\langle b_\perp^2 \right\rangle_{(n)}$

$$x_{\text{av}}^{(n)} = \frac{\int d^2b_\perp\, \int_{-1}^{1} dx\, x \cdot x^{n-1} q(x,\vec{b}_\perp)}{\int d^2b_\perp\, \int_{-1}^{1} dx\, x^{n-1} q(x,\vec{b}_\perp)} = \frac{< x^n >}{< x^{n-1} >}$$

Transverse Distributions: x Dependence (LHPC)

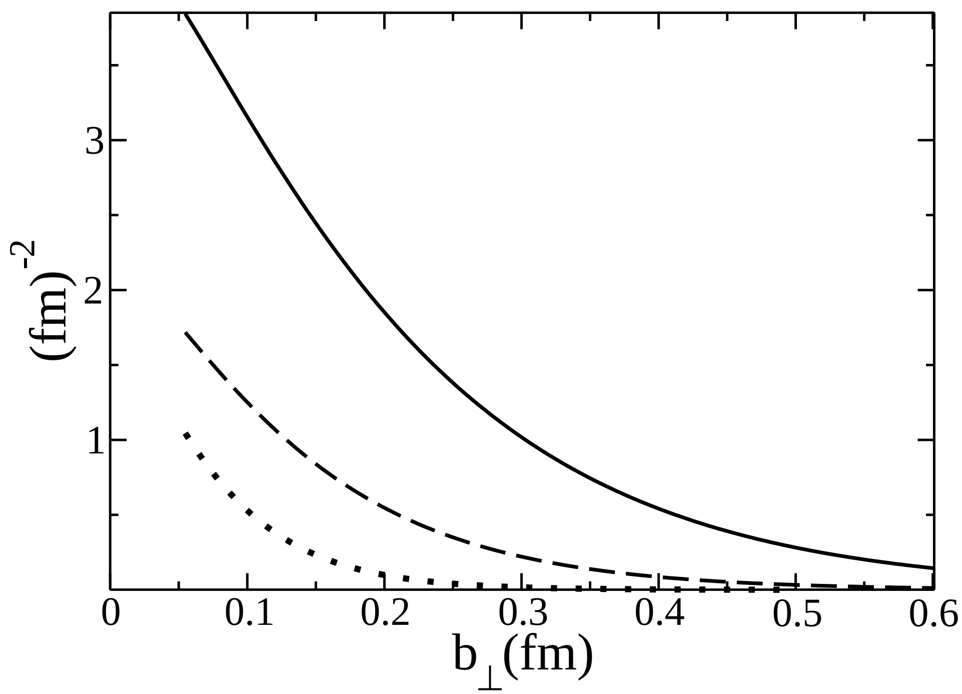


- non-singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(1)}} = 0.38$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u-d}^{(3)}} = 0.15$ fm, 61% decrease
- singlet radius: $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(1)}} = 0.46$ fm & $\sqrt{\left\langle b_\perp^2 \right\rangle_{u+d}^{(3)}} = 0.27$ fm, 41% decrease

Transverse Distributions: $\vec{b}_\perp$ Dependence (LHPC)

$$q_1(\vec{b}_\perp) = \int_{-1}^{1} dx\; q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{10}^q(-\vec{\Delta}_\perp^2)$$

