From Dense-Dilute Duality to Selfduality in high energy evolution.

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With Michael Lublinsky

Evolution of S-matrix at High Energy

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Ancient history: Reggeon field theory (Genya knows...)

Very old stuff: JIMWLK equation (I remember...) - evolution of S - matrix of partonic (dilute) projectile on a dense target, $\rho \sim O(1/\alpha_s^2)$.

Burst of papers in 04-05 : JIMWLK (Gym walk) has no "Pomeron loops", let's fix it (finite density effects in the projectile wave function)/(more accurate gluon emissions in a smaller target) [Mueller + Shoshi, Wang, Levin, Blaizot, Itakura, McLerran, Stasto, Hatta, Iancu, Triantafilloupolos...]

Most complete so far (M. Lublinsky, A.K.) : KLWMIJ (Claw Midge) - evolution of the S-matrix of dilute target for arbitrary projectile.

JIMWLK and KLWMIJ are explicitly dual: **DENSE-DILUTE DUALITY**

The complete evolution (Pomeron loops≡arbitrary target/projectile densities) must be

SELFDUAL

As always we are catching up with Ian Balitsky - he knew it 5 years ago...

JIMWLK

Target	Projectile
$|T\rangle \rightarrow$	$\leftarrow \langle P|$
Large ρ^+	Some ρ^-

Large color charge density in the target, so large target fields (A^+)

$$\alpha^a(x,x^-)T^a \;=\; \frac{1}{\partial^2}(x-y) \quad U^\dagger(y,x^-)\;\rho^{+a}(y,x^-)\,T^a\;U(y,x^-)$$

$$U(x,x^-) \;=\; \mathcal{P}\,\exp\{i\int_0^{x^-} dy^- T^a\alpha^a(x,y^-)\}$$

$U(x,x^-=1)$- eikonal scattering matrix for a fast parton scattering on $|T\rangle$. To calculate the S-matrix of projectile partons:

$$\langle T|U(x_1)...U(x_n)\,|T\rangle \;=\; \quad D\rho^{+a}\;W^T[\rho^+(x^-,x)]U(x_1)...U(x_n)$$

$W^T[\rho]$ - target "probability density" functional (if used carefully)

Increase collision energy $\longrightarrow$ boost the target $|T\rangle$. The probability density changes:

$$\frac{\partial}{\partial Y}W^T \;=\; \chi^{JIMWLK}[\alpha,\frac{\delta}{\delta\alpha}]\;W^T[\rho^+]$$

$$\chi^{JIMWLK}=\frac{\alpha_s}{2\pi^2}\quad {}_{x,y,z}\frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2}\times$$

$$\frac{\delta}{\delta\alpha^a(x,0)}\frac{\delta}{\delta\alpha^a(y,0)}+\frac{\delta}{\delta\alpha^a(x,1)}\frac{\delta}{\delta\alpha^a(y,1)}$$

$$-2\frac{\delta}{\delta\alpha^a(x,0)}U(x,1)^{ab}\frac{\delta}{\delta\alpha^b(y,1)}$$

Operators $\frac{\delta}{\delta\alpha}$ are "isospin rotations"

$$\alpha^c(x,0)U(x,1)^{ab}=-i[T^cU(x,1)]^{ab},\quad \text{left rotation on S matrix}$$

$$\alpha^c(x,1)U(x,1)^{ab}=-i[U(x,1)T^c]^{ab},\quad \text{right rotation on S matrix}$$

KLWMIJ

Contrariwise: assume ρ^+ is small, but projectile is arbitrary

$$\frac{\partial}{\partial Y} W^T \;=\; \chi^{KLWMIJ}[\alpha, \frac{\delta}{\delta\alpha}]\; W^T[\rho^+]$$

$$\chi^{KLWMIJ} = -\frac{\alpha_s}{2\pi^2} \quad {}_{x,y,z} \frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2} \times$$
$$\rho^a(x,0)\rho^a(y,0) + \rho^a(x,1)\rho^a(y,1) - 2\,\rho^a(x,0)R^{ab}(z,1)\rho^b(y,1)$$

with

$$R^{ab}(z,1) = \quad \mathcal{P}e^{\;{}_{-\infty}^{\infty} dx^- T^c \frac{\delta}{\delta\rho^c(x^-,z)}} \quad {}^{ab}$$

R is the eikonal scattering matrix of the whole (POSSIBLY DENSE) projectile on a SINGLE PARTON of the target.

