Chaos in the Color Glass Condensate

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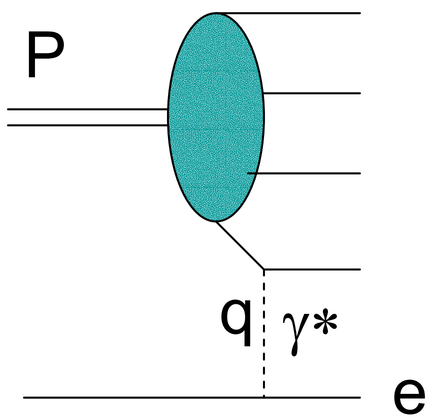
DIS in the Breit frame



$$P = \left(p + \frac{m^2}{2p}, 0, 0, p\right)$$

$$q = (0, 0, 0, -Q)$$

$$x = -\frac{q^2}{2(pq)} = \frac{Q}{2p} << 1$$

- Interaction time $\tau_{int} \sim 1/q_z = 1/Q$
- Life-time of a parton $\tau_{part} \sim k_+/m_t^2$.
- Since $k_z = xp_z$, $\tau_{part} \sim Q/m_t^2$.
- Thus, $\tau_{part} >> \tau_{int}$: photon is a “microscope” of resolution ~1/Q

How many gluons are resolved

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

- Proton's radius $R \sim \ln^{1/2}(s)$
- Density of gluons:

$$\rho(x) = \frac{x^{-\Delta}}{\pi R^2}$$

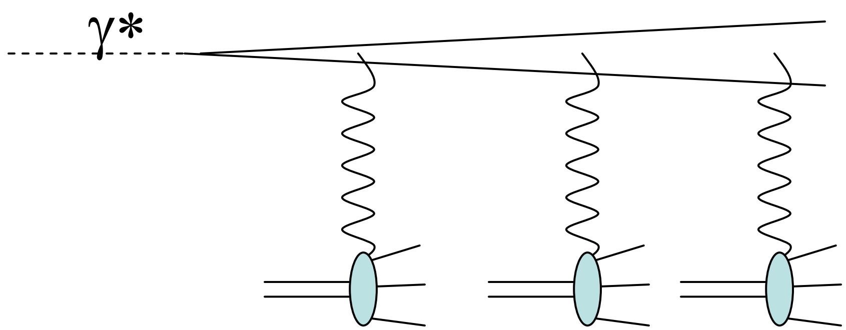
- Number of gluons resolved by a photon:

$$Q_s^2 \quad \frac{x^{-\Delta}}{\pi R^2}\alpha_s^2 \frac{1}{Q^2} \sim 1:$$

High parton density

(Gribov,Levin,Ryskin,82)

Target rest frame



$x << 1 \quad \alpha_s << 1$

- Life-time of a dipole is

$$\tau_{q\bar{q}} \cong \frac{1}{E_{q\bar{q}} - E_{\gamma^*}} \sim \frac{1}{m_N x} >> \tau_{\text{int}}$$

- Total cross section $\sigma(x,Q^2) = \int \frac{d^2 r}{2\pi} dz \Phi^{\gamma^*}(r_\perp, z, Q^2) 2 \int d^2 b \, \mathrm{Im} N(x, r_\perp, b_\perp)$

$\mathrm{Im} N(x, r_\perp, b_\perp)$ is a forward scattering amplitude

- Quasi-classical regime: $\alpha_s \ln(1/x) << 1 \quad \alpha_s^{\,2} A^{1/3} \sim 1$

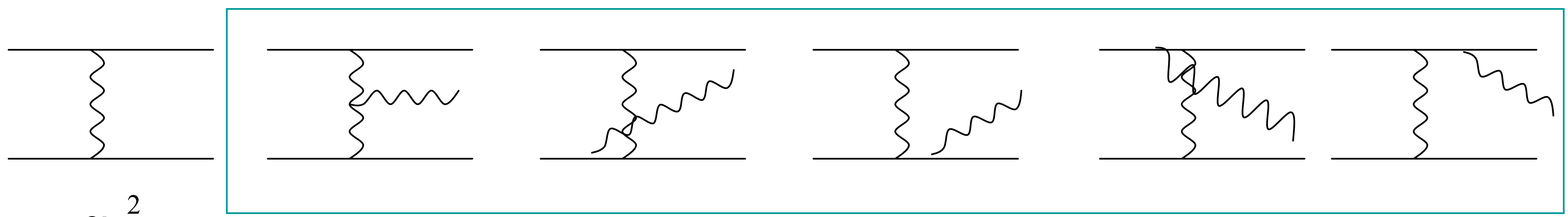
$$N(x, r_\perp, b_\perp) = 1 - \exp\left(-r_\perp^{\,2} Q_s^{\,2}/4\right)$$

