The EMC effect in effective field theory

William Detmold

Department of Physics, Box 351560, University of Washington, Seattle, WA 98195, USA

Abstract. Using effective field theory, we investigate nuclear modification of nucleon parton distributions (for example, the EMC effect). We show that the universality of the shape distortion in nuclear parton distributions (the factorisation of the Bjorken x and atomic number (A) dependence) is model independent and emerges naturally in effective field theory. We present simple fits to experimental data that incorporate this factorisation.

Keywords: Nuclear modification; parton distributions
PACS: 25.30.Mr

The 1983 European Muon Collaboration's (EMC) observation [1] of the deviation of the ratio $R_{Fe}(x)$ of $F_2(x)$ structure functions in iron and deuterium in deep inelastic scattering (DIS) has provoked much analysis and discussion over the years [2] since it shows that the parton distributions are modified in the nuclear environment. Here [3] we employ effective field theory (EFT) to investigate the EMC effect by studying nuclear matrix elements of the twist-two operators which are related to parton distributions and structure functions via the operator product expansion. We find that the universality of the shape distortion of the EMC effect (the factorisation of the x and A dependencies) is a model independent result, arising from the symmetries of QCD and the separation of the relevant scales. The x dependence of $R_A(x)$ is governed by short distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics.

Recently EFT has been applied to the computation of hadronic matrix elements of twist-two operators in the meson and single nucleon sectors [4]. The approach has also been extended to analyse the deuteron system [5] and is readily generalised to the multi-nucleon case. To described the EMC effect observed in F_2 data on isoscalar nuclei, we consider the normalised, spin singlet, isoscalar twist-two operators,

$$\mathcal{O}_q^{\mu_0\cdots\mu_n} = \overline{q}\gamma^{(\mu_0} iD^{\mu_1}\cdots iD^{\mu_n)} q / \left(2M^{n+1}\right), \tag{1}$$

where $(\ldots)$ indicates that enclosed indices have been symmetrised and made traceless, $D^\mu = (\overrightarrow{D}^\mu - \overleftarrow{D}^\mu)/2$ is the covariant derivative and M is the nucleon mass. The matrix elements of $\mathcal{O}_q^{\mu_0\ldots\mu_n}$ in an unpolarised nucleon of momentum P can be parametrised as

$$\langle P|\mathcal{O}_q^{\mu_0\ldots\mu_n}|P\rangle = \langle x^n\rangle_q \widetilde{v}^{\mu_0}\cdots\widetilde{v}^{\mu_n}, \tag{2}$$

where the nucleon velocity $\widetilde{v}^\mu = P^\mu/M$. It is well known that the coefficients $\langle x^n\rangle_q$ correspond to moments of the isoscalar combination of parton distribution functions, $\langle x^n\rangle_q = \int_{-1}^{1} dx\, x^n q(x)$ where $q(x)$ is the isoscalar quark distribution and $q(-x) = -\overline{q}(x)$.

We first consider only nucleonic degrees of freedom (*i.e.*, assume that pions are integrated out of the EFT – they will be reintroduced below) and perform the standard

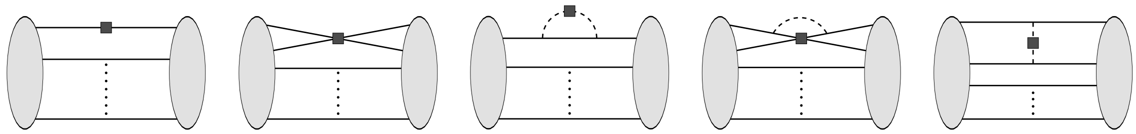

FIGURE 1. Contributions to nuclear matrix elements, labeled (a) to (e) left to right. The dark square represents the various operators in Eq. (3) and the light shaded ellipse corresponds to the nucleus, A. The solid and dashed lines represent nucleons and pions respectively. The dots in the lower part of the diagrams indicate the spectator nucleons.

matching procedure in EFT, equating the quark level twist-two operators to the most general combinations of hadronic operators of the same symmetries [4]. The leading one- and two-body hadronic operators in the matching are

$$\mathscr{O}_q^{\mu_0\cdots\mu_n} = \langle x^n\rangle_q v^{\mu_0}\cdots v^{\mu_n} N^\dagger N[1+\alpha_n N^\dagger N]+\cdots\,, \tag{3}$$

where $v^\mu = \widetilde{v}^\mu + \mathscr{O}(1/M)$ is the velocity of the nucleus. Operators involving additional derivatives are suppressed by powers of M in the EFT power-counting and we have only kept the SU(4) (spin and isospin) singlet two-body operator in the above equation as the other independent two-body operator, $v^{\mu_0}\cdots v^{\mu_n}\left(N^\dagger\tau N\right)^2$, is suppressed [3]. Three- and higher- body operators also appear in Eq. (3), however numerical evidence from other EFT calculations indicates that these contributions are generally small [6].

