Scheme-Invariant Evolution to NNLO

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Abstract. The (Factorization)-Scheme-Invariant analysis of unpolarized DIS structure functions is presented as a method to reduce theoretical errors on the determination of the strong coupling constant α_s, in order to match the accuracy foreseen for determinations coming from future high statistics measurements.

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1. INTRODUCTION

The final HERA-II data on unpolarized DIS structure functions, combined with the present world data, will allow to reduce the experimental error on the strong coupling constant, α_s, to the level of 1% [1]. On the theoretical side the NLO analyzes have limitations due to the scale variations being present which allow no better than 5% accuracy in the determination of α_s. In order to match the expected experimental accuracy analyzes of DIS structure functions need then to be carried out to NNLO level.

To perform a full NNLO analysis the knowledge of the 3-loop β-function coefficient, β_2, the 2-(resp. 3-)loop Wilson coefficients and the 3-loop anomalous dimensions is required. With the calculation of the latter [2], the whole scheme-independent set of quantities is known, thus allowing a complete NNLO study of DIS structure functions.

While pushing the analysis one order further in perturbation theory will help reducing the theoretical errors, we think that other ways of reducing theoretical and conceptual uncertainties should be seeked. In this perspective we are persuaded that combining the standard QCD analysis and fits based on scheme-invariant evolution will provide a valuable tool in high-precision analyzes aiming to 1% accuracy in the determination of α_s. The advantage of scheme-invariant evolution consists in the fact that the input distributions are *physical observables* extracted from data at a reference scale Q_0^2. They are then evolved through evolution equations with physical anomalous dimensions and in the end a one-parameter fit to the data is performed to determine α_s.

The aim of our work is to perform a full NNLO analysis of unpolarized DIS structure functions in order to obtain a high-accuracy determination of α_s and to extract a set of parton distribution functions (PDFs) with fully correlated errors.

A complete study of structure functions data includes taking into account singlet and non-singlet evolution. We refer the reader interested in the non-singlet analysis to [3] and in the present letter we will concentrate on the singlet sector with particular reference to the scheme-invariant approach.

2. PHYSICAL ANOMALOUS DIMENSIONS

The coupled evolution of the singlet quark and gluon parton distribution functions can be mapped into the evolution of a pair of structure functions, related to the PDFs via convolution with the Wilson coefficients:

$$\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right) = \left(\begin{array}{cc} C_{A,\Sigma}^N & C_{A,g}^N \\ C_{B,\Sigma}^N & C_{B,g}^N \end{array}\right) \left(\begin{array}{c} \Sigma^N \\ G^N \end{array}\right). \tag{1}$$

The observables satisfy the matrix evolution equation:

$$\frac{d}{dt}\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right) = -\frac{1}{4}\mathbf{K}^N \left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right), \qquad t = -\frac{2}{\beta_0}\ln\frac{a_s(Q^2)}{a_s(Q_0^2)}, \qquad a_s = \frac{\alpha_s}{4\pi}. \tag{2}$$

The physical anomalous dimensions K_{IJ} are given in terms of the Wilson coefficients $C_{I,k}^N$ and the unpolarized anomalous dimensions γ_{ij}^N by

$$K_{IJ}^N = \left[-4\frac{\partial C_{I,m}^N(t)}{\partial t}\left(C^N\right)_{m,J}^{-1}(t)\frac{\beta_0 a_s(Q^2)}{2\beta(a_s(Q^2))}C_{I,m}^N(t)\gamma_{mn}^N(t)\left(C^N\right)_{n,J}^{-1}(t)\right]. \tag{3}$$

While the anomalous dimensions and the Wilson coefficients are, separately, factorization scheme dependent quantities, the physical anomalous dimensions are factorization scheme invariants. Equation (2) relates physical quantities, namely the structure functions and their slopes. This means that the physical kernels are also physical quantities and thus free of soft or collinear divergences. They can be expanded perturbatively in powers of the coupling constant $a_s(\mu^2)$ and, when computed in fixed-order perturbation theory they retain a dependence on the renormalization scale which can be used to estimate the theoretical error due to higher order corrections.

Different pairs of structure functions can be taken into consideration:

- F_2 and $\partial F_2/\partial t$ [4];
- F_2 and F_L [5, 6].

