Nuclear Structure Functions in Effective Field Theory

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Outline

- Nuclear EFT and twist-two matrix elements
- Universality of the EMC effect: factorisation of x and A dependencies
- EMC effect from lattice QCD
- Factorised nuclear structure functions
- Other probes of nuclear modifications

Deep inelastic scattering

- DIS experiments on nuclear targets [EMC 1983]

$$F_2(x) \to F_2^A(x) \qquad R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} \neq 1$$

- EFT approach via operator product expansion (lose physical picture of underlying mechanism)

- Moments of PDFs ⇔ matrix elements of local operators $\mathcal{O}^{(n)}$:

$$\langle p|\overline{q}\gamma^{\{\mu_0}D^{\mu_1}\dots D^{\mu_n\}}q|p\rangle = \langle x^n\rangle p^{\mu_0}\dots p^{\mu_n}$$

with $\langle x^n\rangle = \int_{-1}^{1} dx\; x^n q(x)$

- Ignore Q^2: EMC ratio very weakly dependent

Nuclear EFT

- Model independent approach to low energy nuclear systems [Weinberg 1990]
- Effective field theory for low energy interactions of pions and nucleons valid for $E,\ |\mathbf{p}|,\ m_\pi \ll \Lambda \sim m_\rho$
- Short distance physics ⇒ higher dimensional operators and corresponding low energy constants
- Power counting determines relevant terms
- Large NN scattering lengths ($a_{^1S_0} \sim -25\ \mathrm{fm}$) make parts of EFT non-perturbative [Kaplan, Savage & Wise 1996; Beane *et al.* 2000]

Nuclear EFT Lagrangian

- Pion and nucleon fields $\Sigma = \xi^2 = \exp\left[\frac{2i}{f}\begin{pmatrix}\pi^0 & \pi^+ \\ \pi^- & \pi^0\end{pmatrix}\right]$
- Heavy baryon formalism: $N_v(x) = e^{iMv\cdot x}N(x)$
- Δ-isobar omitted for simplicity

p= M v +k

$$\mathcal{L} = \frac{f^2}{8}\mathrm{tr}[\partial^\mu\Sigma^\dagger\partial_\mu\Sigma] + \frac{\lambda f^2}{4}\mathrm{tr}[m_Q\Sigma^\dagger + m_Q\Sigma]$$

Velocity

Spin

$$+N^\dagger i\, v\cdot D\ N + g_A N^\dagger S^\mu A_\mu\ N$$

$$+C_0(N^\dagger N)^2 + C_2(N^\dagger D_i N)^2 + \ldots$$

All possible operators constrained by symmetries

where

$$D^\mu = \partial^\mu + \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right) \qquad A^\mu = \frac{i}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)$$

Applications of nEFT

- Successfully used in *A*=1,2,3,4 systems
  - Scattering amplitudes / phase shifts
  - Magnetic and quadrupole moments and FFs
  - Pion photo-production, photo-disintegration
  - Electroweak processes: $\nu\ d \rightarrow n\ p$
- Application to *A*>4 is difficult in general
  - know just enough for twist-two operators

Twist-two operators in EFT

- EFT: match QCD operators to all possible hadronic operators with same symmetries
- Used in π and N sectors to connect lattice PDF moments to experiment [Arndt & Savage; Chen & Ji; Detmold *et al.*]
- Isoscalar, spin independent operators:

$$\overline{q}\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_n\}}q \longrightarrow a_n\frac{1}{\Lambda^n}\mathrm{tr}\left[\Sigma^\dagger D^{\mu_1}\ldots D^{\mu_n}\Sigma + h.c.\right]$$

LECs

$$+c_n N^\dagger \mathcal{V}^{\mu_1\ldots\mu_n}N + c_n' N^\dagger S^{\{\mu_1}A^{\mu_2}\mathcal{V}^{\mu_3\ldots\mu_n\}}N + \ldots$$

$$+\alpha_n N^\dagger\mathcal{V}^{\mu_1\ldots\mu_n}N\; N^\dagger N + \beta_n N^\dagger \mathcal{V}^{\mu_1\ldots\mu_n}\tau_j^{\xi+}N\; N^\dagger\tau_j^{\xi+} + \ldots$$

where

$$\mathcal{V}^{\mu_1\ldots\mu_n} = \left(v + i\frac{D}{M}\right)^{\mu_1}\ldots\left(v+i\frac{D}{M}\right)^{\mu_n} \qquad \tau_j^{\xi\pm} = \frac{1}{2}\left(\xi^\dagger\tau_j\xi \pm \xi\tau_j\xi^\dagger\right)$$