DDD

STARE IN WONDER:

$$\chi^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \quad {}_{x,y,z} \frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2} \times$$
$$\frac{\delta}{\delta\alpha^a(x,0)}\frac{\delta}{\delta\alpha^a(y,0)} + \frac{\delta}{\delta\alpha^a(x,1)}\frac{\delta}{\delta\alpha^a(y,1)}$$
$$-2\frac{\delta}{\delta\alpha^a(x,0)}U(x,1)^{ab}\frac{\delta}{\delta\alpha^b(y,1)}$$

$$\chi^{KLWMIJ} = -\frac{\alpha_s}{2\pi^2} \quad {}_{x,y,z} \frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2} \times$$
$$\rho^a(x,0)\rho^a(y,0) + \rho^a(x,1)\rho^a(y,1) - 2\,\rho^a(x,0)R^{ab}(z,1)\rho^b(y,1)$$

DENSE-DILUTE DUALITY:

$$(\alpha, \frac{\delta}{\delta\alpha}) \leftrightarrow (-i\frac{\delta}{\delta\rho}, i\rho)$$

In fact it is

SELFDUALITY

Assume: EIKONAL SCATTERING

Use: LORENTZ INVARIANCE and PROJECTILE-TARGET DEMOCRACY

Result: For arbitrary dense projectile on arbitrary dense target must be

$$\chi[\alpha, \frac{\delta}{\delta\alpha}] = \chi[i\frac{\delta}{\delta\rho}, -i\rho]$$

S - MATRIX:

To calculate S-matrix, have to average the S-matrix operator over the projectile and target wave functions:

$$S(Y) \;=\; \langle T|\,\langle\, P|\;\hat{s}(\rho^t,\,\rho^p)\;|P\rangle\,|T\rangle$$

Step by step:

Average over the Projectile:

$$\Sigma^P(\rho^t) \;=\; \langle\, P|\;\hat{s}(\rho^t,\,\rho^p)\;|P\rangle$$

Eikonal approximation:

$$\Sigma^P[\rho^t] \;=\; \langle P|\,\mathcal{P}e^{i\quad dx^-\quad d^2x\rho^p(x)\alpha(x,x^-)}\,|P\rangle$$

Σ - is a function of the target degrees of feedom!

A. Kovner

For total rapidity Y, the projectile takes on $Y - Y_0$ and the target Y_0

$$S(Y) \ = \ \int D\rho^t \ \Sigma^p_{Y\,-\,Y_0}[\rho^t] \ W^t_{Y_0}[\rho^t]$$

Boost $\qquad |P\rangle_Y \ \rightarrow \ |P\rangle_{Y\,+\,\delta Y}$

$$\frac{\partial}{\partial Y}\Sigma^P \ = \ \chi^\dagger[\alpha, \frac{\delta}{\delta\alpha}] \ \Sigma^P[\alpha]$$

S-matrix evolves

$$\frac{\partial}{\partial Y}\mathcal{S}_Y = \qquad D\rho^t(x, x^-) \ W^T_{Y_0}[\rho^t(x^-, x)] \quad \chi^\dagger[\alpha, \frac{\delta}{\delta\alpha}] \ \Sigma^P_{Y-Y_0}[\alpha]$$

$$= \qquad D\rho^t(x, x^-) \quad \chi[\alpha, \frac{\delta}{\delta\alpha}] \ W^T_{Y_0}[\rho^t(x^-, x)] \quad \Sigma^P_{Y-Y_0}[\alpha]$$

(Last equality - integration by parts)

LORENTZ INVARIANCE:

$$\partial \mathcal{S}/\partial Y_0 = 0$$

leads to

$$\frac{\partial}{\partial Y} W^T = \chi[\alpha, \frac{\delta}{\delta\alpha}]\, W^T[\rho^t]$$

But $|P\rangle$ has an W too! How does it change?

$$\Sigma^P[\alpha] = \langle P|\, \mathcal{P} e^{i \int dx^- \int d^2x \rho^p(x)\alpha(x,x^-)}\, |P\rangle\,.$$

and

$$\frac{\partial}{\partial Y}\Sigma^P = \chi^\dagger[\alpha, \frac{\delta}{\delta\alpha}]\, \Sigma^P[\alpha]$$

follows:

$$\frac{\partial}{\partial Y} W^P = \chi[-i\frac{\delta}{\delta\rho}, i\rho]\, W^P[\rho^p]$$

PROJECTILE-TARGET DEMOCRACY

No hadron is more equal than other:

$|P\rangle$ and $|T\rangle$ must change the same!

$$\chi[-i\frac{\delta}{\delta\rho}, i\rho] = \chi[\alpha, \frac{\delta}{\delta\alpha}]$$

Do we know χ? - NOT YET.

But we are working on it.

Hopefully when we find it, the selfduality will help us to solve the evolution.