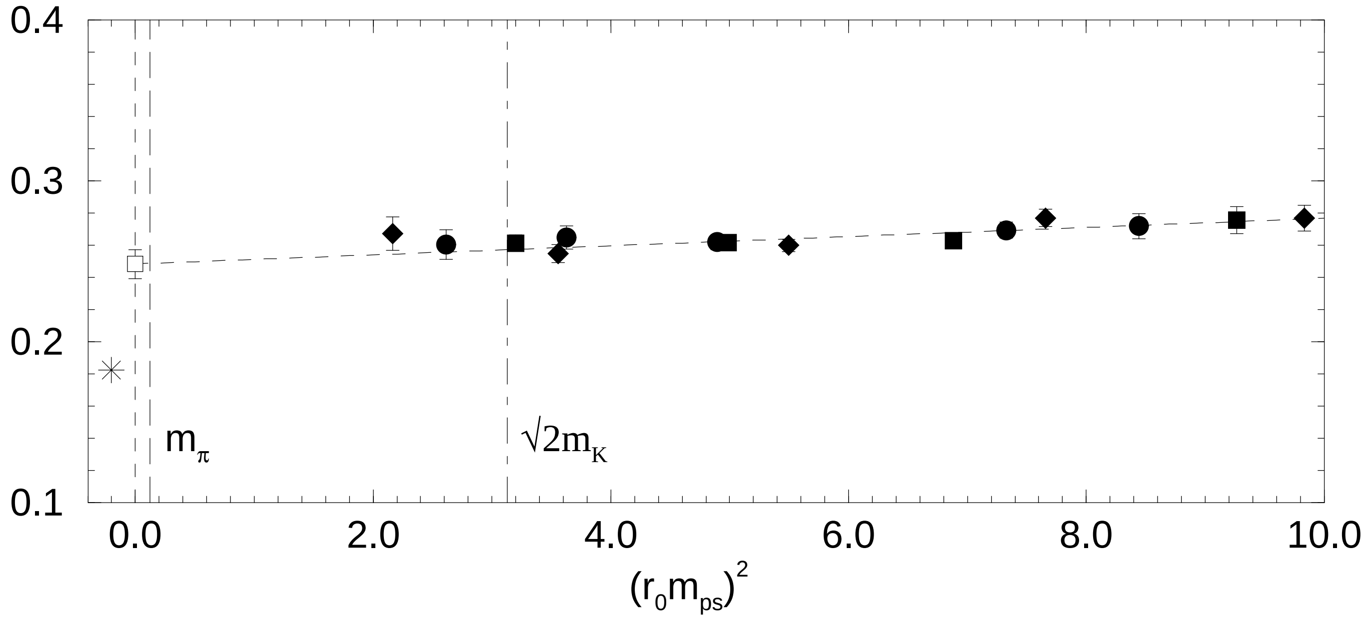
$$q_2(\vec{b}_\perp) = \int_{-1}^{1} dx\; x\, q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} A_{20}^q(-\vec{\Delta}_\perp^2)$$



What do we learn about the momentum fraction of the nucleon?

$\langle x \rangle_{u-d}$: m_π Dependence (QCDSF-Quenched)

