NNLO Scheme Invariant Evolution of Unpolarized DIS Structure Functions

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Talk Outline

- Motivation
- QCD Evolution Equations
- (Factorization-)Scheme-invariant Evolution
- Numerical Results
- Conclusions

Motivation

- The final HERA-II data, together with the world data, will allow to reduce experimental errors on α_s to $\sim 1\%$
- On the other side the theoretical error on the determination of the strong coupling constant in NLO analyses is $\Delta\alpha_\mathrm{s} \sim 5\%$
- In order to match the claimed experimental accuracy NNLO results are necessary on the theoretical side
- Our aim is to perform both a Standard and a Scheme Invariant analysis of unpolarized DIS structure functions in order to:
  - Determine α_s with an accuracy of $\mathcal{O}(1\%)$
  - Extract the parton distribution functions with fully correlated errors

QCD Evolution Equations

- Evolution Equations of DIS Structure Functions do exhibit factorization and renormalization scheme dependencies

- Renormalization scheme dependence is removed only if the perturbative series is summed to all orders

- When considering factorization scheme dependence we have two viable approaches
  - Consider process-independent scheme-dependent evolution equations for PDFs (Standard QCD analysis)
  - Consider process-dependent scheme-independent evolution equations for observables (Scheme Invariant analysis)

Standard QCD analysis

- Introduce a parametrization of the PDFs at a given reference scale
- Evolve the PDFs to the scale Q^2 via evolution equations for mass factorization
- Build structure functions as a combination of PDFs and Wilson Coefficients
- Perform a multi-parameter fit to the data to determine the PDF parameters and α_{s}

Scheme Invariant Evolution

- Extract the parametrization of observable quantities at the initial scale Q_0^2 from data
- Determine the value of the observables at the scale Q^2 using evolution equations with physical anomalous dimensions
- Perform a one-parameter fit to the data to determine α_{s}

The two analyses are complementary and performing both of them will help reduce the theoretical and conceptual errors

Our notation

$$a_s(\mu^2) \equiv \frac{\alpha_s(\mu^2)}{4\pi} \qquad\qquad \mu^2 \frac{d a_s(\mu^2)}{d\mu^2} = -\sum_{n=0}^{\infty} \beta_n a_s^{n+2}(\mu^2)$$

where, for $SU(3)$ we have

$$\beta_0 = 11 - \frac{2}{3}N_f \qquad \beta_1 = 102 - \frac{38}{3}N_f$$
$$\beta_2 = \frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2$$

The expanded form for the coupling constant up to 3-loop is

$$a_s(Q^2) = \frac{1}{\beta_0 L}\left\{1 - \frac{\beta_1 \ln L}{\beta_0^2 L} + \frac{\beta_1^2 \ln^2 L - \beta_1^2 \ln L + \beta_2\beta_0 - \beta_1^2}{\beta_0^4 L^2}\right\}$$

where

$$L = \ln \frac{Q^2}{\Lambda^2_{QCD}}$$

- We work in Mellin space, where the Mellin transform of a function is defined as

$$f(N) = \int_0^1 dx x^{N-1} f(x)$$

while anomalous dimensions are related to DGLAP splitting functions through

$$\gamma_{ij}^N = -2 \int_0^1 dx x^{N-1} P_{ij}(x)$$

- Working in Mellin space is instrumental (almost necessary) for numerical implementation

$$\left(P_{qg}^{(0)}\right)^{-1}(x) \otimes f(x) = 4f(x) - 2xf'(x) - 4p(x) \otimes f(x)$$

with

$$p(x) = \sqrt{x}\left[\cos\left(\frac{\sqrt{7}}{2}\ln x\right) - \frac{3}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}\ln x\right)\right]$$

- The coupled evolution of the singlet and gluon parton distributions can be mapped into the evolution of a pair of structure functions

$$\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t) = \begin{pmatrix} C_{A,\Sigma}^N & C_{A,g}^N \\ C_{B,\Sigma}^N & C_{B,g}^N \end{pmatrix} \begin{pmatrix} \Sigma^N \\ G^N \end{pmatrix}(t)$$

- The observables, then, satisfy the matrix evolution equation

$$\frac{d}{dt}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t) = -\frac{1}{4}\mathbf{K}^N \begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t)\,, \qquad t = -\frac{2}{\beta_0}\ln\frac{a_s(Q^2)}{a_s(Q_0^2)}$$

and the physical anomalous dimensions are

$$K_{IJ}^N = \left[-4\frac{\partial C_{I,m}^N(t)}{\partial t}\left(C^N\right)_{m,J}^{-1}(t) - \frac{\beta_0 a_s(Q^2)}{\beta(a_s(Q^2))} C_{I,m}^N(t)\gamma_{mn}^N(t)\left(C^N\right)_{n,J}^{-1}(t)\right]$$

The physical anomalous dimensions K_{IJ}^N are factorization scheme invariants

Physical Anomalous Dimensions - Properties

- The physical anomalous dimensions are observables and thus free of soft or collinear divergences

- They can be expanded in powers of the coupling $a_s(\mu^2)$

$$K_{IJ}^{N} = \sum_{n=0}^{\infty} \left[a_s(\mu^2)\right]^n K_{IJ}^{N(n)}$$

- When computed in fixed-order perturbation theory the remainig dependence on the renormalization scale can be used to estimate the error due to higher order corrections

- When considering scheme invariant evolution different pairs of structure functions can be chosen:
  - F_2, $\partial F_2/\partial t$
  - F_2, F_L
  - g_1, $\partial g_1/\partial t$ (in polarized DIS)

Leading Order:

[W. Furmanski and R. Petronzio, Z. Phys. C11, (1982), 293]

$$K_{22}^{N(0)} = 0 \qquad K_{2d}^{N(0)} = -4$$

$$K_{d2}^{N(0)} = \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \qquad K_{dd}^{N(0)} = \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}$$

Next-to-Leading Order:

$$
\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right) \\
&\quad -\frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right) \\
&\quad -\frac{2\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) + \frac{\beta_0}{2} C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad -\frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\left(\gamma_{qq}^{N(0)}\right)^2 - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) \\
&\quad -\frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}
$$

