Gluon Distributions and Fits Using Dipole Cross-Sections

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Abstract. I carry out a comparison between the gluon distribution obtained from a dipole picture analysis with structure function data and the standard DGLAP gluon. The former is smaller and steeper, and I explain the approximations that have resulted in this difference.

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There has recently been a lot of work on calculating/modelling dipole cross-sections and using the dipole model [1, 2, 3, 4] to fit to structure function data, and a variety of approaches match data very well. However, the picture of steeply growing quantities at small x tamed by saturation conflicts with the DGLAP picture of a small/negative gluon distribution at small x and Q^2 [5, 6]. We need to understand why this happens. My approach is based on the assumption that QCD factorization theory is correct and quantitative at high Q^2 (and not too small x), and then I work back to the dipole cross-sections. In this way I examine whether the dipole motivated fits are truly quantitative and whether a large/steep dipole cross-section means a large/steep gluon distribution. Any evidence for saturation is only a side issue. More details may be found in [7].

Within LO k_T-factorization theory the $\gamma^\star p$ cross-section can be written as [8]

$$\sigma(x,Q^2) \propto \int dz \int \frac{d^2k}{k^4} \int d^2p\, \tilde{\sigma}(z,p^2,Q^2)\, f(x,k^2)$$

where $f(x,k^2)$ is the unintegrated gluon distribution. In the limit $x \to 0$, i.e. LO in the k_T-factorization theory [9], this formula can be simplified. Integrating over z and p

$$F(x,Q^2) = \int \frac{d^2k}{k^2} \frac{\alpha_S 2N_f}{6\pi} h(k^2/Q^2)\, f(x,k^2).$$

Taking the double Mellin transformation $\int dQ^2 Q^{2-2\gamma}$ and $\int dx\, x^N$ we obtain,

$$\tilde{F}(N,\gamma) = \frac{\alpha_S 2N_f}{6\pi} \tilde{h}(\gamma)\, \tilde{f}(N,\gamma)/\gamma \equiv \alpha_S \tilde{h}(\gamma) \tilde{g}(N,\gamma).$$

$g(x,Q^2) = \int_0^{Q^2} \frac{d^2k}{k^2} f(x,k^2)$ is the integrated gluon distribution. If $g(N,Q^2) \sim (Q^2)^{\gamma(\alpha_S,N)}$

$$F(N,Q^2) = \frac{\alpha_S 2N_f}{6\pi} h(\gamma(\alpha_S,N))g(N,Q^2) \to F(x,Q^2) = \frac{\alpha_S 2N_f}{6\pi} h(\gamma(\alpha_S,\ln(1/x))) \otimes g(x,Q^2),$$

where $\gamma(\alpha_S/N)$ is a positive quantity, e.g. in LO BFKL [10] $\gamma(x) = x + 2.4x^4 + 2x^6 + 17x^7 + \cdots$.

Alternatively, taking a Fourier transformation with respect to p, with r the conjugate variable, integrating over p^2 and z, and letting $x \to 0$ one can equivalently write

$$\sigma = \frac{2\pi}{3}\int_0^1 dz \int d^2r |\Psi(r,z,Q)|^2 \int \frac{d^2k}{k^4}\alpha_S f(x,k^2)(1-J_0(kr)) \equiv \int_0^1 dz \int d^2r |\Psi(r,z,Q)|^2 \hat{\sigma}(x,r^2).$$

Taking the Mellin transformations leads to

$$F_i(N,Q^2) = \frac{\alpha_S 2N_f}{6\pi} h_{id}(\gamma(\alpha_S,N))h_{dg}(\gamma(\alpha_S,N)g(N,Q^2) \equiv \frac{\alpha_S 2N_f}{6\pi} h_i(\gamma(\alpha_S,N)) \otimes g(N,Q^2).$$

So the effective coefficient function for the hard cross-section $h_i(\gamma(\alpha_S,N))$ is the product of a photon-dipole part $h_{id}(\gamma(\alpha_S,N))$ and a dipole-gluon part $h_{dg}(\gamma(\alpha_S,N))$, both of which are calculable. For $dF_2/d\ln Q^2$ [11], expanding in powers of γ

$$h_2(\gamma(\bar{\alpha}_S/N) = 1 + 2.17\gamma + 2.30\gamma^2 + 5.07\gamma^3 + 3.58\gamma^4 + 8.00\gamma^5 + \cdots.$$

$$h_{dg}(\gamma(\bar{\alpha}_S/N) = 1 + 2.23\gamma + 3.49\gamma^2 + 3.95\gamma^3 + 4.22\gamma^4 + 4.06\gamma^5 + \cdots$$

$$h_{2d}(\gamma(\bar{\alpha}_S/N) = 1 - 0.07\gamma - 1.05\gamma^2 + 3.77\gamma^3 - 4.94\gamma^4 + 6.53\gamma^5 + \cdots.$$