- heavy pion world: observables are linear functions of m_π^2

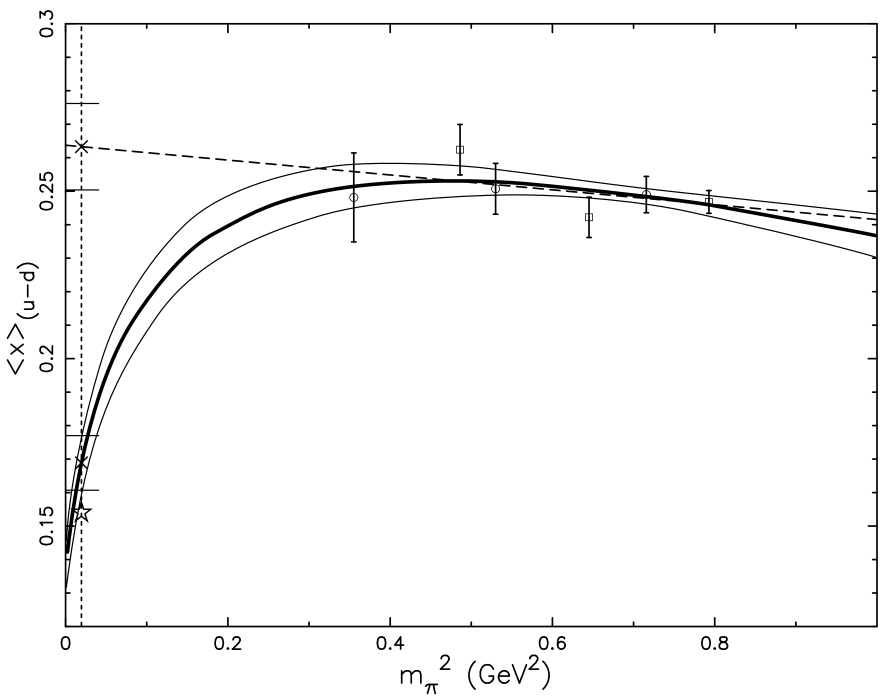


graph from QCDSF hep-lat/0209160

$\langle x \rangle_{u-d}$: Chiral Extrapolations? (LHPC)

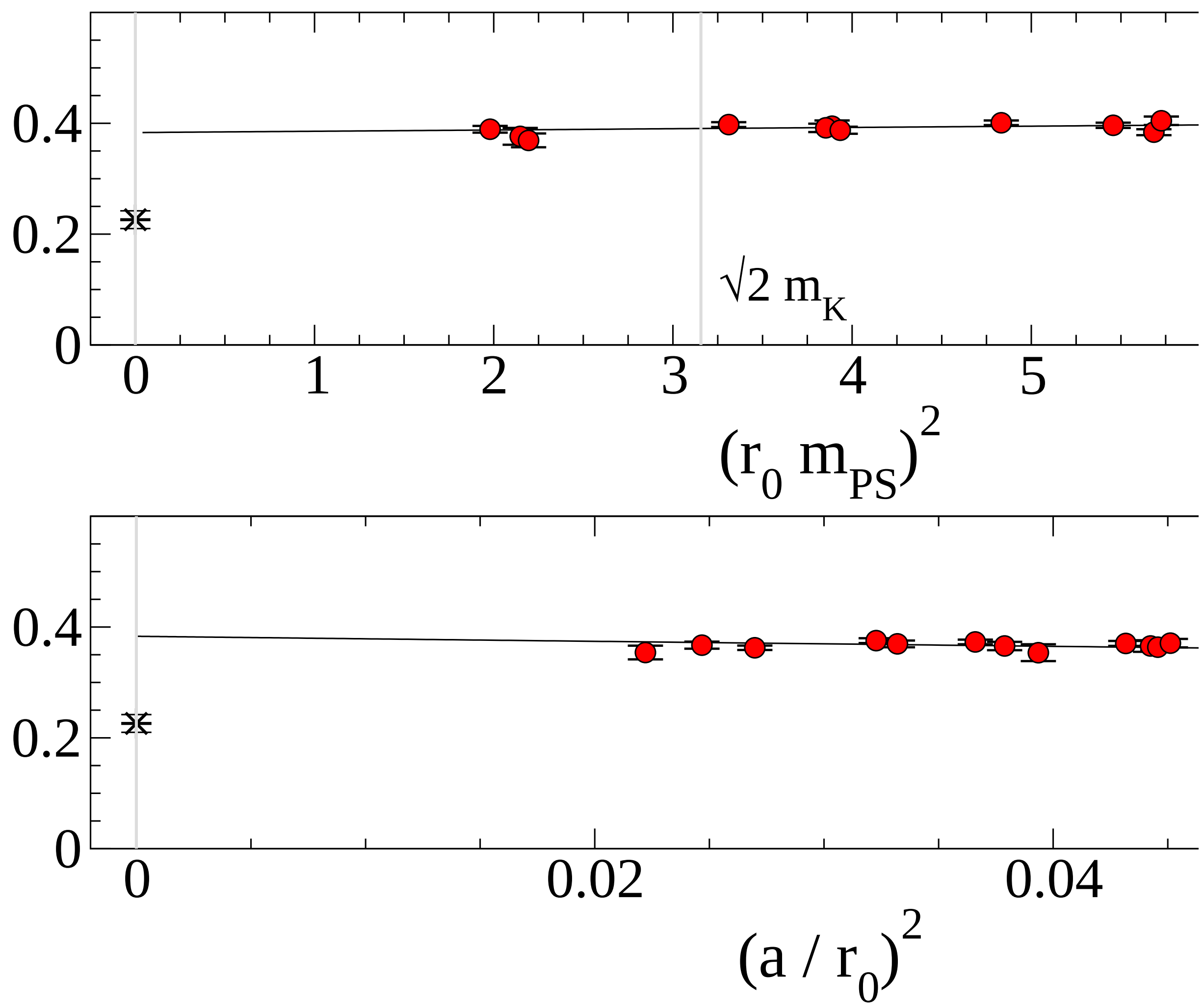
- leading order χPT & heavy quark limit [1]