Twist-two matrix elements

- Nucleon matrix elements

$$v_{\mu_1} \ldots v_{\mu_n} \langle N | \mathcal{O}^{\mu_1 \ldots \mu_n} | N \rangle = \langle x^n \rangle_q$$

Includes pionic and nucleonic terms

- Isoscalar nuclear matrix elements

$$\langle x^n \rangle_{q|A} \equiv v_{\mu_1} \ldots v_{\mu_n} \langle A | \mathcal{O}^{\mu_1 \ldots \mu_n} | A \rangle$$

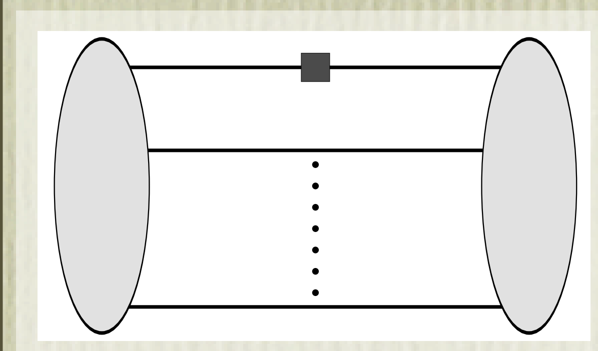
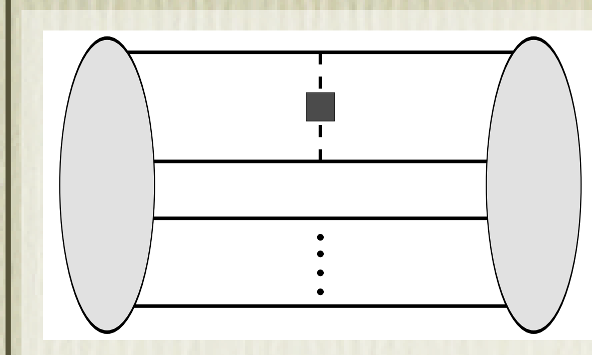
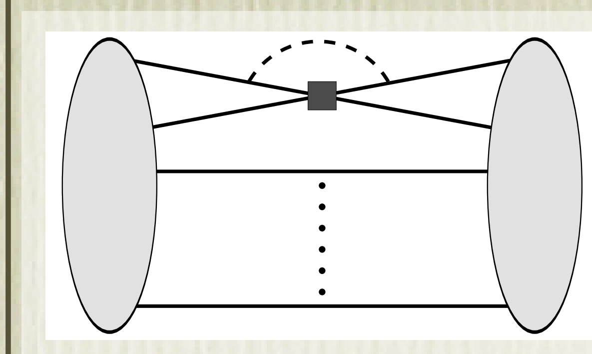
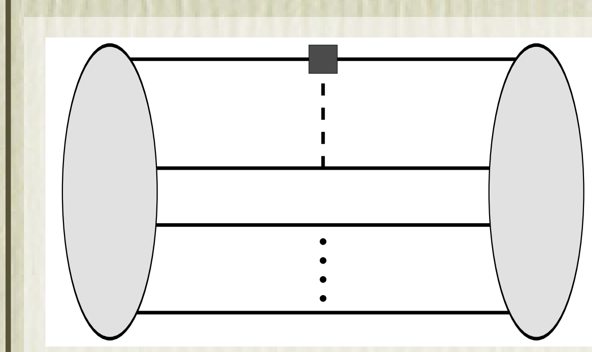
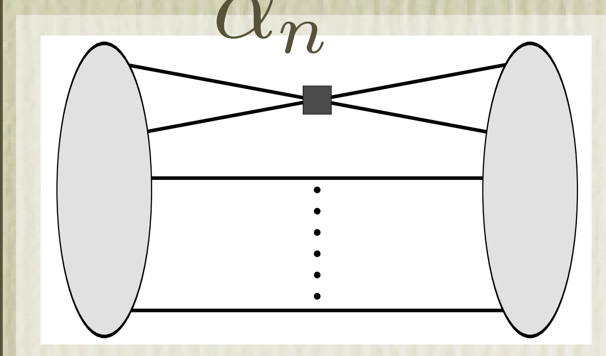
$$= \langle x^n \rangle_q \left[A + \alpha_n \langle A | (N^\dagger N)^2 | A \rangle + \beta_n \langle A | (N^\dagger \tau N)^2 | A \rangle \right] + \ldots$$