In [7] we presented the physical anomalous dimensions for the coupled evolution of the structure functions F_2 and $\partial F_2/\partial t$ up to NNLO. We also computed the ones for the coupled evolution of F_2 and F_L but, due to space limitations, we refer the interested reader to [8], where the complete expressions will be given.

3. HEAVY FLAVOURS CONTRIBUTION

It is known that the heavy flavour contribution to DIS structure functions in the kinematical regime of HERA is sizable. The electromagnetic structure function F_2, for example, receives contributions from heavy flavours of 20 – 40%, depending on the actual event kinematics, which requires a careful account for the heavy flavours contributions.

Since we solve the evolution equation for scheme-invariant evolution in Mellin space we implement heavy flavour contributions using the parametrization derived in [9]. The

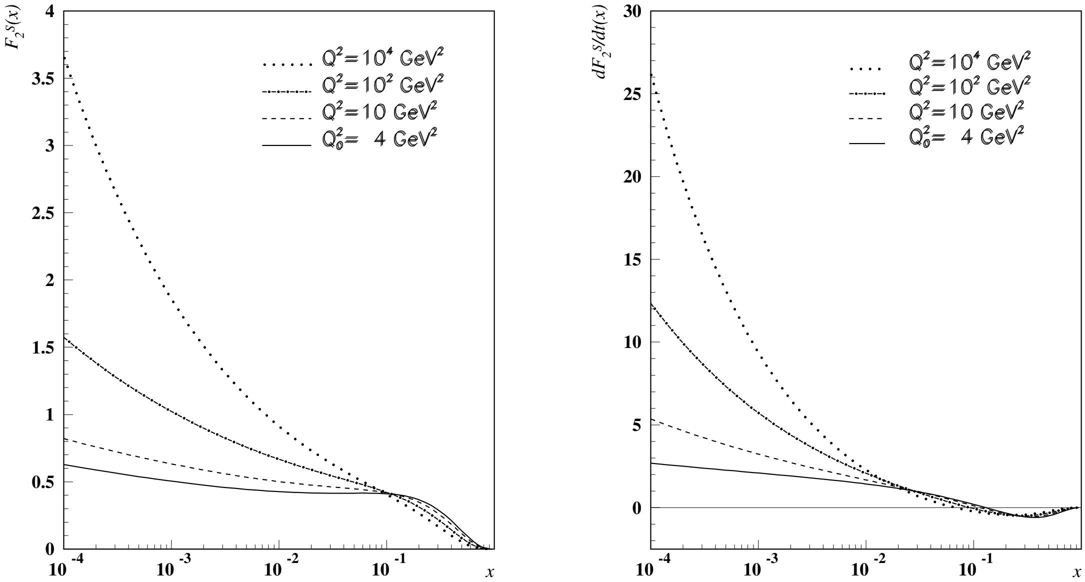

FIGURE 1. NNLO scheme invariant evolution for the singlet part of the structure function F_2 and its slope $\partial F_2/\partial t$ for four massless flavours.

inclusion into a scheme-invariant analysis requires, beyond the one of the heavy flavours Wilson coefficients, the knowledge of their derivatives with respect to the evolution variable t, defined in (2).

4. NUMERICAL RESULTS

As far as the numerical implementation of factorization scheme invariant evolution is concerned, we concentrated on the F_2 and $\partial F_2/\partial t$ system. We implemented the evolution to NNLO for massless flavours and we are on the way to include heavy flavours contributions.

In Fig. 1 we present the scheme invariant evolution for the structure functions F_2 and $\partial F_2/\partial t$ to NNLO. The input distribution at the reference scale are not extracted from data, but rather built up as a convolution of Wilson coefficients and PDFs, the latter being parametrised according to [10]. Comparing the behaviour of the slopes in Fig. 1b to the slopes extracted experimentally in Fig. 12 of [11] points towards a positive gluon density in the small x region.

5. CONCLUSIONS

The future high precision HERA-II data will allow a reduction of the experimental error on the determination of the strong coupling constant α_s to the level of 1%. On the theoretical side, the inclusion of NNLO corrections is therefore mandatory to cope with such a level of accuracy.

In view of a high accuracy determination of α_s we think that combining standard $\overline{MS}$ analysis with fits based on factorization-scheme invariant evolution could provide a method to have a better control on theoretical and conceptual errors.

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