Next-to-next-to-Leading Order:

$$
\begin{aligned}
K_{22}^{N(2)} &= K_{2d}^{N(2)} = 0 \\
K_{d2}^{N(2)} &= \frac{1}{4}\left(\gamma_{qq}^{N(2)}\gamma_{gg}^{N(0)} + \gamma_{qq}^{N(0)}\gamma_{gg}^{N(2)} - \gamma_{qg}^{N(2)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(2)} + \gamma_{qq}^{N(1)}\gamma_{gg}^{N(1)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(1)}\right) \\
&+\frac{\beta_0}{2}\left[C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\right) - \left(C_{2,q}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) - 3C_{2,g}^{N(1)}\gamma_{gq}^{N(1)}\right] \\
&-\beta_0\left[\gamma_{qq}^{N(2)} + 2\gamma_{gq}^{N(0)}\left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right) - C_{2,q}^{N(2)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right)\right] \\
&+\beta_0^2\left[3\left(C_{2,q}^{N(1)}\right)^2 - 4C_{2,q}^{N(2)}\right] + \frac{\beta_1}{2}\left[\gamma_{qq}^{N(1)} - C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} + 2\beta_0\right) + 2C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right] \\
&-\frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)} + \gamma_{qq}^{N(0)}\gamma_{gg}^{N(1)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right) \\
&+\frac{3}{4}\frac{\beta_1^2}{\beta_0^2}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) - \frac{\beta_2}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
&+\frac{1}{\gamma_{qg}^{N(0)}}\left\{\frac{\beta_1}{2}\gamma_{qg}^{N(0)}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right] + 2\beta_0^3 C_{2,q}^{N(1)}C_{2,g}^{N(1)}\right. \\
&+\beta_0^2\left[4\gamma_{qg}^{N(0)}\left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right) - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + C_{2,q}^{N(1)}\gamma_{qg}^{N(1)}\right. \\
&\left.+C_{2,g}^{N(1)}\gamma_{qq}^{N(1)} + \left(C_{2,g}^{N(1)}\right)^2\gamma_{gq}^{N(0)}\right] \\
&+\beta_0\left[C_{2,g}^{N(1)}C_{2,q}^{N(1)}\gamma_{qq}^{N(0)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + \gamma_{qq}^{N(0)}\left(C_{2,g}^{N(1)}\gamma_{gg}^{N(1)} + C_{2,g}^{N(2)}\gamma_{gg}^{N(0)}\right)\right]
\end{aligned}
$$

$$+\frac{\beta_0}{2}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)}+\gamma_{gq}^{N(0)}\gamma_{qg}^{N(1)}-3\gamma_{qq}^{N(0)}\gamma_{qq}^{N(1)}\right)+\gamma_{qq}^{N(1)}\gamma_{qg}^{N(1)}\right.$$

$$\left.+\left(C_{2,g}^{N(1)}\right)^2\left(\gamma_{gg}^{N(0)}\gamma_{gq}^{N(0)}-3\gamma_{gq}^{N(0)}\gamma_{qq}^{N(0)}\right)\right]+\beta_0\gamma_{qg}^{N(2)}\gamma_{qq}^{N(0)}+\beta_1\beta_0C_{2,g}^{N(1)}\gamma_{qq}^{N(0)}-\beta_0C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)}\right)^2\Bigg\}$$

$$+\frac{1}{\left(\gamma_{qg}^{N(0)}\right)^2}\left\{-2\beta_0^3\left(C_{2,g}^{N(1)}\right)^2\gamma_{qq}^{N(0)}+2\beta_0^2\left[\left(C_{2,g}^{N(1)}\right)^2\left(\left(\gamma_{qq}^{N(0)}\right)^2-\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)}\right)-C_{2,g}^{N(1)}\gamma_{qq}^{N(0)}\gamma_{qg}^{N(0)}\right]\right.$$

$$+\beta_0\left[\left(C_{2,g}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)}\right)^2\gamma_{gg}^{N(0)}+C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}\right)^2\gamma_{qg}^{N(1)}-C_{2,g}^{N(1)}\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)}\gamma_{qg}^{N(1)}\right.$$

$$\left.\left.-\frac{1}{2}\left(\left(C_{2,g}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)}\right)^3+\gamma_{qq}^{N(0)}\left(\gamma_{qg}^{N(1)}\right)^2+\left(C_{2,g}^{N(1)}\right)^2\gamma_{qq}^{N(0)}\left(\gamma_{gg}^{N(0)}\right)^2\right)\right]\right\}$$

$$K_{dd}^{N(2)} = \gamma_{qq}^{N(2)}+\gamma_{gg}^{N(2)}-4\beta_0\left[\left(C_{2,q}^{N(1)}\right)^2-2C_{2,q}^{N(2)}\right]-4\beta_2+\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\left(\gamma_{qq}^{N(0)}+\gamma_{gg}^{N(0)}\right)$$

$$-\frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(1)}+\gamma_{gg}^{N(1)}-2\beta_1\right)+\frac{4\beta_0}{\gamma_{qg}^{N(0)}}\left\{4\beta_0\left(C_{2,g}^{N(2)}-C_{2,q}^{N(1)}C_{2,g}^{N(1)}\right)+\gamma_{qg}^{N(2)}\right.$$

$$\left.+\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)}-C_{2,g}^{N(2)}\right)-C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)}-\gamma_{gg}^{N(1)}-2\beta_1\right)-\left(C_{2,g}^{N(1)}\right)^2\gamma_{gq}^{N(0)}\right\}$$

$$+\frac{2\beta_0}{\left(\gamma_{qg}^{N(0)}\right)^2}\left\{-4\beta_0^2\left(C_{2,g}^{N(1)}\right)^2-4\beta_0C_{2,g}^{N(1)}\left[\gamma_{qg}^{N(1)}-C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\right]\right.$$

$$\left.-\left(\gamma_{qg}^{N(1)}\right)^2+2C_{2,g}^{N(1)}\gamma_{qg}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)-\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\right]^2\right\}$$

Instead of F_L we consider

$$\widetilde{F}_L(Q^2) = \frac{F_L(Q^2)}{a_s(Q^2) C_{L,g}^{N(1)}}$$

which is also factorizations scheme independent due to the fact that the first order Wilson coefficients $C_{L,q}^{N(1)}$ and $C_{L,g}^{N(1)}$ are scheme invariants.