Dominant terms

- β_n term suppressed by N_c^2 [Kaplan & Savage 96; K & Manohar 97]
- Ellipsis includes higher-body operators, terms with derivatives: higher-order in power-counting

Twist-two matrix elements

- EFT power counting

LO		N^{3}LO	   … …
NLO	α_n 		
NNLO	None		

Universality

- Ratio of different moments

$$\frac{\dfrac{\langle x^n\rangle_{q|A}}{A\langle x^n\rangle_q}-1}{\dfrac{\langle x^m\rangle_{q|A}}{A\langle x^m\rangle_q}-1}=\frac{\alpha_n}{\alpha_m}$$

Independent of A!

- Write: $q_A(x) = A\left[q(x)+\tilde{g}(x,A)\right]$
- Implies factorisation of $\tilde{g}(x,A) = g(x)\mathcal{G}(A)$
- x dependence: $\alpha_n = \dfrac{1}{\Lambda_0^3\langle x^n\rangle_q}\displaystyle\int_{-\infty}^{\infty} dx\ x^n g(x)$
  - Short distance: strength of two-body current
- A dependence: $\mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle/A\Lambda_0^3$
  - Long distance: from non-rel. nuclear physics

Universality

- Holds for any isoscalar distribution: $F_2(x)$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A)$$

- Factorisation observed in data: [Daté *et al.* 84,..., Gomez *et al.* 95]
- Factorisation requires there be only a single non-trivial source of *A* dependence in EFT operator
  - Holds for two orders or N_c^2: expect ~25%
  - N[3]LO destroys factorisation

Large *x* and higher twist

- Large *x* support:
  - Support of $F_2^A(x)$ is $-A < x < A$
  - Generation in EFT requires higher-body terms
  - Kinematic breakdown of power-counting
- Q^2 dependence and higher twist:
  - EFT matching is by symmetry, not by twist
  - LECs correspond to arbitrary twist: $\alpha_n(Q^2, \mu^2)$
  - Factorisation is therefore $(x, Q^2) \otimes A$

Lattice QCD

- LECs α_n can be determined from experiment

OR

- Lattice QCD can probe EMC effect from first principles
  - Measure $\langle N\ N|\mathcal{O}^{(n)}|N\ N\rangle$ to determine LECs (twist-2 QCD operator)
    - Volume dependence of two particle energy-levels in background twist-two field [WD 05]
    - α_n can be extracted (will be twist-two)
  - EFT needed to extrapolate to physical masses

Parameterisations of $F_2^A(x)$

- Factorised ansatz: $F_2^A(x) = A\left[F_2(x) + g_{F_2}(x)\mathcal{G}(A)\right]$

$$\mathcal{G}(A) \sim \log(A) \sim (1 - A^{-\frac{1}{3}})$$

$$g_{F_2}(x) \sim (a + b\sqrt{x} + cx + dx^2)(1 - x)^f$$

Similar to [Hirai *et al.*]