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln \left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right) + b_n m_\pi^2$$



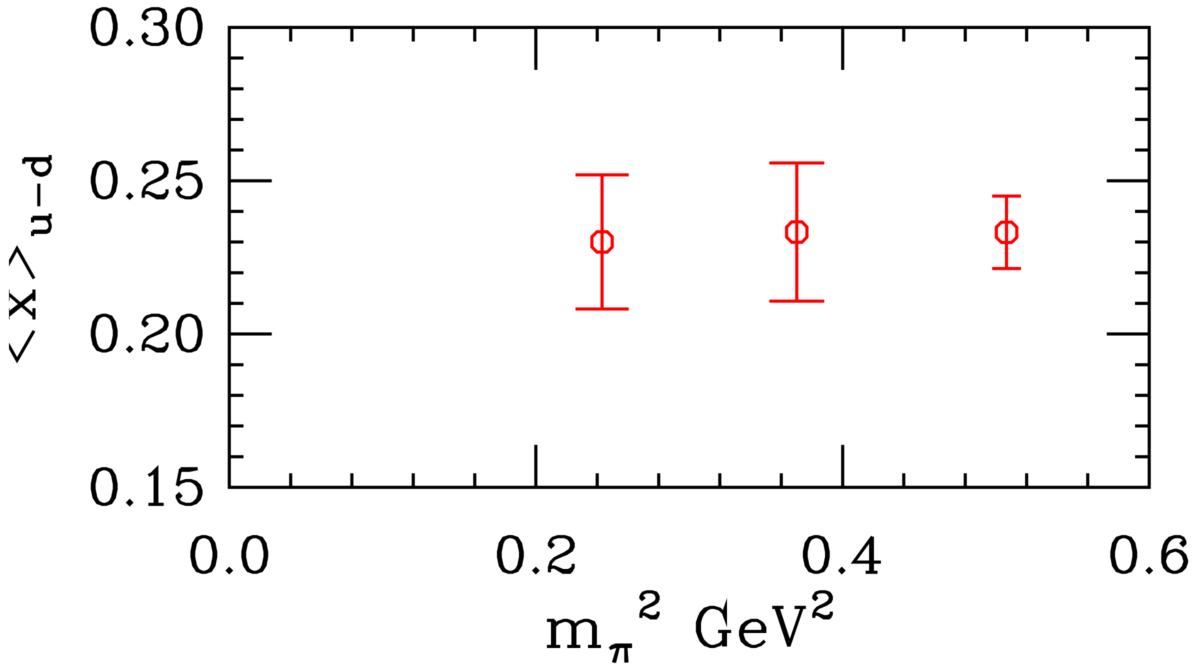
[1] Detmold, Melnitchouk, Negele, Renner, Thomas hep-lat/0103006

$\langle x \rangle_{u-d}$: Chiral Logs? (QCDSF)



graph from QCDSF hep-lat/0409162

$\langle x \rangle_{u-d}$: Chiral Logs? (RBCK)