(McLerran,Venugopalan,94)

Linear evolution

- High energy linear evolution regime $\alpha_s \ln(1/x) \sim 1 \quad \alpha_s^2 A^{1/3} << 1$

(Fadin,Kuraev,Lipatov,Balitsky,75,78)



α_s^2 $\qquad$ $\alpha_s^3 \ln(1/x)$

- Evolution equation:

$$\partial_y N(x_{01}) = \frac{\alpha_s}{2\pi} \int d^2 x_2 \frac{x_{01}^2}{x_{12}^2 x_{20}^2} \left(N(x_{12}) + N(x_{20}) - N(x_{01}) \right)$$

Operator form of BFKL

- Fourier image of the forward amplitude

$$\varphi(k_\perp, y) = \frac{x_\perp^2}{2\pi} \int d^2k \exp\left(i\vec{k}_\perp \cdot \vec{x}_\perp\right) N(x_\perp, y)$$

- The BFKL equation: $$\frac{\partial \varphi(k_\perp, y)}{\partial y} = \bar{\alpha}_s \hat{\chi}(\hat{\gamma}(k_\perp))\varphi(k_\perp, y)$$

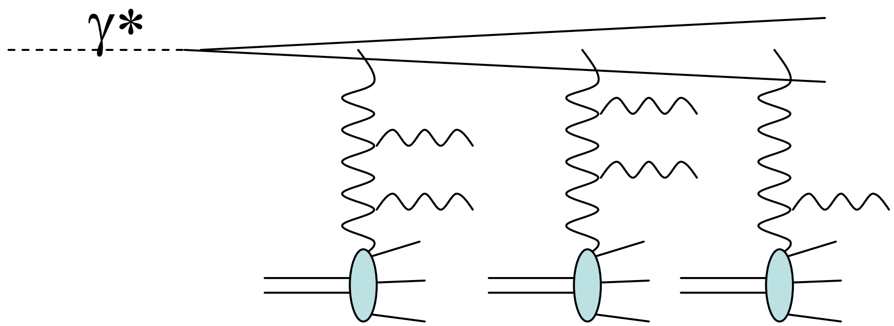
where

$$\hat{\gamma}(k_\perp) = 1 + \frac{\partial}{\partial \ln k_\perp^2}$$

$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma)$$

Evolution in a dense system

- Evolution in a Color Glass Condensate: $\alpha_s \ln(1/x) \sim 1 \quad \alpha_s^2 A^{1/3} \sim 1$



$$\partial_y N(x_{01}) = \frac{\alpha_s}{2\pi} \int d^2 x_2 \frac{x_{01}^2}{x_{12}^2 x_{20}^2} \left(N(x_{12}) + N(x_{20}) - N(x_{01}) - N(x_{12})N(x_{20})\right)$$

(Balitski,Kovchegov,96,00)

- Equivalently:

$$\frac{\partial \varphi(k_\perp, y)}{\partial y} = \overline{\alpha}_s \hat{\chi}(\hat{\gamma}(k_\perp))\varphi(k_\perp, y) - \overline{\alpha}_s \varphi^2(k_\perp, y)$$

(Kovchegov,01)

Discretization of BK equation

(Kharzeev, K.T., 05)

- At small x emission of a gluon into a wave function of a high energy hadron happens when $\alpha_s \ln(1/x) \sim 1$
- Let's impose the boundary condition by putting a system in a box of size $L \sim \Lambda^{-1}$
- We can think of evolution as a *discrete process* of gluon emission when parameter $n = \alpha_s \ln(1/x)$ changes by unity.
- Evolution equation can be written as

$$\varphi_{n+1}(k_\perp) = (\hat{\chi} + 1)\varphi_n(k_\perp) - \varphi_n^{\,2}(k_\perp)$$