Leading Order:

[S. Catani, Z. Phys. C75, (1997), 665]

$$K_{22}^{N(0)} = \gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \gamma_{qg}^{N(0)} \qquad K_{2L}^{N(0)} = \gamma_{qg}^{N(0)}$$

$$K_{L2}^{N(0)} = \gamma_{gq}^{N(0)} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \gamma_{qg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right)$$

$$K_{LL}^{N(0)} = \gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \gamma_{qg}^{N(0)}$$

Next-to-Leading Order:

[J. Blümlein, V. Ravindran and W. L. van Neerven, Nucl. Phys. B586, (2000),349]

$$K_{22}^{N(1)} = \gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)\right] + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}$$

$$-\left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right]\gamma_{qg}^{N(0)} + 2\beta_0\left(C_{2,q}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\right)$$

$$K_{2L}^{N(1)} = \gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} + 2\beta_0) + \left(C_{2,q}^{N(1)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)} - \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right)\gamma_{qg}^{N(0)}$$

$$K_{LL}^{N(1)} = \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)}\right) - C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{gg}^{N(0)}$$

$$-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{qq}^{N(0)} + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)}\right]\gamma_{qg}^{N(0)} + 2\beta_0\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}$$

$$
\begin{aligned}
K^{N(1)}_{L2} = {} & \gamma^{N(1)}_{gq} - \frac{\beta_1}{\beta_0}\gamma^{N(0)}_{gq} + \frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\left(\gamma^{N(1)}_{qq} - \frac{\beta_1}{\beta_0}\gamma^{N(0)}_{qq}\right) - \frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\left(\gamma^{N(1)}_{gg} - \frac{\beta_1}{\beta_0}\gamma^{N(0)}_{gg}\right) \\
& - \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^2\left(\gamma^{N(1)}_{qg} - \frac{\beta_1}{\beta_0}\gamma^{N(0)}_{qg}\right) + \left[\frac{C^{N(2)}_{L,q}}{C^{N(1)}_{L,g}} - \frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}C^{N(1)}_{2,q} + \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^2 C^{N(1)}_{2,g}\right]\gamma^{N(0)}_{qq} \\
& - \left[\left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^3 C^{N(1)}_{2,g} + 2\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\frac{C^{N(2)}_{L,q}}{C^{N(1)}_{L,g}} - \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^2\frac{C^{N(2)}_{L,g}}{C^{N(1)}_{L,g}} - \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^2 C^{N(1)}_{2,q}\right]\gamma^{N(0)}_{qg} \\
& + \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}C^{N(1)}_{2,g} - C^{N(1)}_{2,q} + \frac{C^{N(2)}_{L,g}}{C^{N(1)}_{L,g}}\right)\gamma^{N(0)}_{gq} \\
& - \left[\frac{C^{N(2)}_{L,q}}{C^{N(1)}_{L,g}} + \left(\frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\right)^2 C^{N(1)}_{2,g} - \frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}C^{N(1)}_{2,q}\right]\gamma^{N(0)}_{gg} + 2\beta_0\left(\frac{C^{N(2)}_{L,q}}{C^{N(1)}_{L,g}} - \frac{C^{N(1)}_{L,q}}{C^{N(1)}_{L,g}}\frac{C^{N(2)}_{L,g}}{C^{N(1)}_{L,g}}\right)
\end{aligned}
$$

$$
\begin{aligned}
K_{22}^{N(2)} = {} & \gamma_{qq}^{N(2)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(1)} + \gamma_{gq}^{N(0)}\left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right) + 2\beta_0\left[2C_{2,q}^{N(2)} - \left(C_{2,q}^{N(1)}\right)^2 - \frac{C_{L,q}^{N(2)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right] \\
& + \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right] - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\Bigg[\gamma_{qg}^{N(2)} - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right) \\
& + \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 4\beta_0\right)\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)} - C_{2,g}^{N(2)}\right) + \gamma_{qg}^{N(0)}\left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right)\Bigg] \\
& + \frac{\left(C_{L,q}^{N(1)}\right)^2}{\left(C_{L,g}^{N(1)}\right)^2}\left[\left(C_{2,q}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - C_{2,g}^{N(1)}\gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)}\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)} - C_{2,g}^{N(2)}\right)\right] \\
& + \frac{C_{L,g}^{N(2)}}{\left(C_{L,g}^{N(1)}\right)^2}\left[C_{L,q}^{N(1)}\gamma_{qg}^{N(0)} + C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}C_{L,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right)\right] - 2\frac{C_{2,g}^{N(1)}C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\gamma_{qg}^{N(0)} \\
& - \frac{\beta_1}{\beta_0}\Bigg\{\gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)}\left(C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\right) + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right] \\
& + \frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\gamma_{qg}^{N(0)}\left(C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}\right)\Bigg\} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)\left(\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)
\end{aligned}
$$