Nuclear matrix elements of $\mathscr{O}_q^{\mu_0\cdots\mu_n}$ give the moments of the isoscalar nuclear parton distributions, $q_A(x)$. The leading order (LO) and the next-to-leading order (NLO) contributions to these matrix elements are shown in Fig. 1(a) and 1(b), respectively. For an unpolarized, isoscalar nucleus,

$$\begin{aligned}\langle x^n\rangle_{q|A} &\equiv v^{\mu_0}\cdots v^{\mu_n}\langle A|\mathscr{O}_q^{\mu_0\cdots\mu_n}|A\rangle \\ &= \langle x^n\rangle_q[A+\langle A|\alpha_n(N^\dagger N)^2|A\rangle],\end{aligned} \tag{4}$$

where we have used $\langle A|N^\dagger N|A\rangle = A$. Notice that if there were no EMC effect, the α_n would vanish for all n. Also $\alpha_0=0$ by charge conservation. From Eq. (4) we see that

$$\left(\frac{\langle x^n\rangle_{q|A}}{A\langle x^n\rangle_q}-1\right)\Big/\left(\frac{\langle x^m\rangle_{q|A}}{A\langle x^m\rangle_q}-1\right)=\frac{\alpha_n}{\alpha_m} \tag{5}$$

is independent of A which has powerful consequences. In all generality, the isoscalar nuclear quark distribution can be written as

$$q_A(x) = A\left[q(x)+\widetilde{g}(x,A)\right]. \tag{6}$$

Taking moments of Eq. (6), Eq. (5) then demands that the x dependence and A dependence of $\widetilde{g}$ factorise,

$$\widetilde{g}(x,A) = g(x)\mathscr{G}(A), \tag{7}$$

with

$$\mathscr{G}(A) = \langle A|(N^\dagger N)^2|A\rangle/A\Lambda_0^3, \tag{8}$$

and $g(x)$ satisfying

$$\alpha_n = \frac{1}{\Lambda_0^3 \langle x^n \rangle_q} \int_{-A}^{A} dx x^n g(x) . \tag{9}$$

Λ_0 is an arbitrary dimensionful parameter . Crossing symmetry dictates that the even and odd α_n separately determine the nuclear modifications of valence and total quark distributions. These results apply to any isoscalar combination of parton distributions including $F_2(x)$ for isoscalar nuclei. Thus our result implies that

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathscr{G}(A), \tag{10}$$

which says that the EMC effect (the deviation of $R_A(x)$ from unity) has an universal shape described by $g_{F_2}(x)$ while the magnitude of deviation, $\mathscr{G}(A)$, only depends on A.

The above analysis gives a simple explanation of the observed universal shape of the EMC effect, or equivalently, the factorisation of $\widetilde{g}(x,A)$. The key to establishing this factorisation is that other sources of nuclear modification contributing to the right-hand side of Eq. (3) must be suppressed (higher order in the EFT) such that the A independence of Eq. (5) can be established. We stress that the factorisation persists when pions are included in our analysis. In Fig. 1, examples of the leading pionic contributions are shown. The various single-nucleon diagrams, such as Fig. 1(c), simply renormalise the nucleon moments, $\langle x^n \rangle_q$, without contributing to the EMC effect. Two- and more-nucleon diagrams such as those in Fig. 1(d) and 1(e) contribute to the EMC effect, but only at N^{3}LO and higher (see Ref. [3] for explicit calculations). Other contributions that could upset the factorization include a two-body operator which is similar to that in Eq. (3) but with two more derivatives. However this operator also contributes at N^{3}LO. Consequently, the universality of the EMC effect is preserved to good accuracy. For large x it is clear that the factorisation must break down (simply consider the region $x > 2$ in which only three- and higher- body operators contribute) though the structure function is very small in this region anyway.

It is clear from Eq. (8) that $\mathscr{G}(A)$ is governed by long distance physics which can be computed using a traditional, non-relativistic nuclear physics approach. Information on the shape distortion function $g(x)$ is encoded in the short distance parameters α_n associated with the strength of the two-body currents. One can either fix the α_n from experimental data (to determine all α_n, data on $F_2^A(x)$ and $F_3^A(x)$ are required) or calculate $\langle NN | \mathscr{O}_q^{\mu_0 \ldots \mu_n} | NN \rangle$ in two nucleon systems to extract them. The latter approach, however, is intrinsically non-perturbative and thus requires lattice QCD [7, 3].