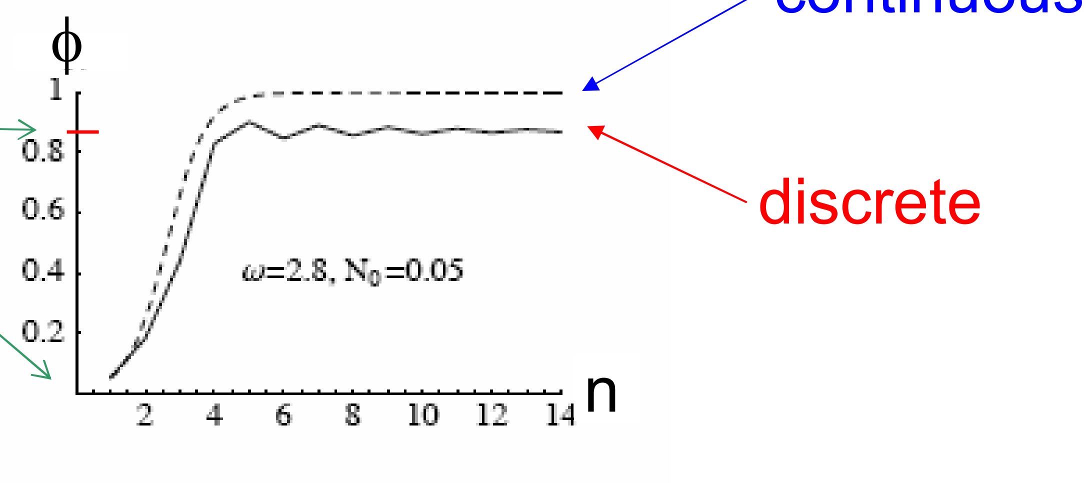
Diffusion approximation

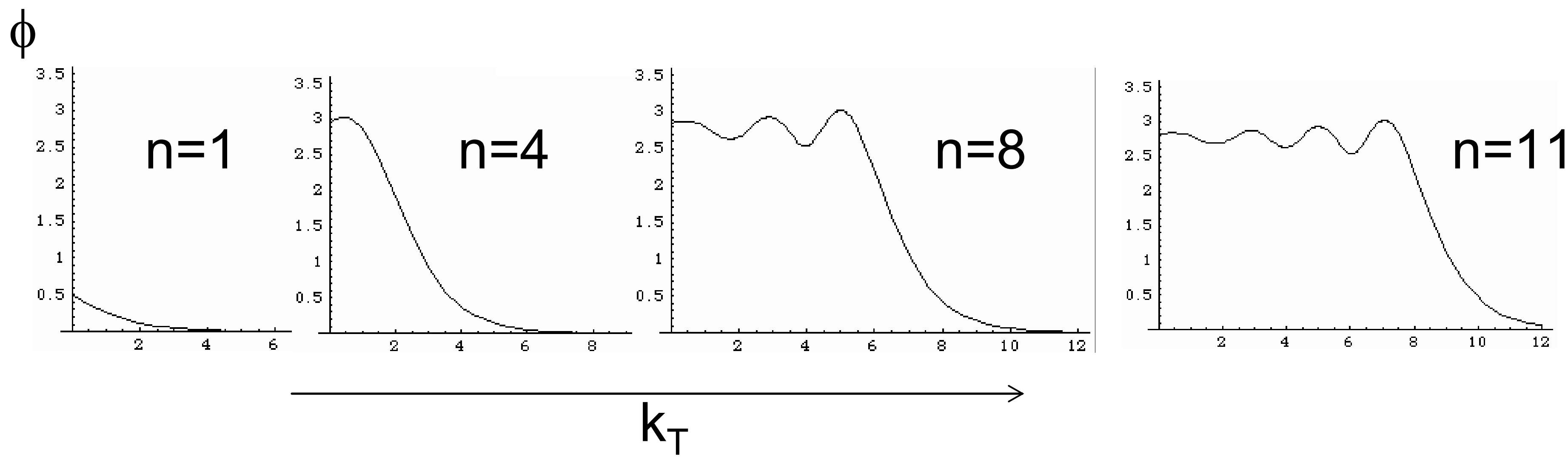
- Diffusion approximation: $\chi(\gamma) \approx 4\ln 2 + 14\zeta(3)(\gamma - 1/2)^2 + \ldots$
- Let's keep only the first term $\chi(\gamma) = \chi(1/2) \equiv \omega - 1.$
- Rescale $\varphi_{Y} = \phi_{Y}\omega$
- Discrete equation: $\phi_{Y}(\vec{k}_\perp) = \omega\phi_{Y}(\vec{k}_\perp)(1 - \phi_{Y}(\vec{k}_\perp))$
- For fixed k_T this is the 'logistic map'.
  - It is used to describe population growth in the ecological systems.
  - It's properties are very different from those of the continuous equation. (von Neumann,47)