$$
\begin{aligned}
K_{2L}^{N(2)} = \; & \gamma_{qg}^{N(2)} + C_{2,q}^{N(1)}\gamma_{qg}^{N(1)} + C_{2,q}^{N(2)}\gamma_{qg}^{N(0)} - 2\beta_0 C_{2,q}^{N(1)} C_{2,g}^{N(1)} - C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
& - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(2)}\gamma_{gq}^{N(0)}\right) + \gamma_{qg}^{N(0)}\left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right) \\
& + \frac{\beta_1}{\beta_0}\left[\frac{\gamma_{qg}^{N(0)}}{C_{L,g}^{N(1)}}\left(C_{L,g}^{N(2)} - C_{2,g}^{N(1)}C_{L,q}^{N(1)}\right) + C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - C_{2,q}^{N(1)}\gamma_{qg}^{N(0)} - \gamma_{qg}^{N(1)}\right] \\
& + \frac{\gamma_{qg}^{N(0)}}{\left(C_{L,g}^{N(1)}\right)^2}\left(C_{L,g}^{N(2)} - C_{2,g}^{N(1)}C_{L,q}^{N(1)}\right)^2 + \frac{1}{C_{L,g}^{N(1)}}\Big[\left(C_{2,g}^{N(1)}C_{L,q}^{N(2)} - C_{2,q}^{N(1)}C_{L,g}^{N(2)} + C_{2,g}^{N(2)}C_{L,q}^{N(1)}\right) \\
& - C_{2,g}^{N(1)}\left(C_{2,g}^{N(1)}C_{L,q}^{N(1)} - C_{L,g}^{N(2)}\right)\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \frac{\gamma_{qg}^{N(1)}}{C_{2,g}^{N(1)}}\Big] \\
K_{L2}^{N(2)} = \; & \gamma_{gq}^{N(2)} - C_{2,q}^{N(2)}\gamma_{gq}^{N(0)} + C_{2,q}^{N(1)}\left(C_{2,q}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{gq}^{N(1)}\right) + \frac{\beta_1}{\beta_0}\Bigg\{\gamma_{gq}^{N(1)} + \left(\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} - C_{2,q}^{N(1)}\right)\gamma_{gq}^{N(0)} \\
& + \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} - C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right] \\
& - \frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\Big[2C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} + C_{L,q}^{N(1)}\gamma_{qg}^{N(1)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - C_{L,q}^{N(1)}C_{2,q}^{N(1)}\gamma_{qg}^{N(0)} \\
& - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{L,g}^{N(2)}\gamma_{qg}^{N(0)}\Big]\Big]\Bigg\} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)\left[\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left\{ \gamma_{qq}^{N(2)} - \gamma_{gg}^{N(2)} - C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} - \gamma_{gq}^{N(1)} \right) + \left(C_{2,g}^{N(2)} - 2C_{2,g}^{N(1)} C_{2,q}^{N(1)} \right) \gamma_{gq}^{N(0)} \right. \\
&\left. + \left[\left(C_{2,q}^{N(1)} \right)^2 - C_{2,q}^{N(2)} \right] \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right) \right\} + \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} \left(\gamma_{gq}^{N(1)} - C_{2,q}^{N(1)} \gamma_{gq}^{N(0)} \right) \\
&+\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} \left[\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} - C_{2,q}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} + 2\beta_0 \right) \right] - 2 \frac{\left(C_{L,q}^{N(2)} \right)^2}{\left(C_{L,g}^{N(1)} \right)^2} \gamma_{qg}^{N(0)} \\
&+2\beta_0 \frac{C_{L,g}^{N(2)}}{\left(C_{L,g}^{N(1)} \right)^2} \left(C_{L,q}^{N(1)} C_{2,q}^{N(1)} - C_{L,q}^{N(2)} \right) + 2 \frac{C_{L,q}^{N(2)} C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)} \right)^2} \left[C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{qq}^{N(0)} + \beta_0 \right) \right. \\
&\left. +C_{2,q}^{N(1)} \gamma_{qg}^{N(0)} - \gamma_{qg}^{N(1)} \right] + \frac{\left(C_{L,q}^{N(1)} \right)^2}{\left(C_{L,g}^{N(1)} \right)^2} \left\{ - \gamma_{qg}^{N(2)} + C_{2,q}^{N(2)} \gamma_{qg}^{N(0)} + C_{2,g}^{N(2)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right) \right. \\
&+C_{2,q}^{N(1)} \left(\gamma_{qg}^{N(1)} - C_{2,q}^{N(1)} \gamma_{qg}^{N(0)} \right) + C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} \right) \\
&\left. -C_{2,q}^{N(1)} C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right) - \left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)} C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}} \right)^2 \gamma_{qg}^{N(0)} \right\} \\
&+\frac{1}{\left(C_{L,g}^{N(1)} \right)^3} \left\{ \left(C_{L,q}^{N(1)} \right)^3 \left[\left(C_{2,q}^{N(1)} C_{2,g}^{N(1)} - C_{2,g}^{N(2)} + \frac{\beta_1}{\beta_0} C_{2,g}^{N(1)} \right) \gamma_{qg}^{N(0)} - C_{2,g}^{N(1)} \gamma_{qg}^{N(1)} \right. \right.
\end{aligned}
$$

$$\left(C_{2,g}^{N(1)}\right)^2 \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)\Bigg] + \left(C_{L,q}^{N(1)}\right)^2 \Big[C_{L,g}^{N(2)}\gamma_{qg}^{N(1)} - \left(C_{2,q}^{N(1)}C_{L,g}^{N(2)} + 3C_{L,q}^{N(2)}C_{2,g}^{N(1)}\right)\gamma_{qg}^{N(0)}$$

$$-C_{L,g}^{N(2)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} + 2\beta_0\right)\Big] + C_{L,q}^{N(1)}C_{L,g}^{N(2)}\left(\beta_0 C_{L,g}^{N(2)} + C_{L,q}^{N(2)}\gamma_{qg}^{N(0)}\right)\Bigg\}$$

$$K_{LL}^{N(2)} = \gamma_{gg}^{N(2)} - C_{2,g}^{N(1)}\gamma_{gq}^{N(1)} + \left(C_{2,g}^{N(1)}C_{2,q}^{N(1)} - C_{2,g}^{N(2)}\right)\gamma_{gq}^{N(0)} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)\left(\gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)$$

$$+\frac{\beta_1}{\beta_0}\Bigg[C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{gg}^{N(1)} - \frac{C_{2,q}^{N(2)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(1)} + \frac{C_{2,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)$$

$$-\frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\left(C_{L,q}^{N(1)}C_{2,g}^{N(1)} - C_{2,g}^{N(2)} + C_{L,g}^{N(2)}\right)\gamma_{qg}^{N(0)}\Bigg] + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2\gamma_{qg}^{N(0)}$$

$$+\frac{1}{C_{L,g}^{N(1)}}\Big[C_{L,q}^{N(1)}\gamma_{qg}^{N(2)} + C_{L,q}^{N(2)}\gamma_{qg}^{N(1)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} + C_{2,q}^{N(1)}\gamma_{qq}^{N(0)}\right)$$

$$-C_{L,q}^{N(2)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - C_{L,q}^{N(1)}C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)\Big]$$

$$\frac{1}{\left(C_{L,g}^{N(1)}\right)^2}\Bigg\{\left(C_{L,q}^{N(1)}\right)^2\Bigg[\left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right)\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}\left(C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \gamma_{qg}^{N(1)}\right)\Bigg]$$

$$+2\beta_0 C_{L,g}^{N(2)}\left(C_{2,g}^{N(1)}C_{L,q}^{N(1)} - C_{L,g}^{N(2)}\right) + C_{L,q}^{N(1)}\Big[2C_{2,g}^{N(1)}C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} + C_{2,g}^{N(1)}C_{L,g}^{N(2)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)$$

$$-C_{L,g}^{N(2)}\left(C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} + C_{L,q}^{N(1)}\gamma_{qg}^{N(1)}\right)\Big]\Bigg\}$$