This leads to a steep growth of $dF_2/d\ln Q^2$ relative to the gluon, but this is all generated by the dipole-gluon cross-section, rather than the photon-dipole coefficient. Indeed, taking the simple Golec-Biernat Wüsthoff model [12] within this picture we find that a flat $\hat{\sigma}(x,r^2)$ comes from a valence-like $xg(x,Q^2)$ and $f_g(x,k^2)$.

In order to see how this works out in practice I perform a fit to data from the starting point of a well-defined gluon distribution that has the same shape in x and Q^2 as a standard LO or NLO gluon distribution and for $Q^2 \gg Q_0^2$ evolves in a quantitatively correct way. Clearly for $Q^2 \sim Q_0^2$ the evolution needs to be modified. In essence I just replace Q^2 by $Q^2 + Q_0^2$. $\alpha_S(\mu^2)$ is also modified in the same way. This expression for the gluon is converted into a dipole cross-section using

$$\hat{\sigma}(x,r^2) = \frac{2\pi}{3}\int \frac{d^2k}{k^4}\alpha_S(k^2) f(x,k^2)(1-J_0(kr)),$$

and put into a fit to data. The comparison of the gluon and the resulting dipole cross-section is shown in fig. 1. Clearly the dipole cross-section is generally steeper, particularly comparing $Q^2 = 0.2\mathrm{GeV}^2$ with $r = 10\mathrm{GeV}^{-1}$.

There are various details of the fit to consider. In reality 3 types of diagram contribute. A gluon can radiate gluons before entering into the scattering, or a quark can fluctuate into a gluon which enters, or a quark can scatter directly off the photon. In the LO k_T-factorization theorem the first two processes combine as $f_g(x,k^2) + 4/9 f_S(x,k^2)$, i.e. the quark also contributes to the dipole cross-section. For the last process I include a term $f \times Q^2/(Q^2+Q_0^2)$, where f is free (and in practice very small). Another very important issue is heavy quarks. These are often ignored in dipole fits, but charm constitutes about 40% of $dF_2/d\ln Q^2$ for $Q^2 > m_c^2$. Its omission leads to $\hat{\sigma}(x,r^2)$ and $g(x,Q^2)$ being up to 5/3 times too big. Since saturation corrections are $\propto g^2(x,Q^2)$ they can be enormously

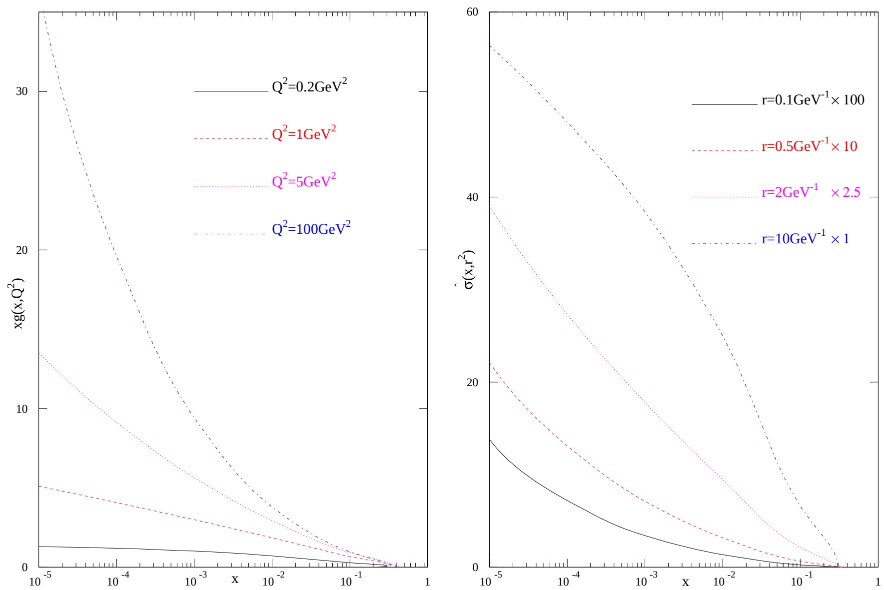

FIGURE 1. Comparison of $xg(x,Q^2)$ at various Q^2 with $\hat{\sigma}(x,r^2)$ at various r