$1<\omega<3$

- Stable fixed point:
- Unstable fixed point:

$$\phi_{n+1} = \phi_n \Rightarrow \quad \phi_{n,fixed} = \begin{cases} (\omega - 1)/\omega \\ 0 \end{cases}$$





$3<\omega<3.442$

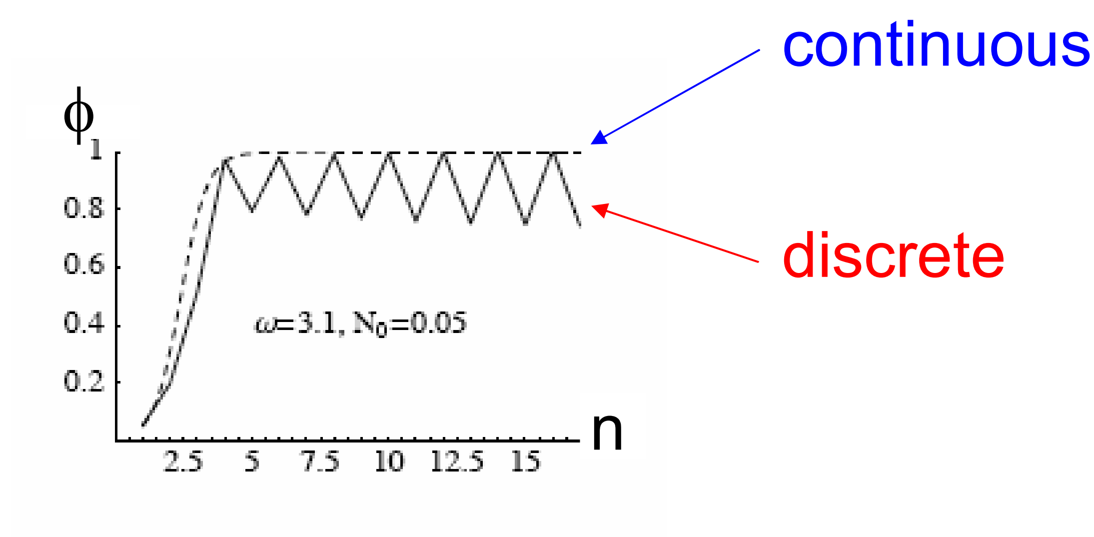
- $\omega=3$ is a bifurcation point: fixed point condition admits two new solutions (period doubling).

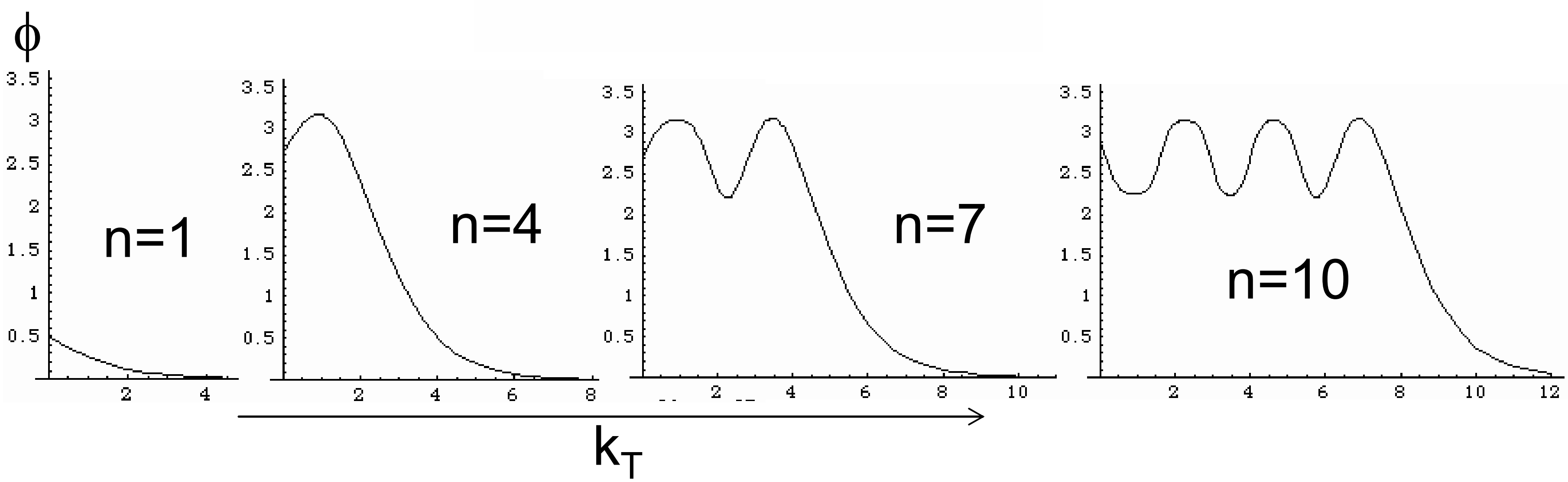
- Unstable fixed points $\phi_{n,fixed} = \begin{cases} (\omega-1)/\omega \\ 0 \end{cases}$