Solution of the Evolution Equation: U-matrix formalism

[J. Blümlein and A. Vogt, Phys. Rev. D58, (1998), 014020]

- We write the solution as a perturbation around the LO solution

$$\mathbf{F}_{LO}(N,a_s) = \left(\frac{a_s}{a_0}\right)^{-\mathbf{K}_0/2\beta_0} \mathbf{F}(N,a_0) \equiv \mathbf{L}(N,a_s,a_0)\mathbf{F}(N,a_0)$$

with

$$\mathbf{F}(N,a_s) \equiv \begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t)$$

- The expansion reads

$$\begin{aligned}
\mathbf{F}(N,a_s) &= \mathbf{U}(N,a_s)\mathbf{L}(N,a_s,a_0)\mathbf{U}^{-1}(\mathbf{N},\mathbf{a_s})\mathbf{F}(N,a_0) \\
&= \left[1+\sum_{k=1}^{\infty} a_s^k \mathbf{U}_k(N)\right]\mathbf{L}(N,a_s,a_0)\left[1+\sum_{k=1}^{\infty} a_0^k \mathbf{U}_k(N)\right]^{-1}\mathbf{F}(N,a_0)
\end{aligned}$$

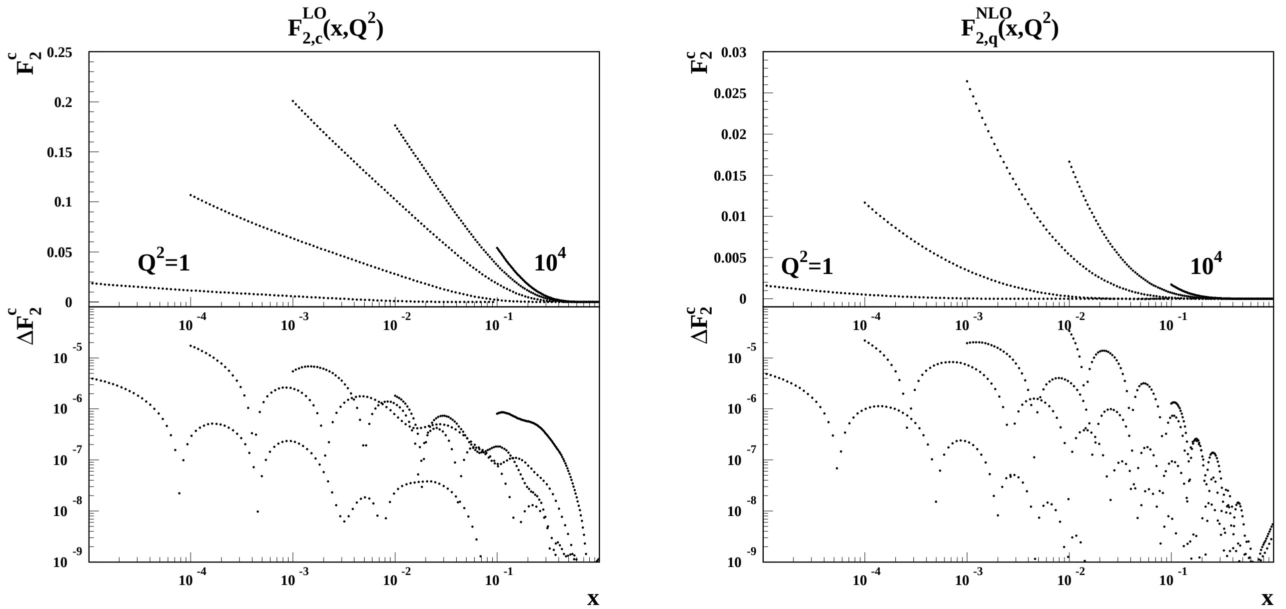
- Up to 3-loops the solution is

$$\begin{aligned}
\mathbf{F}(N,a_s) = [&\mathbf{L} + a_s\mathbf{U}_1\mathbf{L} - a_0\mathbf{L}\mathbf{U}_1 \\
&+a_s^2\mathbf{U}_2\mathbf{L} - a_s a_0\mathbf{U}_1\mathbf{L}\mathbf{U}_1 + a_0^2\mathbf{L}\left(\mathbf{U}_1^2 - \mathbf{U}_2\right)\Big]\,\mathbf{F}(N,a_0)
\end{aligned}$$

- Heavy flavour contribution to DIS structure function in the kinematic regime of HERA is known to be sizable

- Any analysis of DIS structure functions aiming to $\sim 1\%$ accuracy needs to take into account heavy quark effects on F_2 and $\partial F_2/\partial t$

- Implementation of Heavy Flavour contributions in evolution codes in Mellin space is possible thanks to the parametrization of heavy flavour Wilson coefficients derived by Alekhin and Blümlein [S. I. Alekhin and J. Blümlein, Phys. Lett. B594, (2004), 299]