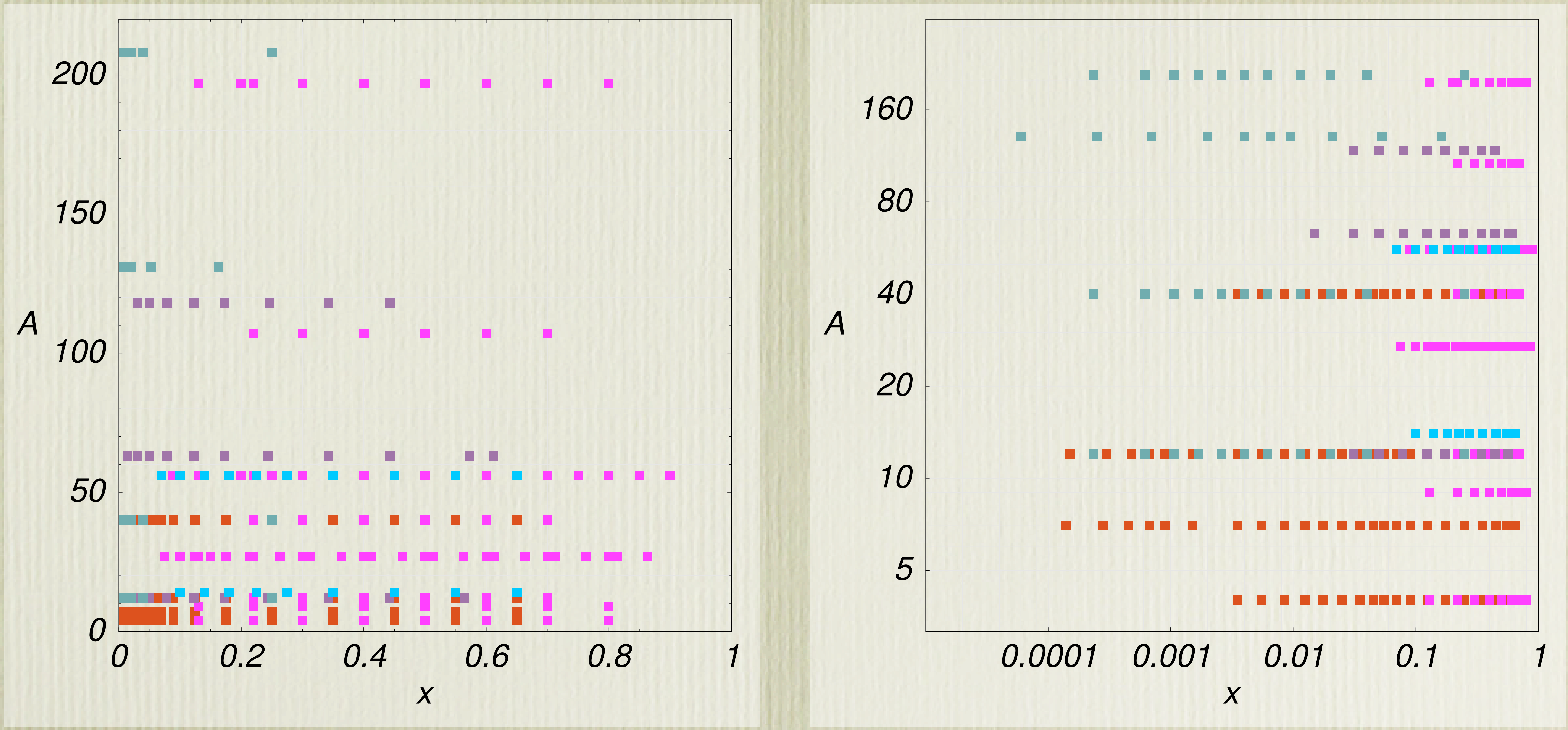
  - Other similar forms equally good

- Fit to world data [EMC, NMC, BCDMS, SLAC, E665] gives $\chi^2/d.o.f. = 1.4 \sim 1.7$ for ~350 data points