- Stable fixed points: $\phi_{n+2} = \phi_{n} \Rightarrow$

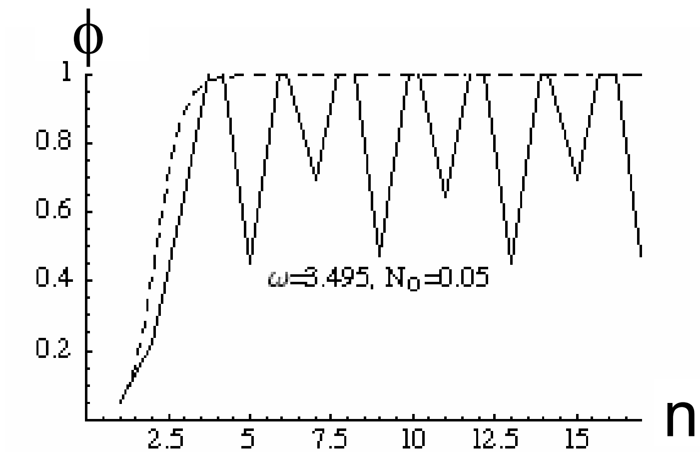
$$\phi_{n,fixed} = \frac{\omega + 1 \pm \sqrt{\omega^2 - 2\omega - 3}}{2\omega}$$