exaggerated. To check the consequences of ignoring charm I performed a DGLAP fit without the charm contribution to $F(x,Q^2)$. The gluon and α_S are bigger, but also the global fit is terrible: $\chi^2 = 2$ per point for 2000 points. One cannot get $dF_2/d\ln Q^2$ consistently correct at all. Hence, NLO and NNLO DGLAP are good enough and constraining enough to determine that charm has to be there. This suggests that one should be suspicious of good qualitative results present without heavy quarks. They should really be wrong until corrected. In my fit I use $m_c = 1.3$GeV.

I fit H1 [13], ZEUS [14] and E665 [15] data from $0.5\text{GeV}^2 \leq Q^2 \leq 50\text{GeV}^2$. The fit does not work for $Q^2 < 0.5\text{GeV}^2$, perhaps suggesting saturation. The quality of fit, $\chi^2 = 1.1$ per point, is very good for 3 different data sets. In fig. 2 the resulting dipole fit gluon is compared to the MRSTNLO gluon. It is approximately $0.65-0.75$ of the size. It should really be compared to $g(x,Q^2)+4/9\,f_S(x,Q^2)$. The biggest change is at high x, and the factor is now $0.5-0.65$. The dipole gluon does not match onto the standard DGLAP gluon at high Q^2. At low Q^2 it is much smaller than the DGLAP gluon at moderate x but eventually becomes bigger at very small x.

This relative behaviour comes from the effective coefficient functions/splitting functions. Consider $dF_2/d\ln Q^2$ controlled by $\gamma^{DIS}(\alpha_S(Q^2),N)$. For my model $\gamma_{gg}(\alpha_s(Q^2),N) = \bar{\alpha}_s(Q^2)(1/N-1)$ (i.e. correct for DGLAP). This means that

$$\gamma^{DIS}_{dip}(\alpha_S(Q^2),N) \approx \frac{\alpha_S(Q^2)2N_f}{6\pi}\left(1+2.17\bar{\alpha}_S(Q^2)\left(\frac{1}{N}-1\right)+2.30\bar{\alpha}_S^2(Q^2)\left(\frac{1}{N}-1\right)^2+\cdots\right).$$

At highish Q^2 and moderate x the first two terms are dominant. The exact result is

$$\gamma^{DIS}_{exact}(\alpha_S(Q^2),N) = \frac{\alpha_S(Q^2)2N_f}{6\pi}\left(1-1.08N+\cdots+2.17\bar{\alpha}_S(Q^2)\left(\frac{1}{N}-3.05+\cdots\right)+\cdots\right).$$

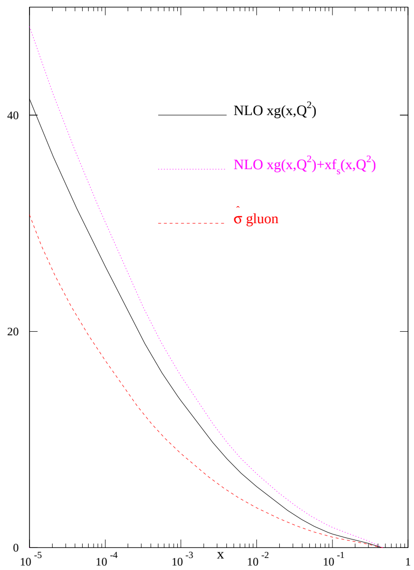

FIGURE 2. Comparison of dipole inspired gluon and standard gluon at $Q^2 = 50\text{GeV}^2$

The first terms are much bigger in the dipole approach, leading to a smaller gluon at all x, and the difference in dipole and DGLAP gluons is mainly due to these effective splitting functions. This is verified by modifying the DGLAP splitting functions in a normal global fit, resulting in a dipole-like gluon [7]. Hence, the good fit at $Q^2 \sim 20-50\text{GeV}^2$ using the dipole model is achieved with a wrong gluon. But the terms in the splitting function are by no means negligible at $Q^2 \sim 1\text{GeV}^2$ – the error in the $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ evolution is more important as gluons get flatter. The DGLAP gluons are not accurate here, but the dipole fits are also incorrect. More sophisticated (beyond LO k_t-factorization theory) calculations are needed for semi-quantitative results. But the simple dipole picture also needs extension beyond LO k_t-factorization theory.

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