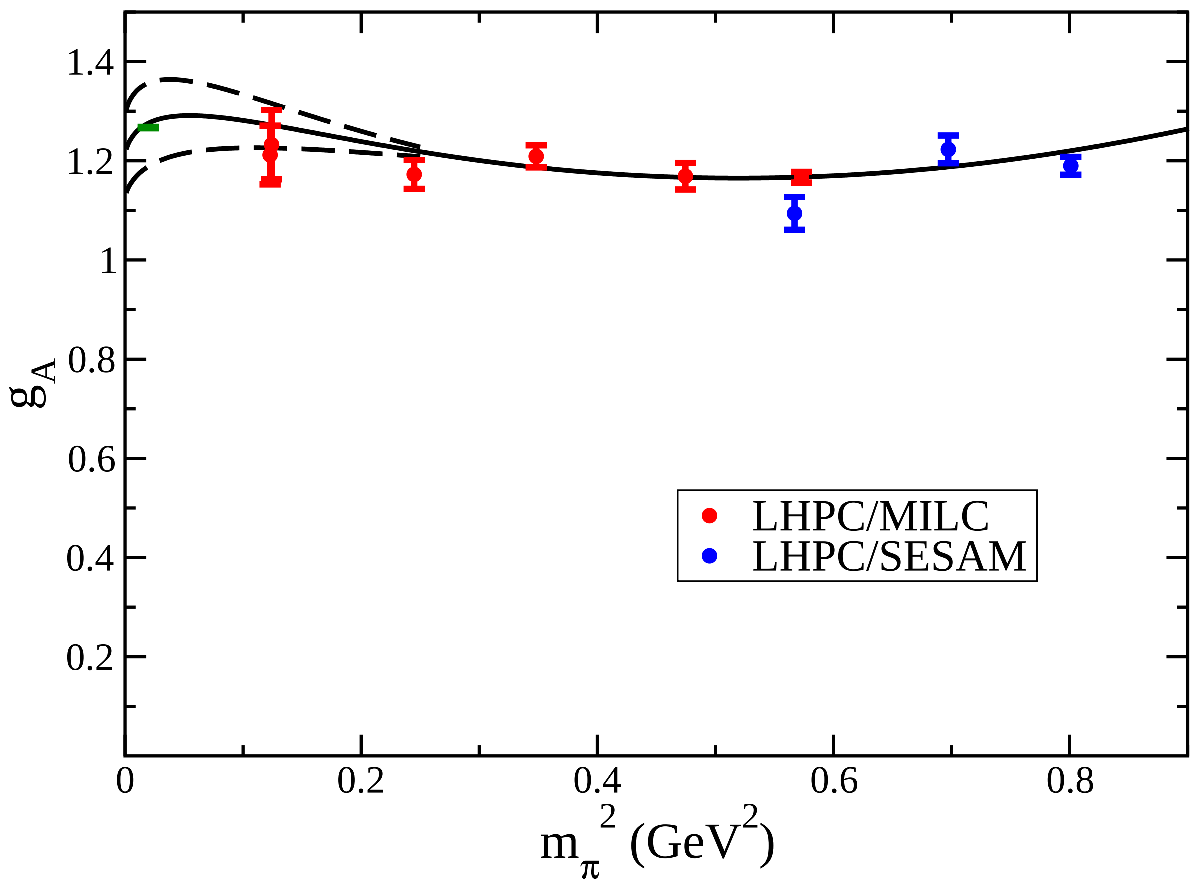


graph from RBCK hep-lat/0411008

What do we learn about the axial coupling of the nucleon?