  - Simplest analysis: Q^2 dependence as for $F_2(x)$

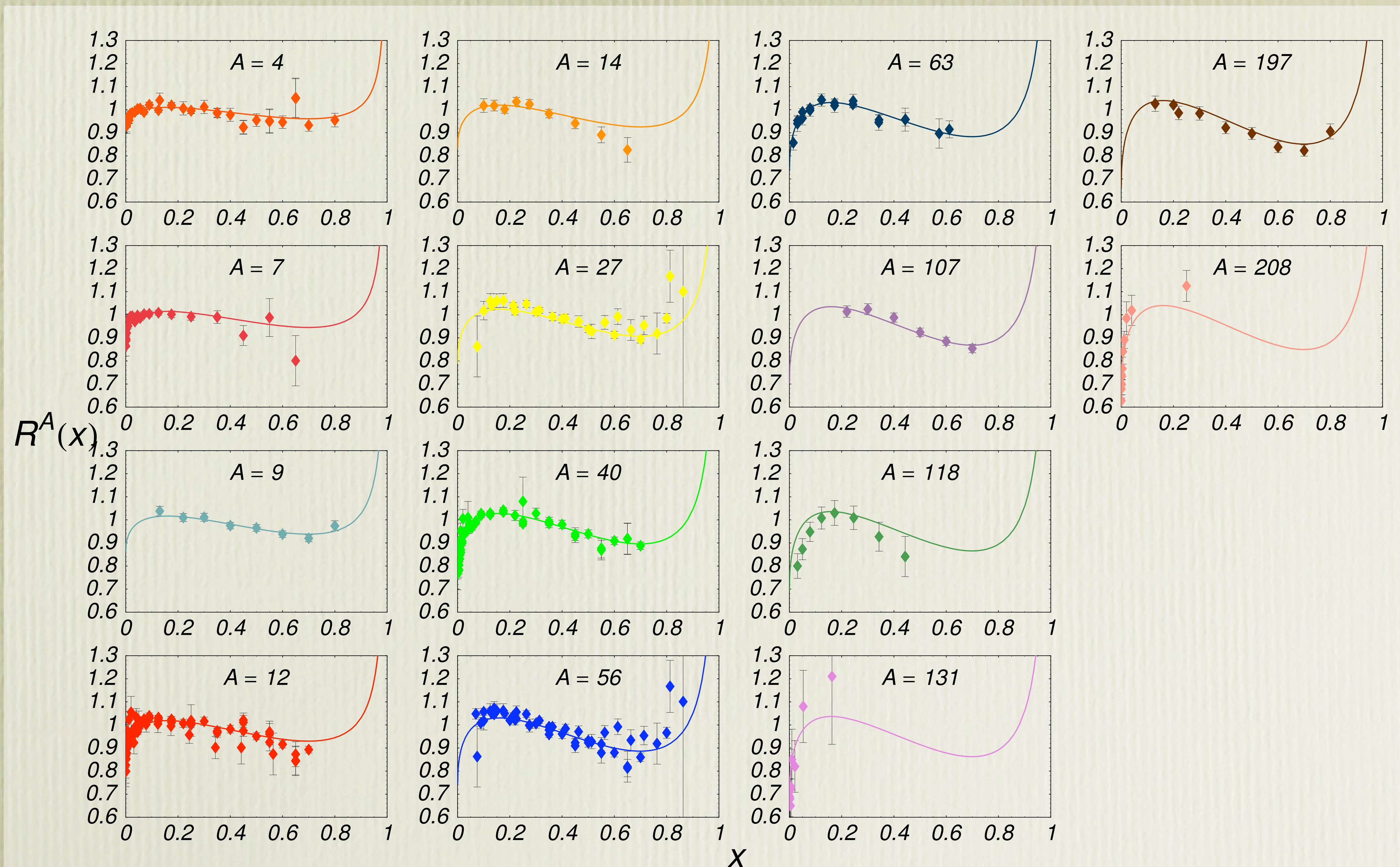
  - Use all available data: no cuts, assume isoscalar