In Figure 2 we present simple fits to the world data on the ratio $R_A(x)$. We choose the simple parameterisation

$$g_{F_2}(x) = (a + b\sqrt{x} + cx + dx^2)(1-x)^f , \qquad \mathscr{G}(A) = 1 - A^{-1/3} \tag{11}$$

though other similar forms also work. This five parameter fit describes the data well in both the small and large x regions, giving a χ^2 per degree of freedom of ~ 1.4. Whilst these fits do not include the (weak) scale dependence of the data, they show consistency with factorisation.

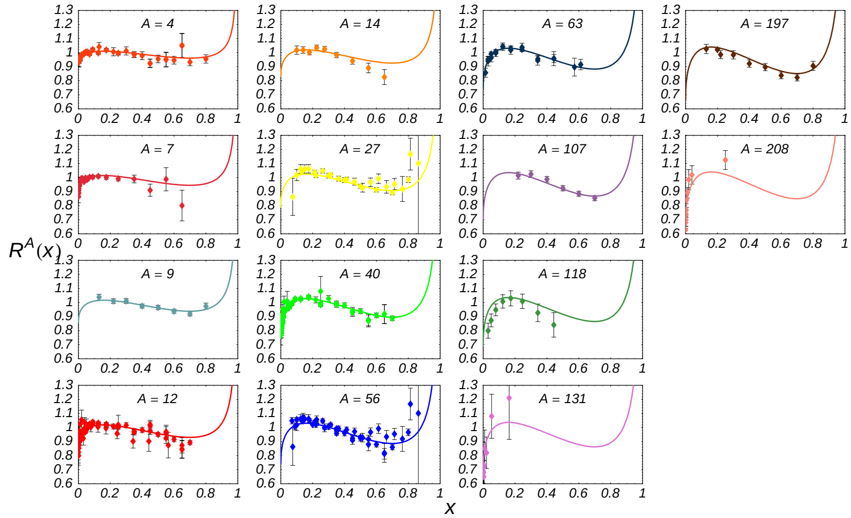

FIGURE 2. Fits to nuclear $R_A(x)$ data.

Similar techniques can also be used to study nuclear modifications of the isovector and spin-dependent parton distributions. Comparable factorisations are expected in EFT. In the isovector case this factorisation can be tested; one can either consider the difference between F_2's in $(Z,N) = (n+m,n)$ and $(n,n+m)$ mirror nuclei [8] and compare it with $F_2^p - F_2^n$, or disentangle $u_A(x)$ and $d_A(x)$ with the upcoming neutrino-nucleus experiment, MINERνA [9]. For spin dependent PDFs, experimental tests are unlikely. Finally, EFT analysis of off-forward matrix elements of the same twist-two operators leads to information about nuclear effects in generalised parton distributions [3].

ACKNOWLEDGMENTS

I thank J.-W. Chen with whom this work was performed. This work was supported by the DOE contract DE-FG03-97ER41014.

REFERENCES

1. J. Aubert *et al.*, Phys. Lett. B **123**, 275 (1983).
2. M. Arneodo, Phys. Rept. **240**, 301 (1994); D. F. Geesaman, K. Saito and A. W. Thomas, Ann. Rev. Nucl. Part. Sci. **45**, 337 (1995); G. Piller and W. Weise, Phys. Rept. **330**, 1 (2000); P. R. Norton, Rept. Prog. Phys. **66**, 1253 (2003).
3. W. Detmold and J. W. Chen, arXiv:hep-ph/0412119; *In preparation.*
4. W. Detmold *et al.*, Phys. Rev. Lett. **87**, 172001 (2001); D. Arndt and M. J. Savage, Nucl. Phys. **A697**, 429 (2002); J. W. Chen and X. Ji, Phys. Lett. **B523**, 107 (2001); *ibid*, 73 (2001); Phys. Rev. Lett. **87**, 152002 (2001). W. Detmold and C.-J. D. Lin, Phys. Rev. D **71**, 054510 (2005).
5. S. R. Beane and M. J. Savage, arXiv:nucl-th/0412025.
6. K. Kubodera and T.-S. Park, Ann. Rev. Nuc. Part. Sci. **54**, 19 (2004);
7. W. Detmold, Phys. Rev. D **71**, 054506 (2005).
8. K. Saito *et al.*, Phys. Lett. B **493**, 288 (2000).
9. D. Drakoulakos *et al.*, hep-ex/0405002.