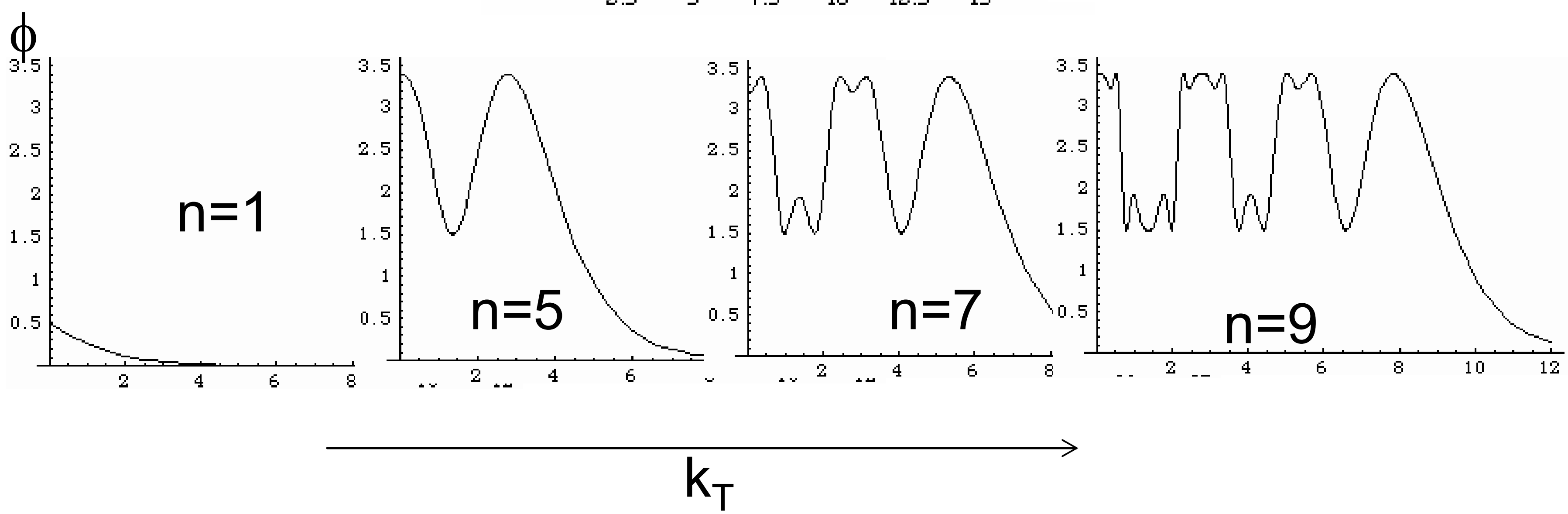
Period doubling





$3.442<\omega<3.56$

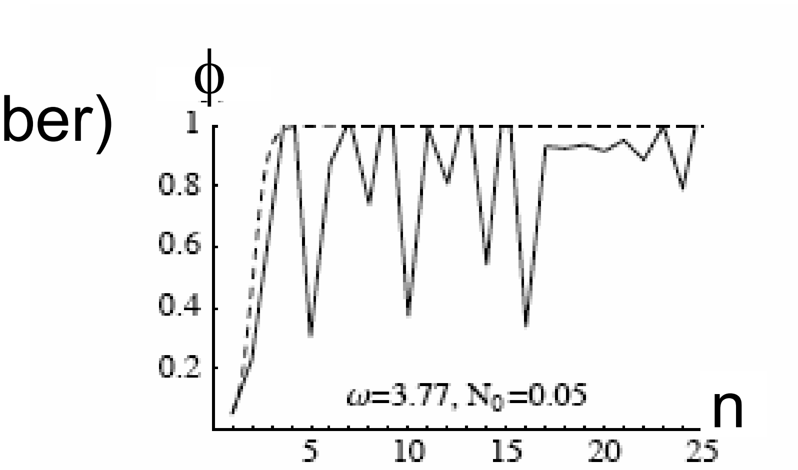


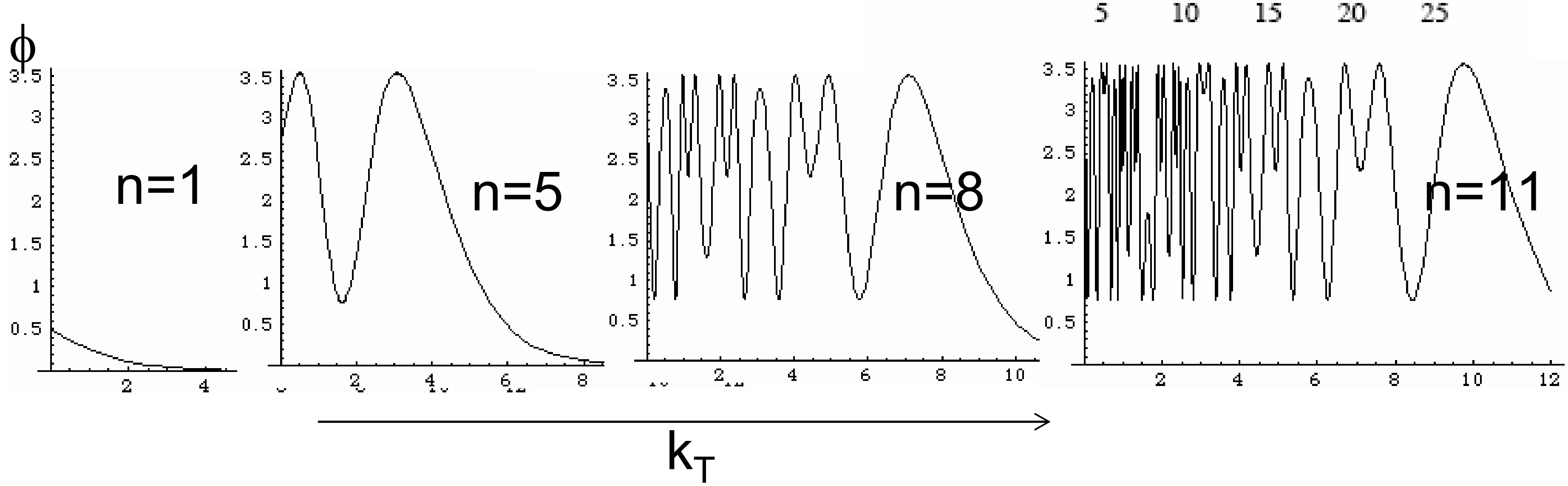


Onset of chaos

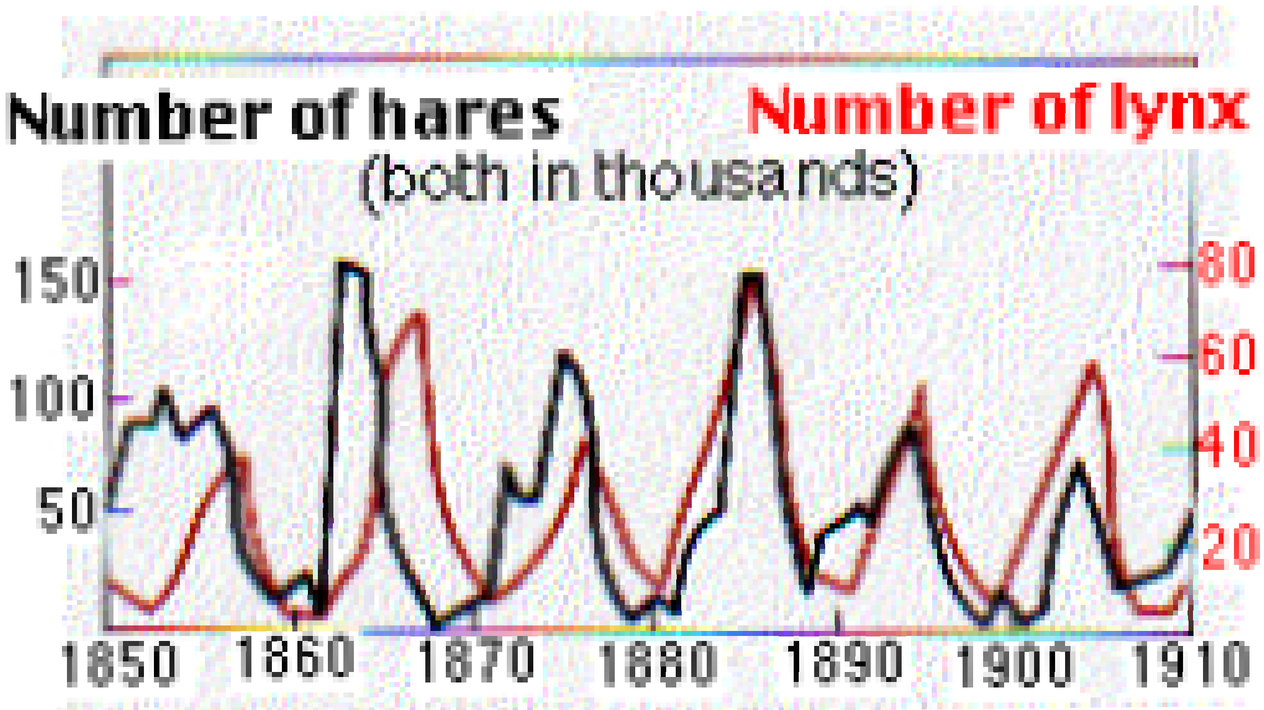
$\omega > \omega_F = 3.569$ (Feigenbaum's number)

- In pertubative QCD:
  $\omega > \omega_{min} = 1 + 4 \times \ln 2 = 3.77 > \omega_F$





Chaos in ecology



Canadian Lynx population (Hudson Bay Company's archives)

Bifurcation diagram

Fixed points

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Note large fluctuations

ω

High energy evolution starts here.

Implication to diffraction

- Diffraction cross section is the statistical dispersion in the absorption probabilities of different eigenstates.

$$\sigma_{diff} = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$

- Large fluctuations in the scattering amplitude imply large target independent diffractive cross sections at highest energies.

Summary

- ✓ Optimistic/pessimistic point of view:
  there are an interesting non-linear effects in the
  Color Glass Condensate beyond the continuum limit.

- ✓ Pessimistic/optimistic point of view:
  appearance of chaos in the high energy evolution
  signals breakdown of a perturbation theory in vacuum.