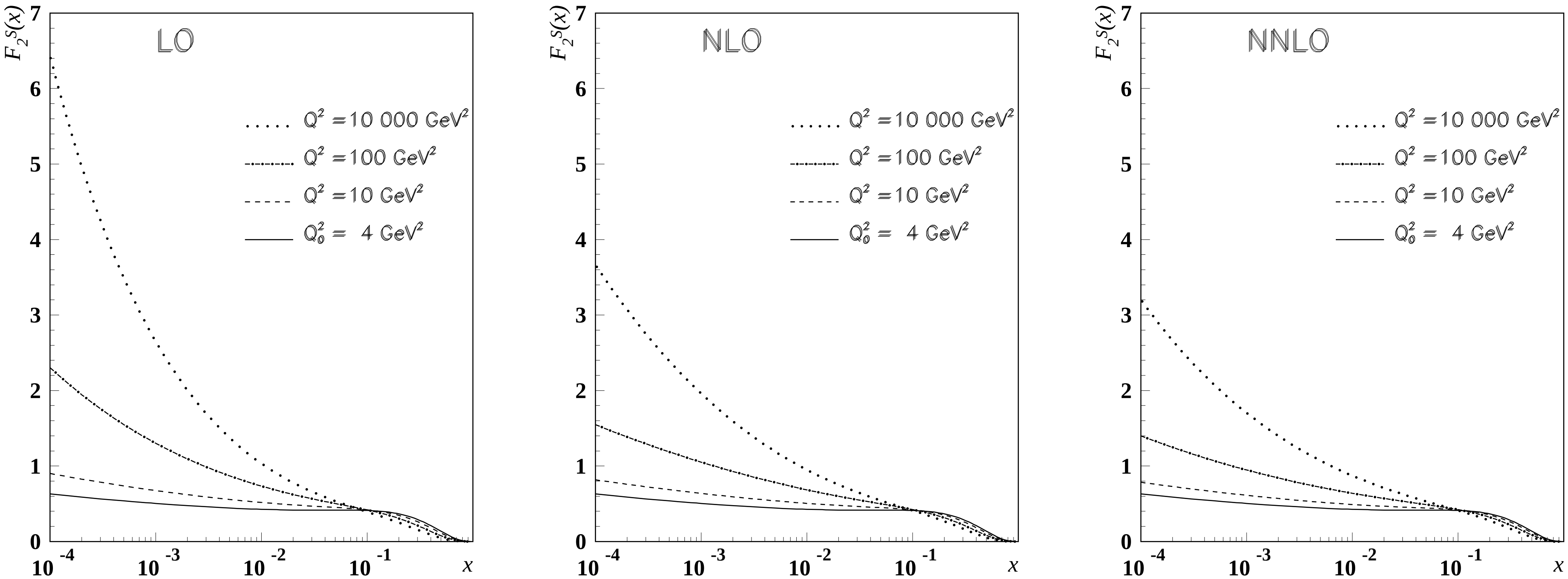


Numerical implementation - Status

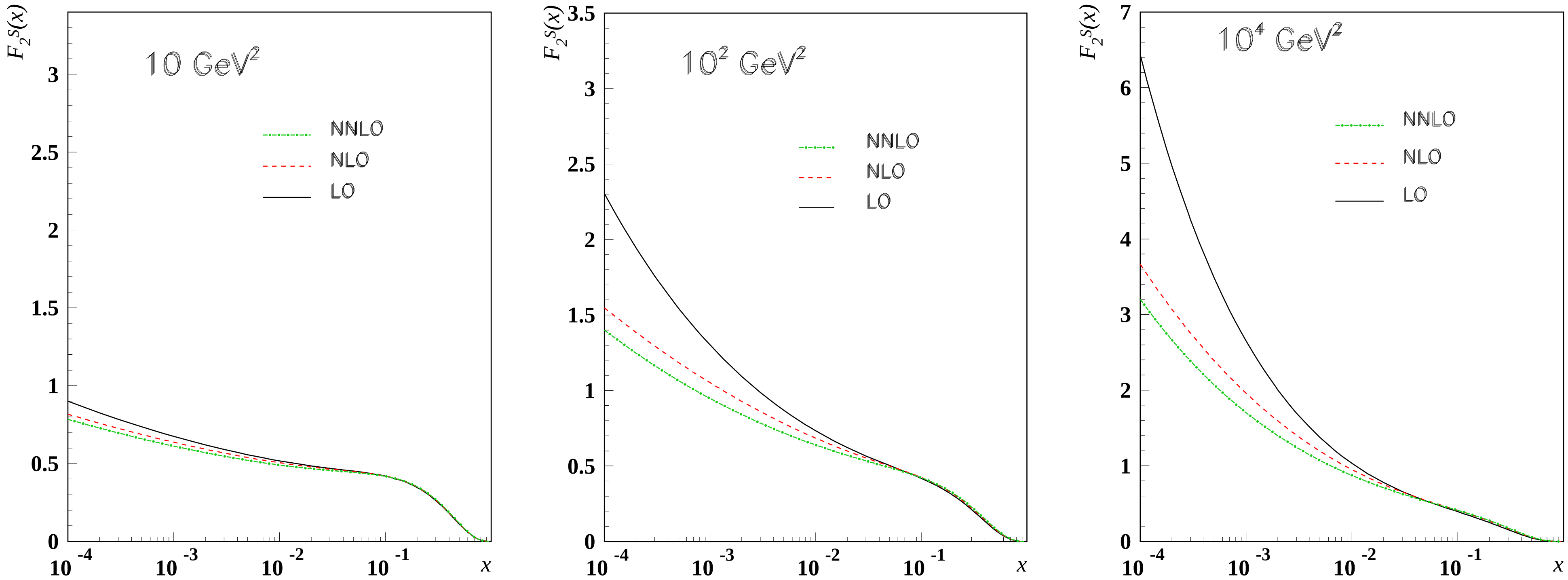
- NNLO evolution with massless flavours is implemented
- Renormalization scale variations are used to estimate the theoretical error
- Inclusion of heavy flavour contributions is on the way
- Incorporation in scheme-invariant analysis requires the computation of the first and second derivative of the heavy flavour Wilson coefficients w.r.t. the evolution variable t
- Structure Functions at the initial scale are not yet extracted from data but parametrized in terms of PDFs and Wilson Coefficients (at the moment we use toy PDFs)