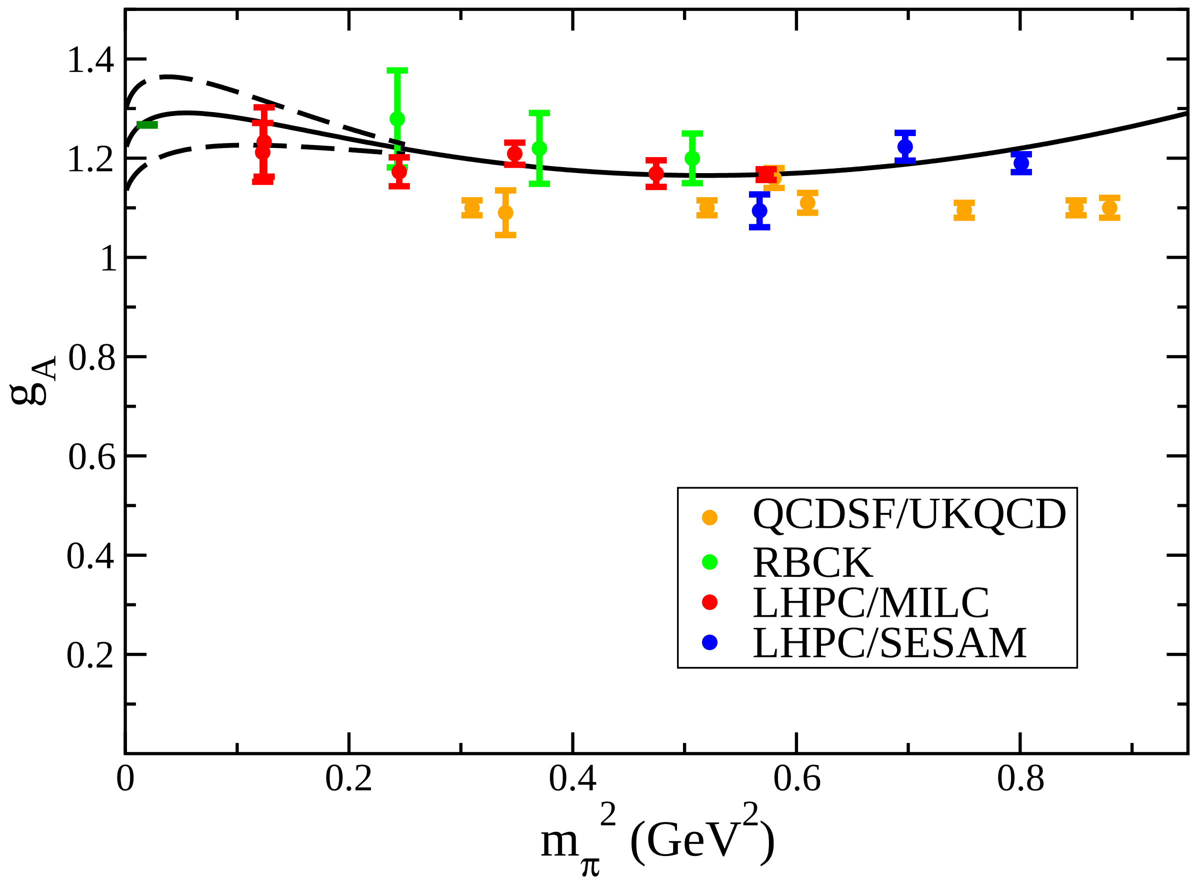
g_A: m_π Dependence (LHPC)

- curve is one loop chiral perturbation theory (including Δ) [1]
- parameters: f_π, g^A_{NN}, $g^A_{N\Delta}$, $g^A_{\Delta\Delta}$, $m_\Delta - m_N$, $B_9 - g_A B_{20}$
- only g^A_{NN} and $g^A_{\Delta\Delta}$ are fit below



[1] Hemmert et al hep-lat/0303002

g_A: Putting it All Together (LHPC,RBCK,QCDSF)



RBCK data from graph in hep-lat/0409161
QCDSF data from graph in hep-lat/0409161

Conclusions

- the transverse size of the nucleon, $\sqrt{\langle r_\perp^2 \rangle}$, in the heavy pion world, shows a significant dependence on the longitudinal momentum fraction $\langle x \rangle$, and presents an opportunity to expand our understanding of QCD

- the momentum fraction, $\langle x \rangle_{u-d}$, shows very little quark mass dependence, is still about a factor of 2 too large, and presents a challenge for future lattice QCD calculations

- the axial charge, g_A, has strong but estimable finite volume effects, allows us to probe various low energy constants in the chiral lagrangian, and will likely come under quantitative control shortly