World Data

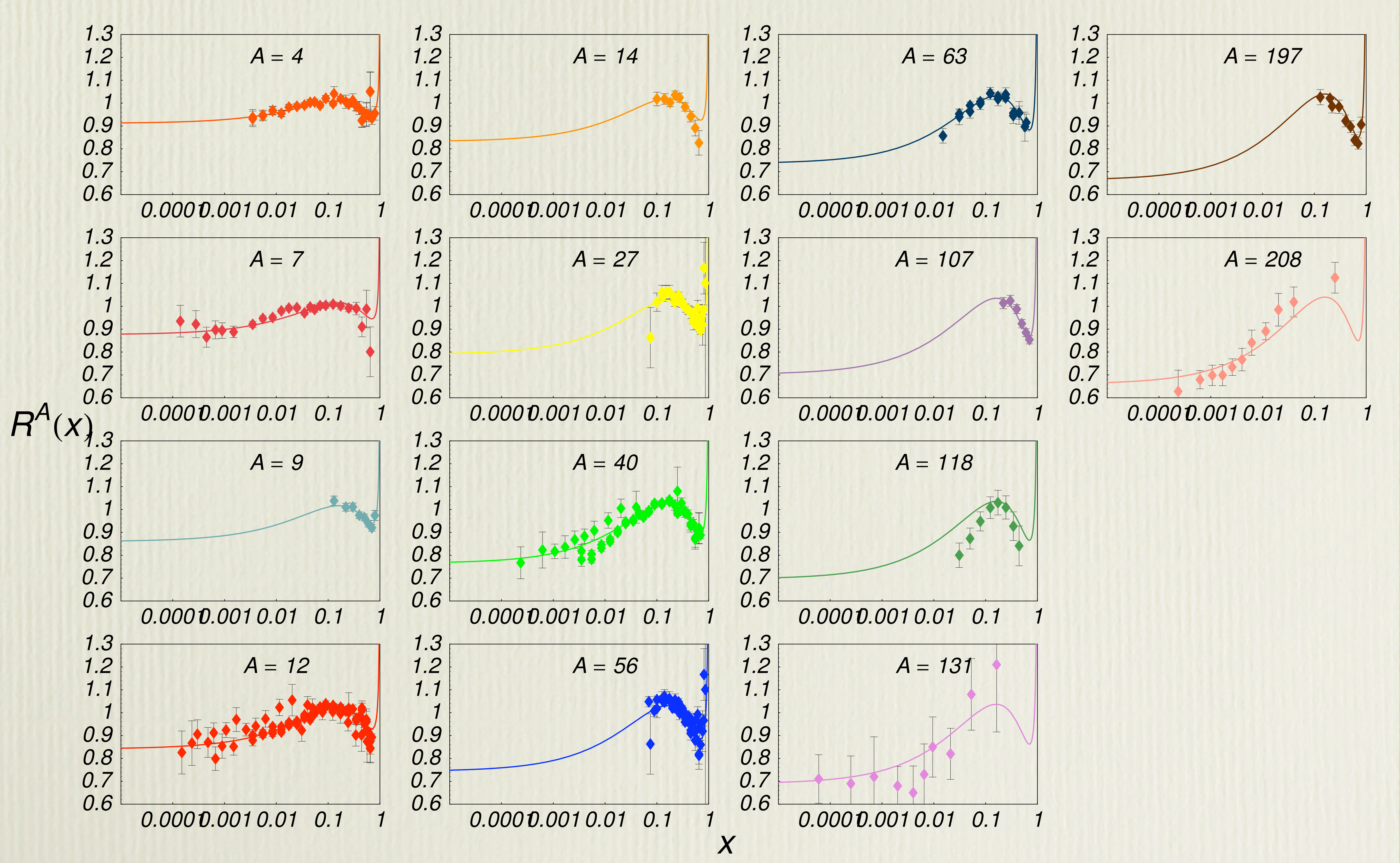


NMC SLAC EMC BCDMS E665

Parameterisations of $F_2^A(x)$



Parameterisations of $F_2^A(x)$



Other nuclear effects

- Independent nuclear modification in
  - Isovector structure function: [MINERνA]
  - Polarised distributions: $g_1(x)$, $b_1(x)$,... [JLab]
  - Generalised parton distributions
- nEFT: similar factorisations occur
- Clear link to nuclear modifications of proton EM form-factors, axial charges

Summary

- EFT applied to EMC effect: twist-two matrix elements
- Find factorisation of x and A dependencies of EMC ratio
- Possible to study EMC effect directly from lattice QCD
- Simple parameterisations support factorisation
- Applies to other PDFs, GPDs and form-factors

Supplementary Slides

Proof of factorisation

$$0 = \frac{d}{dA}\left[\frac{\alpha_n}{\alpha_m}\right] = \frac{d}{dA}\left[\int_{-\infty}^{\infty} dx x^n \tilde{g}(x,A) \Bigg/ \int_{-\infty}^{\infty} dx x^m \tilde{g}(x,A)\right]$$

$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \, (x^m y^n - x^n y^m) \, \tilde{g}(x,A) \frac{\partial}{\partial A} \tilde{g}(y,A)$$

$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \; x^m y^n \left[\tilde{g}(x,A) \frac{\partial}{\partial A} \tilde{g}(y,A) - \tilde{g}(y,A) \frac{\partial}{\partial A} \tilde{g}(x,A)\right]$$

$$\frac{\partial}{\partial A}\left[\frac{\tilde{g}(y,A)}{\tilde{g}(x,A)}\right] = 0$$

$$\widetilde{g}(x,A) = g(x)\mathcal{G}(A)$$

Power Counting