Numerical results - F_2, $\partial F_2/\partial t$

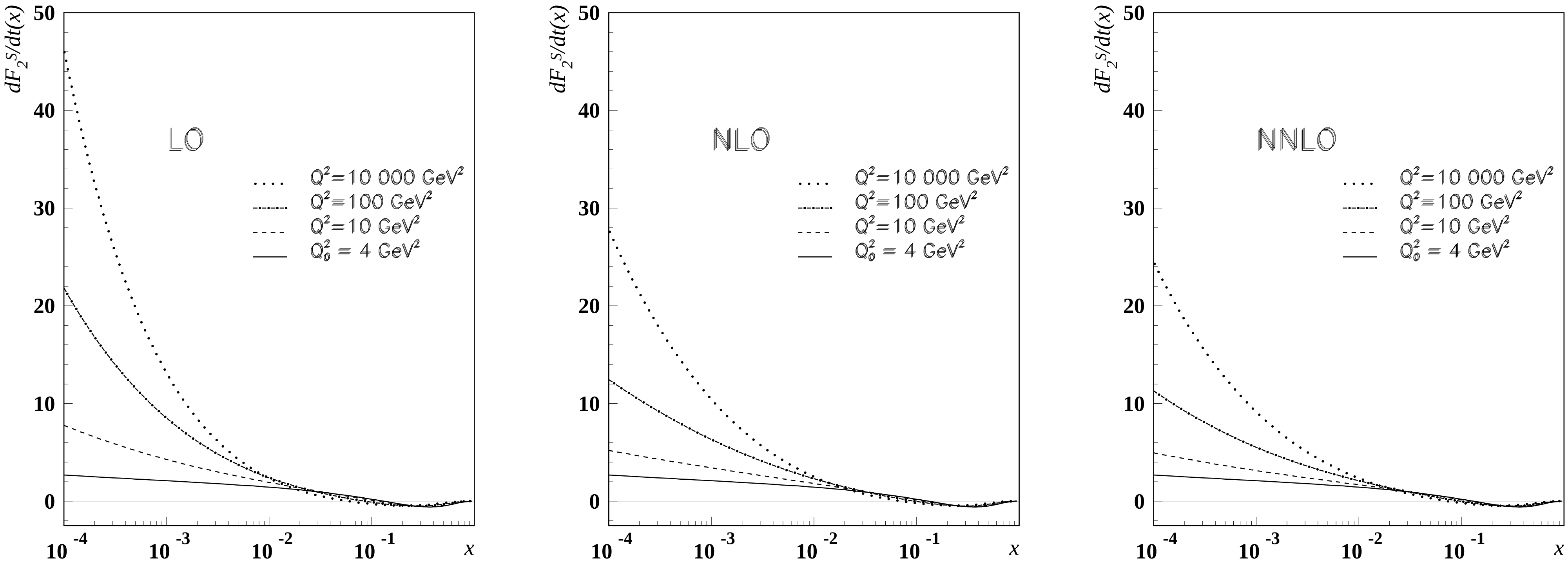
- $F_2^{e.m.}$ as obtained from LO, NLO and NNLO scheme-invariant evolution with 4 massless flavours



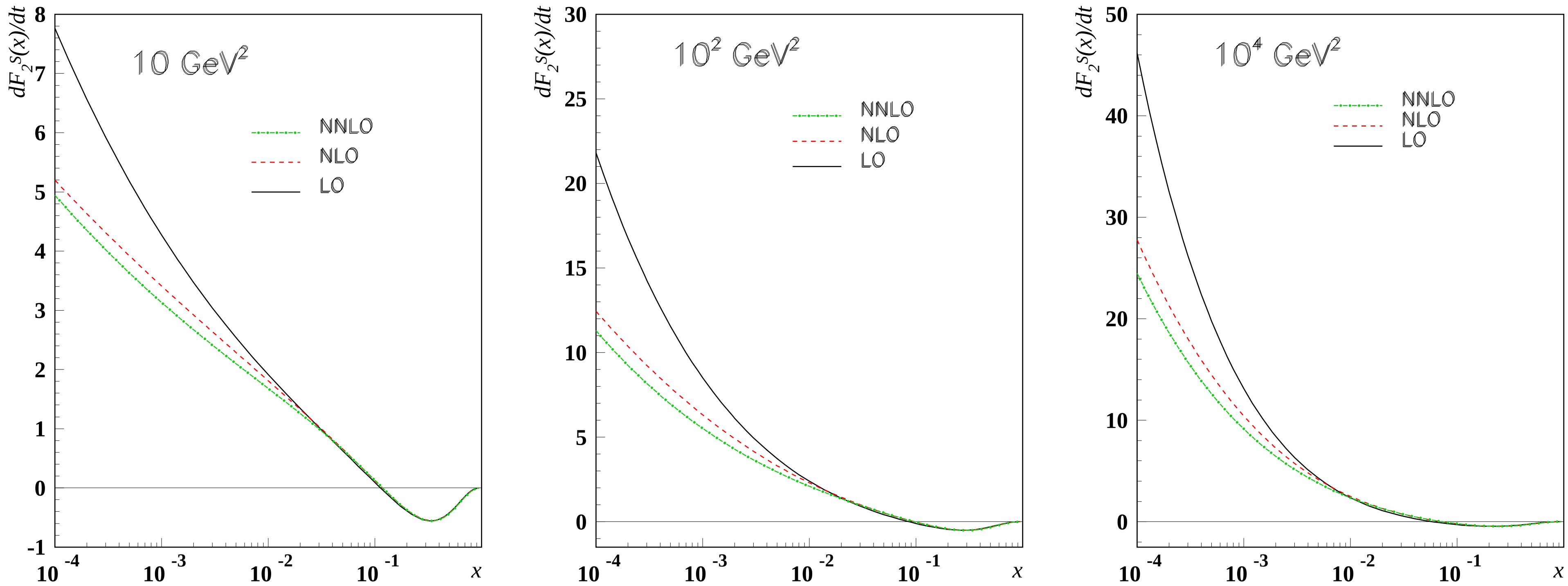
- $F_2^{e.m.}$ in LO, NLO and NNLO at 10, 100 and 10000 GeV^2



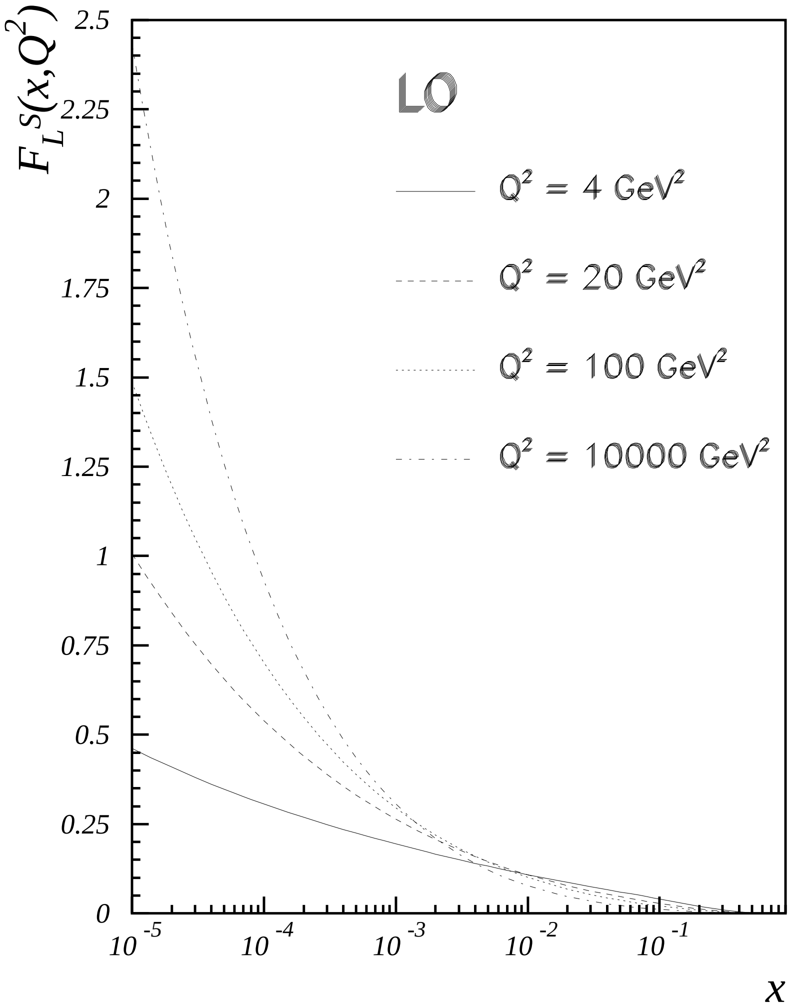
- The slope of $F_2^{e.m.}$ as obtained from LO, NLO and NNLO scheme-invariant evolution with 4 massless flavours

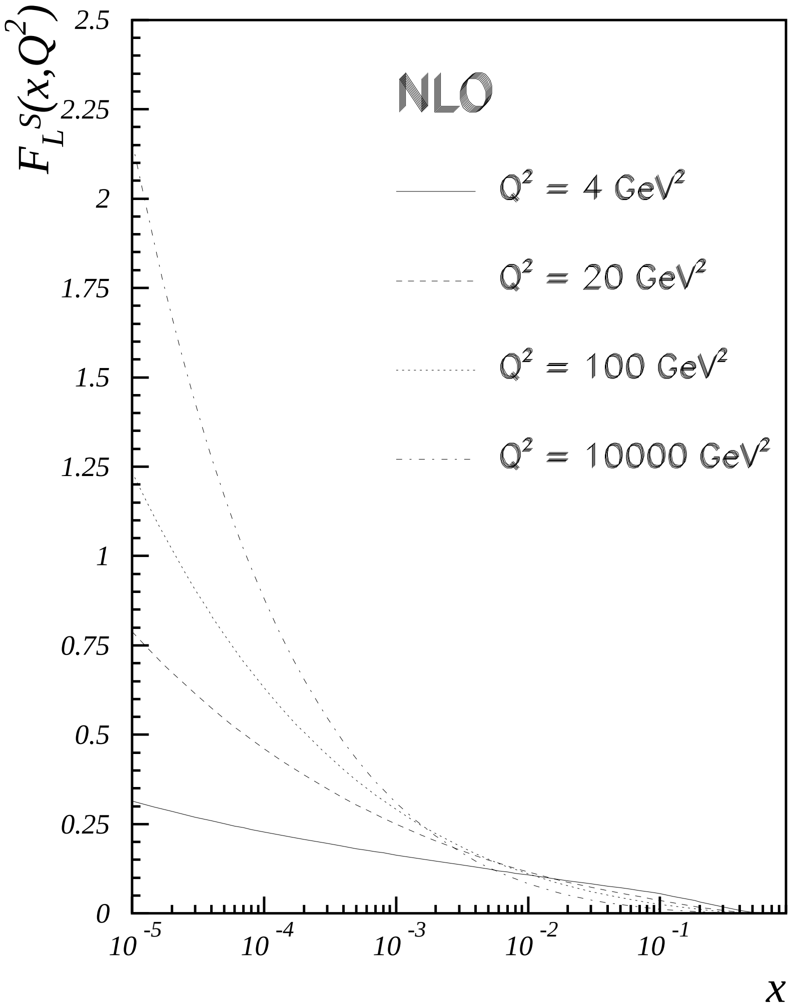


- The slope of $F_2^{e.m.}$ in LO, NLO and NNLO at 10, 100 and 10000 GeV^2



- The longitudinal structure function F_L in LO and NLO as obtained from scheme invariant evolution of F_2, F_L





- Upcoming measurements of DIS structure functions will allow to reduce experimental errors on α_s to the level of 1% $\Longrightarrow$ NNLO analysis is necessary
- Combining standard QCD analysis and fits based on scheme-invariant evolution could provide a method to reduce theoretical and conceptual errors in the determination of α_s
- We aim to perform a combined analysis up to NNLO of DIS structure functions to extract α_s and a set of parton distribution functions with fully correlated errors

Status and Outlook

- Implementation of NNLO scheme-invariant evolution for massless quarks (completed)
- Inclusion of heavy flavours (being performed now)
- 3-loop scheme-invariant fit to data and comparison with $\overline{MS}$ analysis (only F_2)
- One parameter fit to Λ_{QCD} using experimentally measured input for F_2 and $\partial F_2/\partial t$